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Calculating an influence line from direct measurements

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The response of a bridge to a pre-weighed truck can be measured on site. This paper describes a mathematical method for converting the measured response of a load effect into an influence line for that effect. One influence ordinate is calculated for each scan of the data acquisition system. The vector of ordinates is found by solving a large set of simultaneous equations expressed in matrix form. The general form of the matrices is described, and the particular matrices for a three-axle truck are given. The technique is demonstrated using measured strain on two bridges using pre-weighed trucks with different numbers of axles.

**NOTATION**

- $A$: vehicle axle weights
- $L_k^M$: measured load effect in $k$th scan
- $L_k^T$: theoretical load effect in $k$th scan
- $C_i$: number of scans
- $D_i$: distance in metres between axle $i$ and the 1st axle
- $w_k$: measure strain in $k$th scan
- $E$: error function
- $f$: scanning frequency of data acquisition unit
- $I_k$: $R$th influence line ordinate
- $N$: total number of scans
- $V$: number of axles in vehicle
- $v$: vehicle velocity
- $W$: matrix dependent on vehicle axle weights
- $w_{ij}$: component of matrix $W$

**1. INTRODUCTION**

The influence line (IL)—the load effect at a point due to unit load at different positions across a bridge—is a useful measure of the characteristic response of the bridge to load. Although ILs can readily be found by analysis, the results often do not correspond to measurements on site, and it is sometimes useful to derive the IL from direct measurements of the load effect in response to a vehicle of known weight. As vehicles invariably have more than one wheel, this paper describes a method to determine the IL from the measured multi-wheel response. In existing bridges such so-called ‘measured’ ILs can be used to determine and understand the real behaviour, to monitor changes over time, to accurately predict responses to various load patterns and to calibrate a bridge weigh-in-motion system.

\[ L_k^I = \sum_{i=1}^{N} A_i [I_k^M - C_i] \]

where $N$ is the number of axles, $I_k^I$ is an influence ordinate, and $C_i$ is the number of scans corresponding to $D_i$, the distance between axle $i$ and the first axle, given by $C_i = \frac{D_i f}{v}$.

\[ E = \frac{K}{\sum_{k=1}^{K} (L_k^M - L_k^T)^2} \]

where $K$ is the total number of scans.

The starting-point for the influence line is usually taken as a fixed reference—for example, the instant the first axle reaches a specified point prior to the bridge. There is no need to know the exact position at which the applied load causes the bridge to start bending. Therefore the uncertainty surrounding the real boundary conditions and the very small strains generally induced near the supports is avoided.
The set of influence ordinates, \( I \), that minimise \( E \) are those for which the partial derivatives are zero. For the important case of a three-axle truck, the partial derivative of \( E \) with respect to the \( k \)th influence ordinate, \( I_k \), can be written as:

\[
\frac{\partial E}{\partial I_k} = 2\left[ I_{k+k_1} - (A_{ik} + A_{ik+c_1} + A_{ik+c_2}) \right] \left[ (A_k) \right] + 2\left[ I_{k+k_1} - (A_{ik} + A_{ik+c_2} + A_{ik+c_3}) \right] \left[ (A_k) \right] + 2\left[ I_{k+k_1} - (A_{ik} + A_{ik+c_1} + A_{ik+c_2} + A_{ik+c_3}) \right] \left[ (A_k) \right]
\]

where \( \mathbf{C} < \mathbf{K} \), that is, the general case when all axles are on the bridge structure. Setting all partial derivatives to zero gives a set of \( \mathbf{K} \) simultaneous equations:

\[
\mathbf{W}_{\mathbf{K}, \mathbf{K}} = \mathbf{I} = \mathbf{e}_{\mathbf{K}, \mathbf{K}}
\]

where \( \mathbf{W} \) is a sparse symmetric matrix dependent on the vehicle axle weights, \( \mathbf{I} \) is a vector containing the sought influence ordinates, and \( \mathbf{e} \) is a vector dependent on the vehicle axle weights and the measured load effect readings.

The main diagonal of \( \mathbf{W} \) consists of the sum of the squares of the axle weights. The number of off-diagonals either side of this main diagonal is then equal to the number of unique axle pairs—that is, equal to \( \sum_{j=1}^{N-1} 2 \). For example, for a two-axle truck there is one unique pair, \( 1-2 \), and for a three-axle truck there are three: \( 1-2, 1-3 \), and \( 2-3 \). The products of such pairs (e.g., \( A_1A_2, A_1A_3, A_2A_3 \)) appear on these off-diagonals, at distances from the main diagonal proportional to the distance between their axles (e.g., \( C_2 - C_1, C_3 - C_1, C_3 - C_2 \)). However, as \( C_1 = 0 \), \( \mathbf{W} \) appears, for the three-axle case, as in equation (6), where the main diagonal elements

\[
w_{ij} = \sum_{k=1}^{N} A_k^2
\]

and the upper triangular elements are given by

\[
w_{ij+k} = A_2A_3; \quad i + [C_3 - C_2] \leq k - C_2
\]

\[
w_{ij} = A_1A_2; \quad i + C_2 \leq k - C_2
\]

\[
w_{ij} = A_1A_3; \quad i + C_3 \leq k - C_3
\]

Symmetry in the matrix gives the corresponding lower triangular elements. The \( \mathbf{e} \) vector is dependent on the vehicle axle weights and the measured load effect readings

\[
\mathbf{e}_{\mathbf{K}, \mathbf{K}} = \begin{pmatrix} A_1I_{1+1} & + & A_1I_{1+2} & + & \cdots & + & A_1I_{1+M-1} \\ \\
A_1I_{2+1} & + & A_1I_{2+2} & + & \cdots & + & A_1I_{2+M-1} \\ \\
\vdots & & \vdots & & \ddots & & \vdots \\ \\
A_1I_{M-1+1} & + & A_1I_{M-1+2} & + & \cdots & + & A_1I_{M-1+M-1} \\ \\
A_1I_{M+1} & + & A_1I_{M+2} & + & \cdots & + & A_1I_{M+M-1} \\
\end{pmatrix}
\]

The elements of \( \mathbf{W} \) for a three-axle truck have been given in equations (6)–(8). Various combinations of axles have been simulated using the MAPLE software program. The pattern is similar to that described in the three-axle sample case, and these equations can be easily extended to cater for a calibration vehicle with any number of axles.

The vector, \( \mathbf{I} \), of influence ordinates can be solved by inverting the square matrix \( \mathbf{W} \), or by Cholesky factorisation (the \( \mathbf{W} \) matrix is symmetric positive definite).

In Liljencrantz et al., a revised form of this matrix method has been used to determine the influence line for railway bridges.

### 3. Field Trials

The IL was calculated from field measurements of mid-span strain in two integral frame-type concrete bridges in Sweden. A three-axle pre-weighed truck was used for the 10.5 m span Östermalms IP bridge illustrated in Fig. 1(a). At a scanning frequency of 1024 Hz, the dimensions of the \( \mathbf{W} \) matrix were of the order of 1500–2000. The IL derived from equations (6)–(8) is illustrated in Fig. 2(a), and a comparison of measured strain with strain predicted from the IL shows an excellent match in Fig. 2(b). Similar results were found for a two-axle calibration truck.

A seven-axle calibration truck (Fig. 1(b)) was used to find the IL for the 14 m span Kramfors bridge (Fig. 3). Strain data were collected at a frequency of 512 Hz using the SIWIM system, which applied a lowpass filter to the measured signals.
In this case, there are small differences between measured and predicted results (Fig. 3(b)), probably due to the increased number of axles. Nevertheless, the match between measured and predicted strains is very good.

4. CONCLUSIONS
A matrix equation is presented for the calculation of influence ordinates for a bridge from measured load effects. The method is shown to be very effective at finding the IL directly from strain measurements, particularly for two- and three-axle calibration trucks.

REFERENCES

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