RECOMMENDATIONS ON THE USE OF A 3-D GRILLAGE MODEL FOR BRIDGE DECK ANALYSIS

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Abstract—This paper highlights areas of inaccuracy in current modelling techniques for bridge decks with wide cantilevers. A 3-D grillage model known as the upstand grillage is assessed by comparing its behaviour with that of a three-dimensional finite-element model. Recommendations are made for the implementation of such a technique. The paper concludes with a comparison of results from analyses of a bridge deck using the standard and upstand grillage analogies and a three-dimensional finite-element method.

1. INTRODUCTION

Reinforced and prestressed concrete road bridges often include a transverse cantilever portion along their edges. This type of construction is chosen in part for its slender appearance to underpassing traffic. A second advantage is the reduction in self-weight of the bridge deck. The latter becomes more significant where footpaths are provided, as the relatively small footpath loading allows quite slender cantilevers to be used. Figure 1(a) shows a cross-section which might be used for a bridge deck carrying vehicular traffic only, while Fig. 1(b) shows how the incorporation of footpaths might alter the cross-section.

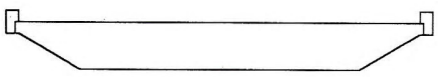
2. ANALYSIS OF BRIDGE DECKS

2.1. *Neutral axis location*

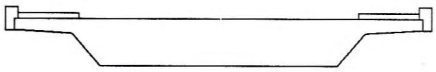
An important concept in the theory of bending is that all bending occurs about the neutral axis. In simple rectangular beams in bending, the neutral axis is located at the centroid of the beam. Bridge decks with short cantilevers tend to behave as T-beams and again their neutral axes will be located at their centroids. However, it has been reported by Hambly[1] amongst others that in bridge decks with longer cantilevers the neutral axis location may vary across the width. This is due to the tendency of the cantilever to bend about its own centroid, thus raising the neutral axis locally. Figure 2(a) shows the neutral axis location in a bridge deck with short cantilevers which is uniform across its width and coincides with the centroid. Figure 2(b) shows how the neutral axis location of a bridge deck with longer cantilevers might vary locally in the vicinity of the edge cantilevers.

2.2. *Plane grillage analysis*

Many analysis techniques are available to today's bridge designers. Of these techniques, bridges of the type considered here are commonly analysed in the design office using the plane (two-dimensional) grillage analogy. A comprehensive background to

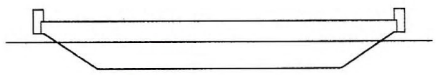


(a) Bridge deck carrying vehicular traffic only

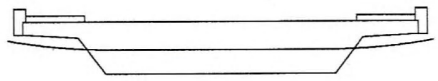


(b) Bridge deck incorporating footpaths

Fig. 1. Bridge deck cross-sections: (a) bridge deck carrying vehicular traffic only; (b) bridge deck incorporating footpaths.



(a) Bridge deck with short cantilevers



(b) Bridge deck with longer cantilevers

Fig. 2. Neutral axis locations in bridge decks: (a) bridge deck with short cantilevers; (b) bridge deck with longer cantilevers.

the development of this technique, and why it has been chosen in favour of other techniques, is given by Best[2] and Taylor.[3] It involves the discretization of the bridge deck slab into a number of longitudinal and transverse beams lying in the same plane. Each grillage beam represents a strip of slab and its properties are determined relative to the neutral axis of the bridge deck. The theory governing this technique has been well documented by many authors such as Cussens[4] and Bhatt.[5]

The plane grillage technique has been widely tested and found to be robust for many shapes of structure, loading conditions and support arrangements as reported by Best[2] and West.[6] Although these authors used the technique primarily for concrete slab and pseudo-slab bridges, it has been found to have applications in the modelling of concrete box girder bridges as reported by Maisel *et al.*[7] and Hambly.[8] It has been reported by Cussens[4] that the recommendations of West[6] are also applicable to steel and composite bridge decks.

Guidance on the implementation of a grillage analysis has been given by a number of authors. West[6] gives recommendations on the idealization of the deck, calculation of member properties, application of loads and interpretation of results. He gives particular guidance on calculating member torsion constants and concludes with examples of grillages for nine specific forms of construction.

Hambly[1] presents nine guidelines to assist the analyst in setting up a grillage mesh and gives detailed recommendations on choosing member properties. He also gives guidance on the use of a shear flexible grillage for multi-cellular decks. Clark[9] gives general guidance on grillage analysis from consideration of the recommendations of West[6] and Hambly[1] and his own views. Cope and Clark[10] give guidance on how the results from a grillage analysis should be interpreted for design purposes.

The accuracy of the plane grillage technique clearly depends on a knowledge of the location of the bridge deck neutral axis which, as mentioned earlier, may vary in bridges with wide cantilevers and is not easily determined. Consequently the plane grillage analogy may be inaccurate for this type of bridge deck.

2.3. *Upstand grillage analysis*

As computing power increases more elaborate numerical models become possible in the design office and it may now be feasible for three-dimensional grillage or space frame models to be used for bridge deck analysis in place of plane grillages. Some work has been carried out in this area with varying opinions on its suitability. Cussens[4] presents a technique which may be suitable for box girder bridges but comments on the disadvantages of such techniques as using considerable computer time and never being more than an approximation of the true structure.

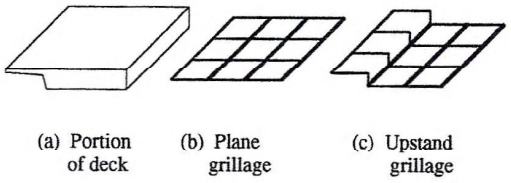


Fig. 3. Portion of a bridge deck with plane grillage and upstand grillage models: (a) portion of deck; (b) plane grillage; (c) upstand grillage.

A number of three-dimensional techniques have been suggested by Hambly.[1] The first of these is a space frame which would be most suited to skeletal structures such as steel girder bridges. A second technique favoured by Hambly[1] is the cruciform space frame. He demonstrates how this can be used to model in-plane shear distortion but it would not be capable of modelling in-plane bending without modification. With the increased complexity of this technique much of the similarity between the model and the real structure which is inherent in the grillage model is lost. A third technique suggested by Hambly[1] is the downstand grillage which is a variation of the space frame technique. The analogy as presented is best suited to beam and slab decks but it will be seen that the upstand grillage technique presented in this paper is of a similar form.

Upstand grillage modelling removes the constraint of having all the grillage beams in the one plane, which allows the modelling of a variation in neutral axis location. The technique involves placing each beam at the centroid of the portion of the real structure which it represents. This technique should give improved accuracy for bridge decks where the neutral axis varies significantly such as those with wide cantilevers. Figure 3 shows part of a bridge deck along with its discretized plane grillage and upstand grillage models.

As has been mentioned, plane grillage models have been used extensively for many years and there is a considerable amount of guidance available in the literature on their implementation. Unfortunately, detailed guidance does not appear to be available on upstand grillage modelling. The aim of this paper is to address this shortfall by providing insight into the mechanisms by which an upstand grillage models the behaviour of a three-dimensional bridge deck.

2.4. *Three-dimensional finite-element analysis*

In order to assess the accuracy of the plane and upstand grillage models it was necessary to compare the stress distributions with an acceptable correct

solution. For this purpose the NIKE3D[11] finite-element analysis program was used, which supports full 3-dimensional brick elements.

The authors have used this program extensively and are confident that when used correctly it can very accurately predict the behaviour of structures such as the bridge decks considered here. This confidence comes both from analysing simple structures, the behaviour of which is known, and from comparing NIKE3D analyses to results from experimental work.[12,13]

Three-dimensional finite-element analysis programs (3-D FEA) such as NIKE3D are seen primarily as research tools by bridge designers. This is due in part to the dependency of the results on the finite-element mesh chosen, the computer resources required and the time required to become competent in the use of such programs.

3. UPSTAND GRILLAGE MODELLING

3.1. *Member properties for upstand grillages*

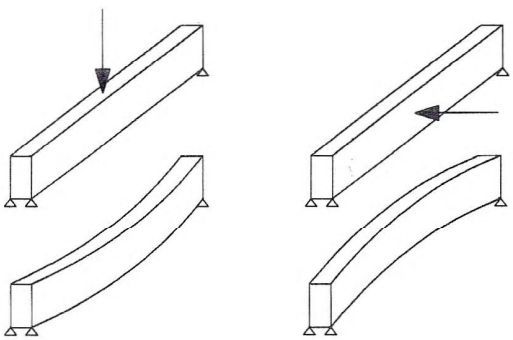
Plane grillage analyses do not allow any in-plane displacements or rotations. Consequently the only member properties which are required are, area (A), out-of-plane second moment of area (I), and torsion constant (J). Upstand grillage analysis due to its full three-dimensional nature will allow deformations in any plane or direction. Consequently in-plane bending and in-plane shear deformations must also be considered. This results in one extra member property, namely in-plane second moment of area (I'). In-plane shear deformation is determined by the same member area as out-of-plane shear deformation. Most three-dimensional grillage computer programs will allow two independent shear factors to be specified, one for in-plane and one for out-of-plane shear deformations. In this work a standard shear factor of 0.85 is used for both directions. Figure 4(a) shows, for a rectangular beam, the type of deformation associated with out-of-plane bending while Fig. 4(b) shows that associated with in-plane bending.

The recommendations of the authors mentioned in Section 2.2 should also be applied to upstand grillage models. The one addition to this is the calculation of properties for the vertical members which connect the upstand members to the remainder of the deck. The properties of these members are discussed later.

3.2. *Simplified model*

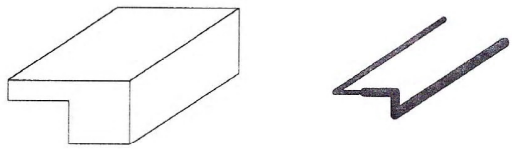
There are a number of uncertainties involved in upstand grillage modelling, possibly the most obvious of these being the interaction of the main part of the bridge deck with the cantilever part. This is modelled in the upstand grillage by a number of vertical and transverse beams. It was decided to start by modelling this region alone rather than attempting to model the entire bridge deck. Figure 5 shows the part of the bridge deck involved along with that part of the upstand grillage which models its behaviour.

3.2.1. 3-D FEA of simplified model. The 3-D FEA mesh used to model the portion of the bridge deck under consideration can be seen in Fig. 6. All the elements in the mesh are cubic with a 0.2 m side length. One end face of the model was completely fixed by restraining all of the nodes in the x, y and z directions. A concentrated moment was applied to the opposite face by means of a pair of nodal forces as illustrated.



(a) Out of plane bending (b) In-plane bending

Fig. 4. Deformations associated with member properties for upstand grillages: (a) out-of-plane bending; (b) In-plane bending.



(a) Portion of deck (b) Upstand grillage

Fig. 5. Portion of bridge deck at junction of cantilever and main deck: (a) portion of deck; (b) upstand grillage.

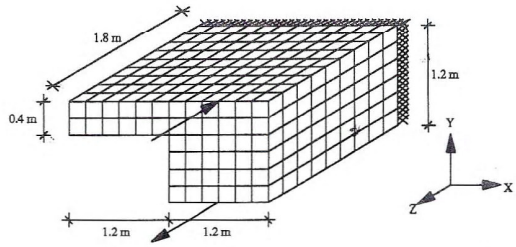


Fig. 6. NIKE3D finite-element mesh showing nodal fixities and applied loads.

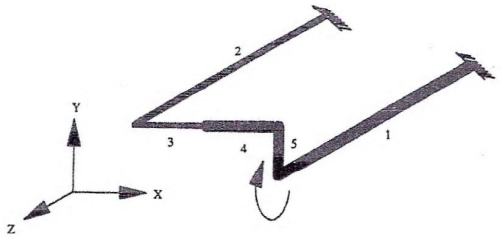


Fig. 7. Upstand grillage model showing member numbers, nodal fixities and applied loading.

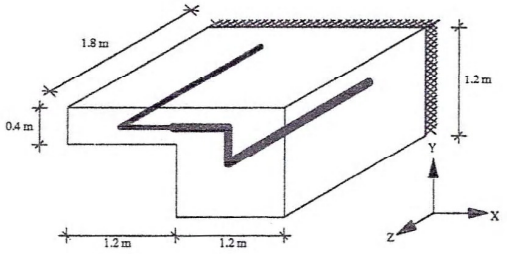


Fig. 8. Portion of bridge deck with upstand grillage model superimposed.

After this model was analysed a new mesh was generated which had twice as many elements in each direction. This new model was also analysed and it was found that the results of both analyses were practically identical. From this it was concluded that the original mesh formation was sufficiently refined to give accurate results and this mesh was used as the basis for all subsequent 3-D FEA modelling in this paper.

3.2.2. Upstand grillage analysis of simplified model. The upstand grillage model corresponding to the portion of bridge deck under consideration was analysed using the STRAP computer program.[14] Again, one end of the upstand grillage was completely fixed in the x, y and z directions and also in this case against rotations about these three axes. A concentrated moment was applied to the opposite end of the upstand grillage. Figure 7 shows this upstand grillage with members labelled 1–5.

As previously mentioned each beam in the upstand grillage represents a portion of the real bridge deck. In Fig. 8 the bridge deck is seen with the upstand grillage superimposed to show which portion is attributed to each member. Table 1 gives dimensions of members 1–4 along with their calculated properties.

For the vertical upstand grillage member (member 5 in Fig. 7), it was decided to adopt member properties initially which would not allow any deformation in that member. For this purpose the area, second moments of area and torsion constant were all given very large values. After repeated analyses it was determined that a suitable very large value which did not result in significant round-off

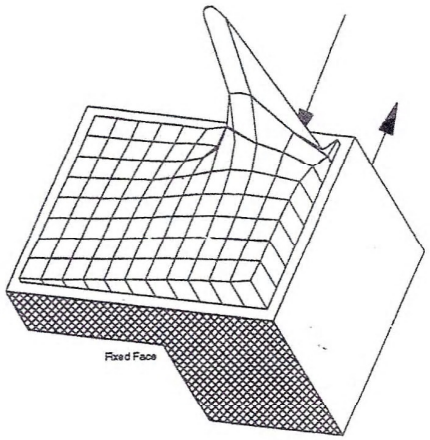


Fig. 9. Normal stress in direction of applied loads.

errors was 10^3–10^4 times the maximum member property in any other member in the model. The properties chosen for the vertical member can be seen in Table 2.

3.2.3. Comparison of results of simplified model. The results obtained from the 3-D FEA enabled the direct plotting of a three-dimensional stress contour along the top surface of the portion of the bridge deck considered. The contour of normal stress in the direction of the applied loads can be seen in Fig. 9.

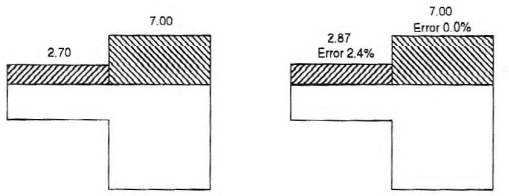
It is clear from Fig. 9 that the localised peaks of stress close to the applied loads have diminished sufficiently at the fixed face. It was therefore decided to use the results at this location to determine the accuracy of the upstand grillage relative to the 3-D FEA model. Figure 10 shows the normal stress (in N/mm^2) in the direction of the applied load at the fixed end, again along the top surface for both the 3-D FEA model and the upstand grillage model. In the case of the 3-D FEA model an average stress has been calculated for each

Table 1. Member dimensions and properties (in metres)

Member	1	2	3	4
Depth	1.2	0.4	0.4	1.2
Breadth	1.2	1.2	1.8	1.8
Area	1.44	0.48	0.72	2.16
$I = \frac{1}{12}\, bd^3$	0.1728	0.0064	0.0096	0.2592
$I' = \frac{1}{12}\, db^3$	0.1728	0.0576	0.1944	0.5832
$J = \frac{1}{6}\, bd^3$	0.3456	0.0128	0.0192	0.5184

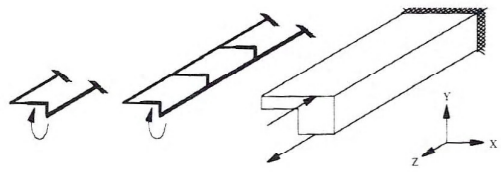
Table 2. Member properties (in metres) of vertical member

Member	Area	I	I'	J
5	3000	3000	3000	3000



(a) 3-D FEA (b) Upstand grillage

Fig. 10. Normal stress (in N/mm^2 perpendicular to page) for 3-D FEA and upstand grillage analyses: (a) 3-D FEA; (b) upstand grillage.



(a) Original upstand grillage (b) Extended upstand grillage (c) Extended 3-D FEA model

Fig. 11. Extended model: (a) original upstand grillage; (b) extended upstand grillage; (c) extended 3-D FEA model.

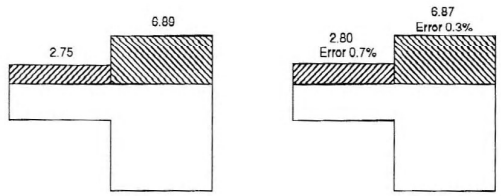
of the two regions shown. Stresses in the upstand grillage model were calculated using the bending moments and axial forces in the members. Percentage differences in stress between the two models are also shown. These percentages have been expressed in terms of the maximum stress at the section under consideration.

3.3. *Extended models*

3.3.1. Simple extended model. The aim of this work was to model a full bridge using the upstand grillage technique. The small portion of bridge considered above was extended in a number of steps taking experience gained from one step to the next. The first step in this process was to extend the length of the portion of bridge. Figure 11 shows the original upstand grillage and the new extended upstand grillage. The 3-D FEA model of the new lengthened structure is also shown.

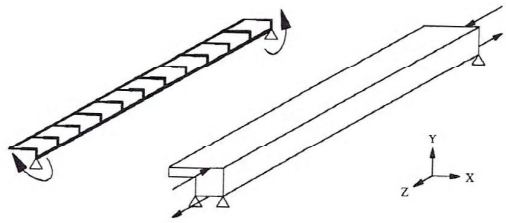
As in the previous upstand grillage model the new extended version was completely fixed at one end and had a concentrated moment at the opposite end. The member properties were all calculated in the same way as those of the previous upstand grillage.

It was found that the extended upstand grillage accurately modelled the behaviour which was predicted by the 3-D FEA model. Figure 12 shows the



(a) 3-D FEA (b) Upstand grillage

Fig. 12. Normal stress (in N/mm^2 perpendicular to page) for 3-D FEA and extended upstand grillage analyses: (a) 3-D FEA; (b) upstand grillage.



(a) Upstand grillage (b) 3-D FEA model

Fig. 13. Full-length model: (a) upstand grillage; (b) 3-D FEA model.

normal stress in the direction of the applied load (in N/mm^2) at the fixed face, along the top surface for both the 3-D FEA and upstand grillage models.

3.3.2. Full-length extended model. The second stage in extending the upstand grillage model was to extend its length to 24.8 m. This length was chosen because it was convenient for setting up the 3-D FEA mesh and because it was typical of the span length of a bridge deck of the type being considered here. The support conditions were chosen as pinned at both ends to simulate a single span simply supported bridge deck. The applied loading remained of the same form, but due to the support conditions, equal and opposite concentrated moments were applied at each end of the model. Figure 13 shows both the 3-D FEA model and the new upstand grillage model with support conditions and applied loads. It was also necessary to provide a support against rotation about the z axis to one of the two support nodes of the upstand grillage in order to make the structure stable.

The new model again confirmed the accuracy of the upstand grillage technique for this type of structure. The correlation between the 3-D FEA and upstand grillage models was once again excellent as can be seen in Table 3. Unlike the previous models, results are presented at 1/8, 1/4, 3/8, and 1/2 span locations along the length of the model. The results presented are once again the normal stress in N/mm^2 at the top surface, in the longitudinal direction, i.e. along the span length of the bridge deck.

Table 3. Stress (in N/mm^2) from 3-D FEA and upstand grillage models

Location	3-D FEA	Upstand grillage	% error
Cantilever			
1/8 span	2.84	2.81	0.4
1/4 span	2.83	2.81	0.3
3/8 span	2.83	2.81	0.3
1/2 span	2.83	2.81	0.3
Main deck			
1/8 span	6.86	6.89	0.4
1/4 span	6.85	6.86	0.1
3/8 span	6.85	6.86	0.1
1/2 span	6.85	6.86	0.1

3.4. *Full-width model*

The next step in this work was to extend the modelling to full-width bridge decks. As before, 3-D FEA and upstand grillage models were generated and analysed and the results compared. The same simply supported conditions were used and a series of concentrated moments were applied across the width of the bridge deck. Figure 14 shows both of these models along with their supports and applied loads.

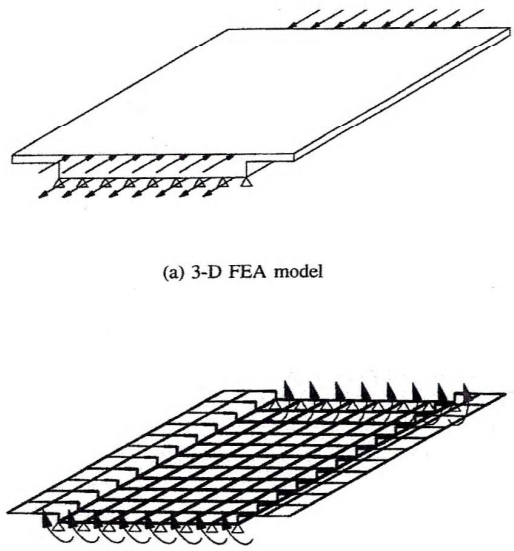
As before, results were compared at 1/8, 1/4, 3/8 and 1/2 span locations. The results did not compare as well with each other as in previous cases with disagreement between the two models of up to 5.7%.

3.4.1. Further investigation of in-plane bending. The disagreement appears to be attributable to poor modelling in the upstand grillage of in-plane bending in the main body of the deck. To test this hypothesis, a comparison was made of a simple beam in bending with a grid of beams subjected to in-plane bending. Figure 15 shows both of these models. Midspan deflections are also shown for both beams. It can be seen that the grid overestimates the midspan deflection by some 31%. It can be concluded from this that the model is too flexible and is not capable of modelling in-plane bending correctly.

Bridge decks of the type being investigated here are not generally subject to substantial in-plane bending. In an attempt to find an analysis procedure simple enough for everyday design, the effect was investigated of eliminating in-plane bending completely from the upstand grillage model. This was achieved by restraining all of the nodes in the upstand grillage against in-plane rotation. With these modifications it was found that the maximum disagreement between the upstand grillage and 3-D FEA models was reduced from 5.7 to 4.4%.

3.5. *Full-size model with extended cantilever*

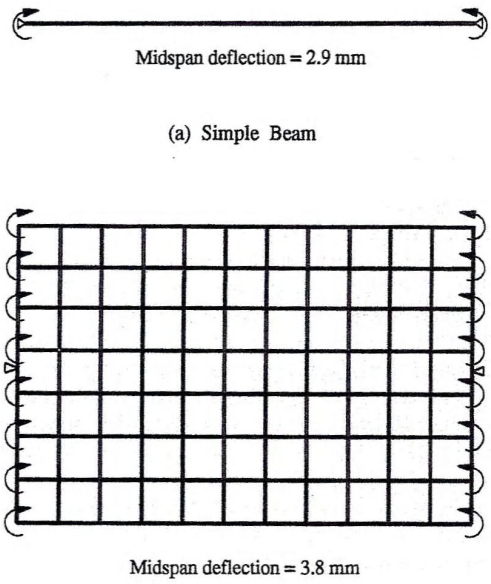
It was proposed at the beginning of this paper that the upstand grillage could be a suitable method of modelling bridge decks with long slender cantilevers. To test this the length of the cantilever was then increased from 1.2 to 2.4 m. This new can-



(a) 3-D FEA model

(b) Upstand grillage model

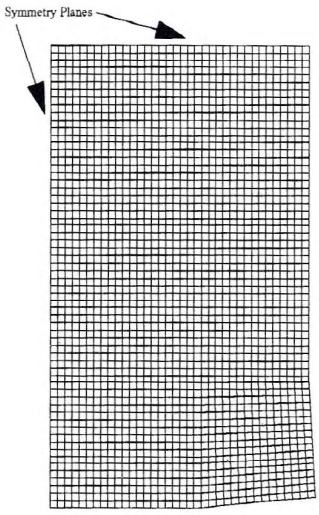
Fig. 14. Full-width model: (a) 3-D FEA model; (b) upstand grillage model.



(a) Simple Beam

(b) "Equivalent" grid

Fig. 15. In-plane bending of beam and "equivalent" grid: (a) simple beam; (b) "equivalent" grid.



(a) Plan

(b) Cross section

Fig. 16. 3-D FEA deflected shape showing in-plane bending: (a) plan; (b) cross-section.

tilever length represented the greatest that would be practical, with a span-to-depth ratio of 6.

The conclusions made from the previous upstand grillage models were all adopted in this new model. All of the vertical member properties were made very large and all of the nodes were restrained from in-plane bending. It was found that the results of this new upstand grillage model compared poorly with those from the equivalent 3-D FEA. The greatest disagreement of 15.1% was found to occur in the cantilever portion at 1/8 span.

3.5.1. In-plane bending in 3-D FEA Model. Figure 16 shows the deflected shape of the 3-D FEA model in plan. Only 1/4 of the model is shown as it is symmetrical about both axes as shown. The cross-section of the bridge deck is also shown for clarity. It is quite clear from this deflected shape that in-plane bending does occur in the cantilever portion towards the ends of the model. This is indeed the area where the greatest disagreement with the upstand grillage results occurred.

3.5.2. In-plane bending in the upstand grillage model. The imposition of fixities against in-plane bending in the upstand grillage does not allow the bending which was seen in Fig. 16 and leads to the inaccuracies observed. It was therefore decided to release some of the rotational fixities. These changes were carried out only in the regions where in-plane bending was observed in Fig. 16. The measures improved the correlation between the 3-D FEA model and the upstand grillage model from a maximum disagreement of 15.1%, with no allowance for in-plane bending to a maximum disagreement of just 3.6% when in-plane bending was allowed at the ends of the cantilever.

3.5.3. Results of full-size model with extended cantilever. The complete results from the upstand grillage model, firstly with no allowance for in-plane bending, and secondly with an allowance for in-plane bending at the ends of the cantilevers only are presented in Fig. 17 along with the results from the 3-D FEA. A plane grillage analysis was also carried out for this bridge deck, the results of which can also be seen in this figure.

It can be concluded from this figure that the upstand grillage model with an allowance for in-plane bending in the ends of the cantilevers only, gives the best agreement with the 3-D FEA model.

3.6. *Upstand grillage modelling of a bridge deck with typical loading*

The effects of typical loading were considered by analysing for self-weight and code-based live loading. The self-weight consisted of the weight of the deck and all superimposed dead loads. The live loading was determined in accordance with Eurocode 1 using load model 1.[15]

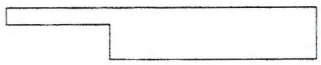
The cross-sectional layout of the bridge deck showing footpaths and road surfacing is illustrated in Fig. 18. The dimensions of the bridge deck were the same as those used in the full-size model with extended cantilevers. Consequently the same grillage models were used, with and without an allowance for in-plane bending as before. The full results of this analysis are shown in Fig. 19 along with those of a plane grillage and 3-D FEA. It can be concluded from this figure that the upstand grillage model with an allowance for in-plane bending in the ends of the cantilevers only, gives the best agreement with the 3-D FEA model.

4. CONCLUSIONS AND RECOMMENDATIONS

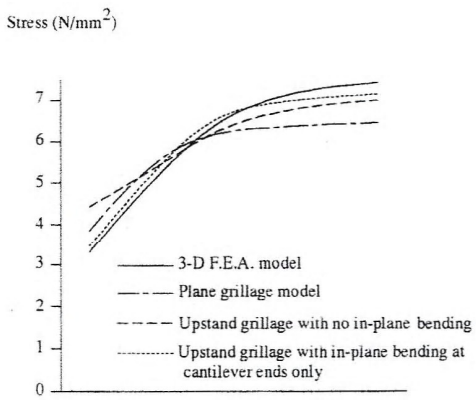
4.1. *Conclusions*

The inaccuracy of the plane grillage analogy for bridge decks with wide cantilevers has been highlighted. The upstand grillage technique has been shown to constitute an improvement when compared to a 3-D FEA model. The upstand grillage is effectively a number of plane grillages located on different planes which are connected by stiff vertical members.

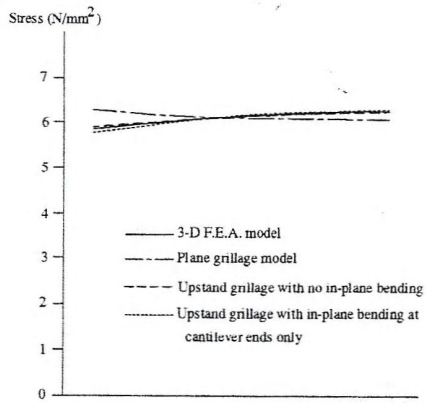
The phenomenon of in-plane bending has been shown to be of significant importance in the upstand grillage model. It was shown that providing restraint



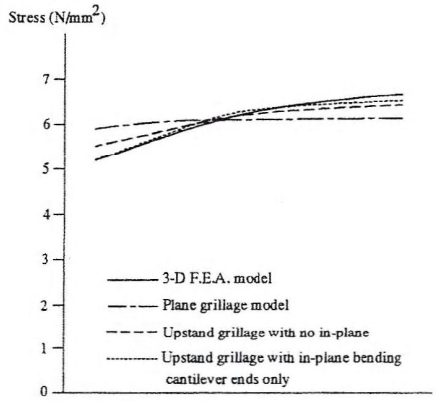
(a) Cross section



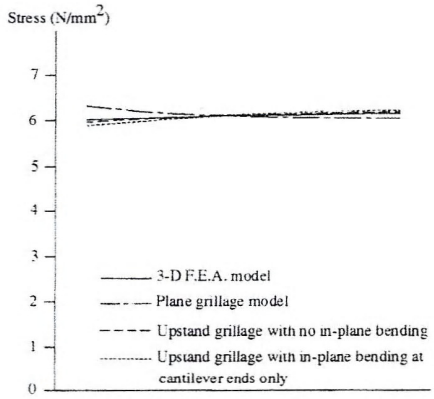
(b) 1/8 span



(d) 3/8 span



(c) 1/4 span



(e) 1/2 span

Fig. 17. Normal stresses (in N/mm^2 perpendicular to page) for 3-D FEA, plane grillage and upstand grillage with and without any allowance for in-plane bending: (a) cross-section; (b) 1/8 span; (c) 1/4 span; (d) 3/8 span; (e) 1/2 span.

against in-plane distortion gave the best results. An exception to this was encountered in the edge cantilevers towards the ends of the bridge deck. The 3-D FEA confirmed that in-plane distortion does occur in this area. By releasing the restraints in this area the upstand grillage was shown to give better agreement with the 3-D FEA model.

An understanding of the behaviour of the bridge deck prior to analysis is necessary to enable the analyst to obtain the most from the model. Failing this, if in-plane distortion is restrained throughout the entire deck accurate modelling will be achieved except locally where the in-plane bending occurs. Nevertheless this fully restrained upstand grillage

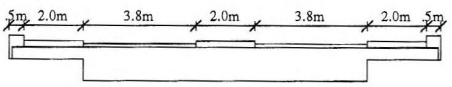


Fig. 18. Cross-sectional layout of bridge deck.

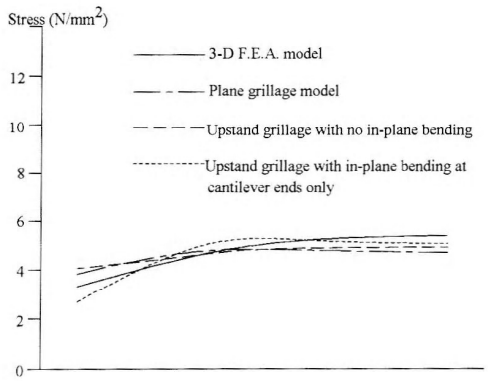
model has been found to give better results than the plane grillage.

The upstand grillage was shown to model typical bridge loading well. A similar improvement over the plane grillage was observed for this type of loading.

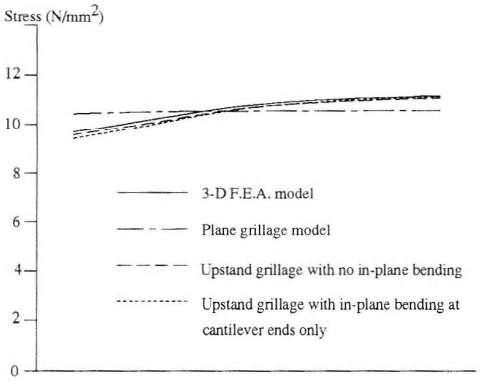
4.2. *Recommendations for upstand grillage modelling*

The recommendations of West,[6] Hambly,[1] Cope and Clark[10] and Clark[9] for plane grillage modelling should be used for upstand grillage modelling. In addition to these recommendations the properties of the vertical members in the upstand grillage should be chosen so as to make these members very stiff. This can be implemented by making their areas, second moments of area and torsion constants very large. A suitable value would appear to be 10^3–10^4 times the next greatest member property.

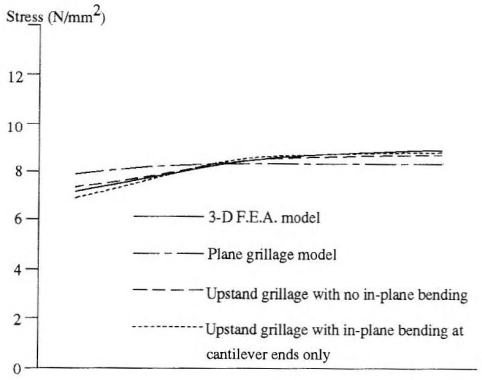
(a) Cross section



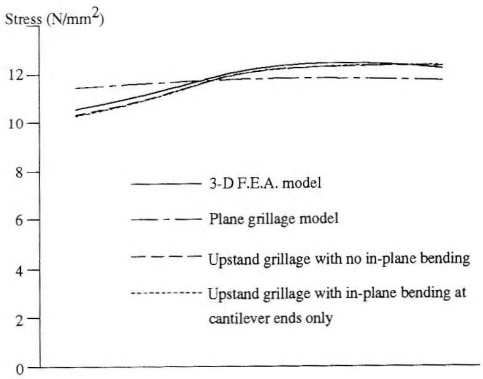
(b) 1/8 span



(d) 3/8 span



(c) 1/4 span



(e) 1/2 span

Fig. 19. Typical bridge loading, normal stresses (N/mm^2) for 3-D FEA, plane and upstand grillage: (a) cross-section; (b) 1/8 span; (c) 1/4 span; (d) 3/8 span; (e) 1/2 span.