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Application of the Wavelet Multi-resolution Analysis and Hilbert Transform for the Prediction of Gear Tooth Defects

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Abstract - In machine defect detection, namely those of gears, the major problem is isolating the defect signature from the measured signal, especially where there is significant background noise or multiple machine components. This article presents a method of gear defect detection based on the combination of Wavelet Multi-resolution Analysis and the Hilbert transform. The pairing of these techniques allows simultaneous filtering and denoising, along with the possibility of detecting transitory phenomena, as well as a demodulation. This paper presents a numerical simulation of the requisite mathematical model followed by its experimental application of acceleration signals measured on defective gears on a laboratory test rig. Signals were collected under various gear operating conditions, including defect size, rotational speed, and frequency bandwidth. The proposed method compares favourably to commonly used analysis tools, with the advantage of enabling defect frequency isolation, thereby allowing detection of even small or combined defects.

Key-words – *Vibratory analysis/ Gears defects/ Wavelet multi-resolution analysis/ Hilbert transform/ Demodulation*

1. Introduction

Gears play an important role in industry. They are of capital importance when transmitting significant torque. Like all machine components, gears are often subjected to various degradations, namely those that contact the gear teeth. Failing to achieve timely detection can cause serious accidents. As such, gear damage detection is a major predictive maintenance priority. To address this, several methods have been developed, in particular vibratory analysis [1] and acoustic emission measurement [2].

In the time domain, the use of specific statistical scalar indicators, such as the kurtosis and the crest factor have shown strong promise in several studies [3,4]. These indicators are very sensitive to the impacts present in a signal caused when a non-defective tooth comes into contact with a defective one. The results indicate values higher than normal (i.e. 3 for the kurtosis and 6 for the crest factor). In several studies, the kurtosis was shown to be more sensitive than the crest factor and was used as a detection criterion for discovering gear teeth defects as a function of operating hours [5].

The problem with the scalar indicators is that they are often ineffective in the presence of intense levels of background noise or in the presence of other machine components, which tend to mask the effects, especially of incipient defects. One time domain solution consists of measuring the signals in narrow bands at high frequencies by band-pass filtering [3,4]. A second approach consists of using modern signal denoising tools such as the Soft Thresholding Denoising as developed by Donoho [6] for discrete wavelet transformation. Another denoising approach based on the Morlet wavelet was proposed by Jing [7], which when applied to defective bearings and gears signals was shown to be more effective than Donoho Denoising. Adaptive filtering has been also proposed by using the Morlet wavelet with parameter optimization based on the kurtosis maximization principle [8]. Indeed, the application of these methods leads to filtered signals with more significant kurtosis, thereby aiding in defect detection.

In the frequency domain, spectrum analysis is the oldest method. The amplitude of the meshing frequency increases gradually with crack size growth. Moreover, in the case of chipping, sidebands appear around the meshing frequency, the spacing of which corresponds to the rotational frequency of the shaft carrying the defective gear [9,10]. In certain cases, these sidebands are not visible on the spectrum. Because of this invisibility, use of a cepstrum analysis is necessary. The cepstrum analysis allows separation of lines of families. The approach consists of calculating a vector, named the cepstrum, which represents the inverse Fourier transform of the spectrum logarithm. Several applications of this approach have been proposed [11-14].

Since spectrum analysis is not adaptable to non-stationary signals, the time-frequency methods (in particular wavelet analysis) are currently favoured in gear defect detection. The time-frequency map allows a local visualization of the signal, which enables the detection of transitory phenomena. This is contrary to the spectrum, which accommodates only a global vision. In its continuous version, the wavelet analysis has been applied by several researchers to this problem [15-18]. Discrete analysis (or multi-resolution) has been also used to extract the defect signature from the globally measured signal. Based on a succession of high-pass and low-pass filtering, it has been shown to be more reliable than the Short-time Fourier Transform [19]. An interesting and highly effective application of this was developed by Chinmaya and Mohanty [20], which consisted of the Wavelet Multi-resolution Analysis (WMRA) of the current signal. The wavelet packet analysis, being a generalization of the WMRA, has been proposed as a detection tool especially well-suited for non-stationary signals [21]. The WMRA has been also applied in the detection of other mechanical faults, namely rolling bearings. An optimisation of this technique was proposed using the kurtosis as its principal criterion [22,23].

The objective of this article is to propose a hybrid detection method based on a synthesis of WMRA and the Hilbert transform. Section 2 is devoted to the numerical simulation and the analysis of signals generated by defective gears. Section 3 includes the presentation of the WMRA theory. The proposed method is then applied on simulated

signals in section 4. Finally, an experimental validation carried out on defective gears on a laboratory test rig is presented in section 5.

2. Numerical simulation and analysis of defective gear signal

The signal collected on an undamaged pair of gears is dominated by the meshing frequency and its harmonics. Mathematically it can be represented by equation (1) [21,24]:

$$x(t) = \sum_{m=1}^M X_m \cos(2\pi m F_m t + \phi_m) \quad (1)$$

where F_m is the meshing frequency given by equation (2):

$$F_m = Z_1 \cdot f_1 = Z_2 \cdot f_2 \quad (2)$$

with Z_1 and Z_2 the number of teeth of the gear 1 and 2, respectively, f_1 and f_2 are their rotation speeds (frequencies), respectively. M is the number of meshing frequency harmonics. X_m and ϕ_m are the amplitude and the phase of the m th meshing harmonic, respectively. Frequency components corresponding to the shafts' rotational speeds can also be added to equation (1). Figure 1 represents a typical example of a spectrum of an undamaged pair of gears. Rotational frequencies were taken as 25 Hz and 40 Hz, the corresponding number of teeth were 40 and 25, respectively, the meshing frequency was then equal to 1000 Hz. Under normal conditions, shafts rotational frequencies, the affiliated meshing frequency, and the related harmonics are normally visible on the spectrum.

2.1. Case of a defect localised on one tooth

A localised defect on one gear tooth will produce periodic shocks with each contact of the defective tooth against any other. The corresponding signal will modulate in amplitude and phase the meshing signal of the equation (1). This modulation will be translated on the spectrum by sidebands around the meshing frequency and its harmonics. These sidebands are spaced by the rotational frequency of the shaft carrying the defective gear.

The amplitude and phase modulating functions $a_m(t)$ and $b_m(t)$ can be generally represented as equations (3.a) and (3.b) [21,24]:

$$a_m(t) = \sum_{n=1}^N A_{mn} \cos(2\pi n F_r t + \alpha_{mn}) \quad (3.a)$$

$$b_m(t) = \sum_{n=1}^N B_{mn} \cos(2\pi n F_r t + \beta_{mn}) \quad (3.b)$$

where N is the number of sidebands around the meshing frequency harmonics, F_r is the rotational frequency of the shaft carrying the defective gear. The terms A_{mn} and B_{mn} represent amplitudes at the n th sideband of amplitude and phase modulating signal, respectively, around the m th meshing frequency harmonic. The terms α_{mn} and β_{mn} represent phases at the n th sideband of amplitude and phase modulating signal, respectively, around the m th meshing frequency harmonic. As such, the resulting signal, highlighting a gear with a defect localised in one tooth, is given by equation (4):

$$y(t) = \sum_{m=1}^M X_m (1 + a_m(t)) \cos(2\pi m F_m t + \phi_m + b_m(t)) \quad (4)$$

For the same example of figure 1, figure 2 represents a signal in the case of a small, localised tooth defect on the gear turning at 25 Hz. The signal highlights an impulse train in which the period is equal to 0.04 s (1/25 Hz). The corresponding spectrum of figure 3 [0-2000] Hz shows sidebands around the meshing frequency. These sidebands are spaced by 25 Hz; the modulation phenomenon is, thus, highly visible. The amplitudes of the modulating functions are considered too small to simulate the vibration signal of a weak gear defect. Instead, three sidebands around the three meshing frequency harmonics have been considered. Table 1 contains the whole model signal data.

The cepstrum, being the most used tool in gears defects detection, was calculated to highlight the simulated defect. On figure 4 one notes a quefrency corresponding to 0.04 s (1/25 Hz) and several of its rhamonics. The simulated defect is, thus, clearly detectable by the cepstrum. Actually the cepstrum of a signal $s(t)$ represents the inverse Fourier Transform of the spectrum $S(f)$ logarithm as defined in equation (5).

$$Cepstrum\ s(t) = TF^{-1}[\log(S(f))] \quad (5)$$

Unfortunately, in practice defect detection is rarely obvious directly with one of the aforementioned tools. Background noise and other machine components produce a mask effect, which makes detection difficult and, for incipient defects, often impossible. Figure 5 shows the same signal as in figure 2 with the addition of a significant level of white Gaussian noise as defined in equation 6.

$$y_n(t) = y(t) + background\ noise \quad (6)$$

Different noise ratios were used in the investigations, from high to low. In every case good results were obtained. The noise ratio was calculated according to equation (7) as defined in [26]:

$$k = \frac{\sum_{n=1}^N |z_n|}{\sum_{n=1}^N |y_n|} \times 100 \% \quad (7)$$

where z_n is the Gaussian noise signal, N is the number of samples, and y_n is the noisy signal. The signal ratio used in this simulation is equal to 55%; that means an SNR of 1.44. In this case, the impacts caused by the defect are masked, and the kurtosis is equal to 3.1 – a common value. The spectrum of the noised signal (Figure 6) is contaminated by the random noise. Similarly, no information on the defect is discernable in the cepstrum of figure 7, specifically because there is no component corresponding to the quefrency 0.04 s (1/25 Hz). In general, the cepstrum is influenced by random noise and, thus, becomes unreliable.

2.2. Case of two defects localised on a tooth of each gear (combined defect)

From equation (3), it is easy to simulate two defects to test the approach. In this case, one simulated defect is on a gear tooth turning at 25 Hz and another is on one turning at 40 Hz. Figure 8 shows the noisy signal corresponding to the two defects. As in the case of one defect, the cepstrum of figure 9 is unable to detect the simulated defects, because the two quefrencies and their rhamonics are not obviuos; the one equal to 0.04 s corresponds to the first defect (25 Hz) and the other equal to 0.025 s corresponds to the second defect (40 Hz).

This simulation shows that the detection of one or multiple gears defects is sometimes impossible by frequency domain analysis tools, such as the spectrum and the cepstrum, or other time domain analysis tools such as the kurtosis, although these tools have been effective in other applications.

3. Theory of the Wavelet Multi-resolution Analysis

To overcome the above-noted limitations, a hybrid approach was developed. The first component of this was the application of Wavelet Multi-resolution Analysis (WMRA). WMRA is a discrete version of the continuous wavelet analysis, well known in the literature. Mallat [25] proposed a fast waterfall algorithm of this transform in 1989 (fig. 9). It consisted of decomposing the original signal $s(t)$ into several under-signals of various scales using two filters: high-pass and low-pass. From these two vectors were obtained: (1) the detail coefficients (cD_i) corresponding to the high frequencies and (2) the approximation coefficients (cA_i) corresponding to the low frequencies. After the first level, only the approximation coefficients were filtered. During decomposition, the signal $s(t)$ and vectors (cA_i) underwent down sampling. This is why the approximation and detail coefficients passed through two reconstruction filters (LR) and (HR). From this, two vectors were obtained: (A_i) approximations and (D_i) details. This then satisfied the relation in equations (8.a) and (8.b):

$$A_{j-1} = A_j + D_j \quad (8.a)$$

$$s = A_j + \sum_{i \leq j} D_i \quad (8.b)$$

where i and j were integers.

Noting F_{max} the maximum frequency of the original signal, the frequency bands of each level (i) returned to $\left[0 - \frac{F_{max}}{2^i}\right]$ for the approximations and $\left[\frac{F_{max}}{2^i} - \frac{F_{max}}{2^{i-1}}\right]$ for the details.

4. Hilbert transform

The Hilbert transform is a linear operator which takes a signal $s(t)$ and produces a transform $H(t)$ in the same domain. The operator can be considered as a time domain convolution defined by equation (9):

$$H(s(t)) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{s(\tau)}{t - \tau} d\tau \quad (9)$$

The Hilbert transform produces a complex time series. The envelope is the magnitude of this complex time series and represents an estimate of the modulation present in the signal due to sidebands, as defined in equation (10) [21]:

$$a(t) = \sqrt{(s^2(t) + H^2(t))} \quad (10)$$

The proposed method consists of the Hilbert transform of the wavelet coefficients of the selected detail. From this, an envelope spectrum of these coefficients is then calculated, the amplitude and phase modulating functions $a_m(t)$ and $b_m(t)$ can then be extracted from the original signal, and the modulating frequency (defect frequency) is then isolated.

5. Proposed approach

WMRA has several properties that make it very effective for the identification of defective gear signals. Specifically, it enables the following:

1. Separation between the high and low signal frequencies;
2. Having several filtered signals of various frequency bands with good resolutions;
3. Denoising of the original signal.

The method enables extraction of the defect signature from the original signal. This approach was first proposed as early as 2000 [19,20] but without a clear argumentation as to parameter selection. As part of the contribution presented herein, selection of those parameters is explained below.

5.1. Optimal choice of the maximum frequency

During WMRA, the bands of the details and the approximations are automatically deduced starting from the maximum frequency of the original signal. As the proposed approach is based on filtering and demodulation, it is imperative that the band of one of the details covers the meshing frequency modulated by the defect frequency. The solution relies, therefore, on a rational choice of the maximum frequency, which mathematically must satisfy equation (11):

$$F_m = \frac{\frac{F_{\max}}{2^i} + \frac{F_{\max}}{2^{i-1}}}{2} \quad (11)$$

Equation (11) must, then be set equal to equation (12):

$$F_{\max} = \frac{2^{i+1}}{3} F_m \quad (12)$$

with (i) as the index of the selected detail (exp. $i=1$ for the first detail ...etc.).

The Hilbert transformation of the wavelet coefficients of the selected detail is calculated to allow the demodulation. From this, an envelope spectrum of these coefficients is then calculated to highlight the modulating frequency and its harmonics (the defect frequency).

The number of the decomposition levels is very crucial in a WMRA. The number must be large enough to contain the information in one search, but not so large as to require excessive computation. The approach developed in this article is based on the choice of a maximum frequency so that one detail covers the meshing frequency. Moreover, the localization of the defect frequency is possible in the detail covering it. Since this frequency is generally much lower than the meshing frequency **one need low level that must contain it**. Mathematically one can say that the defect frequency must be included in the bandwidth $\left[\frac{F_{\max}}{2^n} - \frac{F_{\max}}{2^{n-1}} \right]$, where n is the decomposition level number that one can simply evaluate it.

5.2. Optimal choice of the analysis wavelet

The approach proposed for the wavelet choice is based on its filtering characteristic. As one searches for a reconstructed signal more filtered than the original one, the optimal wavelet will be that which gives a good result (i.e. a reconstructed signal in which the impulsions due to the defect can be readily localized). Since the kurtosis is the most sensitive indicator to shocks, it was chosen to help select the optimal analysis wavelet. Experimentally, it was established that the wavelet that gives the highest kurtosis was the optimal one.

The kurtosis is the ratio between the fourth statistical moment and the squared second statistical moment, as given by equation (13):

$$Kurtosis = \frac{\int_{-\infty}^{+\infty} |x|^4 p(x) dx}{\left(\int_{-\infty}^{+\infty} |x|^2 p(x) dx \right)^2} \quad (13)$$

In several works [19-21], the Daubechies family was used as a wavelet mother. Elsewhere [22,23], it has been shown that ‘db5’ (Daubechies 5), ‘db6’, ‘db10’ and ‘db12’ are generally the best adapted in WMRA of shock signals. After computing the kurtosis of the reconstructed (filtered) signal with various Daubechies wavelets, maximal values are taken. Table 2 gives the wavelets adapted for each sampling rate (F_s) and shock frequency (F_c) (or the ratio F_s/F_c) [23]. Moreover, table 3 gives the kurtosis values of the reconstructed signal for a sampling frequency of 10000 Hz for different wavelet mothers.

The proposed approach is applied in the following section on the noised signals simulating single and combined defects developed in sections 2.1 and 2.2. Note that with these signals, defect detection was not possible by the most useful frequency domain tool, the cepstrum, or the most sensitive scalar indicator, the kurtosis.

5.3. Application on simulated signals

Figure 10 shows the envelope spectrum of the wavelet coefficients after the application of the proposed approach to the noisy signal shown in figure 5. In this case, detail 2 [750-1500] Hz covers the simulated meshing frequency taken equal to 1000 Hz. The Daubechies wavelet ‘db5’ is used as the analysis wavelet. The envelope spectrum clearly shows a component corresponding to the rotational frequency of the defective gear, that is to say 25 Hz and one of its harmonics.

Detection is also possible at low frequencies. Detail 7 [23.437-46.8750] Hz (fig. 11) covers the rotational frequency of the defective gear and shows a sinusoid that corresponds perfectly to this frequency. In this case, because of the local visualisation of the WMRA, the defect can be detected even if the modulating phenomenon is not stationary; an interesting simulation using the wavelet packet transform was presented in [21].

When applying the same approach to the noisy signal of figure 7 that includes two simulated defects, the envelope spectrum of the wavelet coefficients highlights two components and their harmonics. These components correspond exactly to the rotational frequencies of the two defective gears, namely 25 Hz and 40 Hz (fig. 12). To further verify this approach, a laboratory experiment was devised as reported below.

6. Experimental validation

6.1. Experiment set up

In order to validate the proposed method, several measurements were collected on defective gears assembled on a laboratory test rig (fig. 13). A normally lubricated gear drive consists of a pinion of 28 teeth assembled on a driving shaft and of a wheel of 35 teeth assembled on a output shaft, where there is also an electromagnetic brake to simulate a variable load. Several defects, localised on the wheel or combined wheel-pinion, were intentionally introduced by a diamond tipped tool with a very high rotation speed. Small, medium, and large defects were all investigated. The small defect corresponds to the gear deteriorated to 25%, the medium defect to 50% and the large defect to 100%. Figure 14 depicts an example of a medium defect on the wheel and a great defect on the pinion; small defects were too difficult to see in photographs.

Measurements were collected in several configurations; namely three rotation speeds (14 Hz, 24 Hz and 33.5 Hz), and four frequency bandwidths ([0-800] Hz, [0-1600] Hz, [0-3200] Hz and [0-6400] Hz). The acceleration signals were measured by a bi-channel analyzer and two accelerometers placed on the rolling bearing of the driving and driven shafts in vertical direction.

6.2. Results and discussion

6.2.1. Case of a defect on the wheel

Figure 15 shows the spectrum [0-1600] Hz of the acceleration signal corresponding to a small defect on the wheel. In this case, the rotation speeds are equal to 33.5 Hz for the pinion (driving shaft) and 26.8 Hz for the wheel (driven shaft), the meshing frequency is thus equal to 938 Hz. No information on the defect is discernible from this spectrum, because the modulations are not identifiable. The cepstrum of figure 16 is not useful in locating the quefrequency corresponding to the rotational frequency of the defective wheel (26.8 Hz), as the component corresponding to 0.037 s ($1/26.8$) is nonexistent.

The proposed method was applied, and db5 was used as the analysis wavelet. In this case, the first detail (D1) [800-1600] Hz covers the meshing frequency. Figure 17 shows the envelope spectrum of the wavelet coefficients. On the spectrum, there is a component corresponding to 26.6 Hz and several of its harmonics. This component corresponds to the rotational frequency of the wheel, thereby, confirming that it is defective. Actually, the variation between this frequency and the theoretical (28.8 Hz) is due to the difference of 2.2 Hz between the programmed and the real rotation speed. Note that the kurtosis of the reconstructed signal is equal in this case to 4.73, which confirms that a shock defect is present.

6.2.2. Case of a combined defect wheel-pinion

For the case of a combined defect of wheel and pinion, figure 18 shows the spectrum [0-3200] Hz, for which the rotation speeds are equal to 24 Hz for the pinion and 19.2 Hz for the wheel. The meshing frequency is thus equal to 672 Hz. The corresponding cepstrum of figure 19 shows a quefrequency of 0.041 s corresponding to the rotational frequency of the pinion (24 Hz) and one of its rhamonics. From this the defect on the wheel is not visible. After the application of the proposed method, the envelope spectra of figures 20 and 21 computed starting from the third detail (D3) [400-800] Hz, highlights two components and several of their harmonics. These two components are equal to 24 Hz and 19 Hz, which are very close to the rotational frequencies of the pinion and the wheel, respectively, which confirms that one is in the presence of a combined defect wheel-pinion. In this case, the kurtosis of the reconstructed signal is equal to 3.41.

7. Conclusions and Summary

In this article, a method of gear defect detection, based on the Wavelet Multi-resolution Analysis and Hilbert transform was proposed. Several WMRA parameters were optimised to obtain reliable results. The kurtosis, being the most sensitive indicator to the shocks, was used as an optimisation and evaluation parameter. The proposed approach uses WMRA for its filtering and de-noising qualities, as well as its time-frequency visualisation, especially for non-stationary signals. The Hilbert transform was used as a demodulation tool of the reconstructed signal obtained by the WMRA. The combination of these two tools seems very useful in the gears defects detection. Through a numerical simulation and an experimental validation, the results show the efficacy of this method compared to other methods, which are based solely on either spectrum or cepstrum analysis. The validity of the proposed approach was proven in various configurations, including the case of an isolated defect and combined wheel-pinion defect. The detection is even evident for incipient defects. Moreover, the proposed

method can be integrated in an automatic monitoring system for machine gearbox defect detection.

8. References

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Figures captions

- Fig. 1. Typical spectrum of an undamaged pair of gears
- Fig. 2. Signal simulating defect on the gear turning at $f_1=25$ Hz
- Fig. 3. Spectrum in the case of small localised defect on the gear turning at $f_1=25$ Hz
- Fig. 4. Cepstrum in the case of localised defect on the gear turning at $f_1=25$ Hz
- Fig. 5. Noisy signal simulating defect on the gear turning at $f_1=25$ Hz
- Fig. 6. Spectrum of the noisy signal
- Fig. 7. Cepstrum of the noisy signal in the case of a localised defect ($f_1=25$ Hz)
- Fig. 8. Noisy signal in the case of a combined defect ($f_1=25$ Hz and $f_2=40$ Hz)
- Fig. 9. Cepstrum of the noisy signal in the case of a combined defect ($f_1=25$ Hz and $f_2=40$ Hz)
- Fig. 10. Envelope spectrum of the wavelet coefficients in the case of one defect ($f_1=25$ Hz)
- Fig. 11. Signal of detail 7 [23.437-46.875] Hz
- Fig. 12. Envelope spectrum of the wavelet coefficients in the case of two defects ($f_1=25$ Hz and $f_2=40$ Hz)
- Fig. 13. Experimental set up
- Fig. 14. (a) Medium defect on the pinion, (b) Great defect on the wheel
- Fig. 15. Spectrum [0 – 1600] Hz, a small defect on the wheel ($N=33.5$ Hz)
- Fig. 16. Cepstrum in the case of a small defect on the wheel ($N=33.5$ Hz)
- Fig. 17. Envelope spectrum of the wavelet coefficients in the case of a small defect on the wheel ($N=33.5$ Hz)
- Fig. 18. Spectrum [0-3200] Hz, a combined defect wheel-pinion ($N=24$ Hz)
- Fig. 19. Cepstrum in the case of a combined defect wheel-pinion ($N=24$ Hz)
- Fig. 20. Envelope spectrum of the wavelet coefficients in the case of a combined defect wheel-pinion ($N=24$ Hz)
- Fig. 21. Zoom [0 – 300] Hz of the envelope spectrum of the wavelet coefficients in the case of combined defect

Tables titles

- Table 1. Signal model data
- Table 2. Choice of the optimal wavelet
- Table 3. Kurtosis of the reconstructed signal for different wavelet mothers

Tables

Sampling period	Number of samples	Meshing frequency F_m	Amplitude of :	Modulating functions frequencies	Modulating functions Amplitudes
0,00016 s	8192	1000 Hz	Fm: 1 2Fm: 0,8 3Fm: 0,5	Single defect: 25 Hz Combined defect: 25 Hz and 40 Hz	First sideband: 0,3 Second sideband: 0,2 Third sideband: 0,1

Table 1. Signal model data

	F_s/F_c				
Sampling rate [Hz]	50	100	200	500	1000
10000	db5	db5	db5	db5	db6
30000	db5	db12	db5	db6	db5
50000	db6	db12	db10	db5	db10

Table 2. Choice of the optimal wavelet [23]

Types of wavelets										
F_c	db5	db6	db4	db10	db12	db20	Bior 3.7	Coif 5	Coif 3	Sym 4
50	5.37	4.89	4.86	4.87	4.99	4.79	4.86	4.54	4.81	4.53
100	5.72	5.33	5.08	5.16	5.34	5.1	5.09	4.93	5.22	4.92
125	4.6	4.51	4.1	4.35	4.25	4.19	4.08	4.17	4.4	4.24
200	5.27	5.18	4.52	4.68	4.77	4.69	4.51	4.69	5.04	4.7
400	4.24	4.08	4.07	3.85	3.99	3.63	3.99	3.75	4.08	4.08

Table 3. Kurtosis of the reconstructed signal for different wavelet mothers

Figures

Fig. 1. Typical spectrum of an undamaged pair of gears

Fig. 2. Signal simulating defect on the gear turning at $f_1 = 25 \text{ Hz}$

Fig. 3. Spectrum in the case of small localised defect on the gear turning at $f_1=25$ Hz

Fig. 4. Cepstrum in the case of localised defect on the gear turning at $f_1=25$ Hz

Fig. 5. Noisy signal simulating defect on the gear turning at $f_1=25$ Hz

Fig. 6. Spectrum of the noisy signal

Fig. 7. Cepstrum of the noisy signal in the case of a localised defect ($f_1=25\text{Hz}$)

Fig. 8. Noisy signal in the case of a combined defect ($f_1=25\text{Hz}$ and $f_2=40\text{Hz}$)

Fig. 9. Cepstrum of the noisy signal in the case of a combined defect ($f_1=25\text{Hz}$ and $f_2=40\text{Hz}$)

Fig. 10. Envelope spectrum of the wavelet coefficients in the case of one defect ($f_1=25\text{Hz}$)

Fig. 11. Signal of detail 7 [23.437-46.875] Hz

Fig. 12. Envelope spectrum of the wavelet coefficients in the case of two defects ($f_1=25\text{Hz}$ and $f_2=40\text{Hz}$)

Fig. 13. Experimental set up

Fig. 14. (a) Medium defect on the pinion, (b) Great defect on the wheel

Fig. 15. Spectrum [0 – 1600] Hz, a small defect on the wheel ($N=33.5$ Hz)

Fig. 16. Cepstrum in the case of a small defect on the wheel ($N=33.5$ Hz)

Fig. 17. Envelope spectrum of the wavelet coefficients in the case of a small defect on the wheel (N=33.5 Hz)

Fig. 18. Spectrum [0-3200] Hz, a combined defect wheel-pinion (N=24 Hz)

Fig.19. Cepstrum in the case of a combined defect wheel-pinion (N=24 Hz)

Fig. 20. Envelope spectrum of the wavelet coefficients in the case of a combined defect wheel-pinion (N=24 Hz)

Fig. 21. Zoom [0 – 300] Hz of the envelope spectrum of the wavelet coefficients in the case of a combined defect