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<td><strong>Authors(s)</strong></td>
<td>James, C. S.; Birkhead, A. L.; Jordanova, A. A.; O'Sullivan, J. J.</td>
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<tr>
<td><strong>Publication date</strong></td>
<td>2004</td>
</tr>
<tr>
<td><strong>Publication information</strong></td>
<td>Journal of Hydraulic Research, 42 (4): 390-398</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>Taylor and Francis</td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/4099">http://hdl.handle.net/10197/4099</a></td>
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<tr>
<td><strong>Publisher's statement</strong></td>
<td>This is an electronic version of an article published in Journal of Hydraulic Research, 42 (4) 2004-01, pp.390-398. Journal of Hydraulic Research is available online at: <a href="http://www.tandfonline.com/doi/abs/10.1080/00221686.2004.9641206">http://www.tandfonline.com/doi/abs/10.1080/00221686.2004.9641206</a></td>
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<td><strong>Publisher's version (DOI)</strong></td>
<td>10.1080/00221686.2004.9641206</td>
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Flow Resistance of Emergent Vegetation

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Abstract

Conventional resistance equations (such as those of Manning, Chézy and Darcy-Weisbach) are inappropriate for flow through emergent vegetation, where resistance is exerted primarily by stem drag throughout the flow depth rather than by shear stress at the bed. An alternative equation form is proposed, in which the resistance coefficient is related to measurable vegetation characteristics and can incorporate bed roughness when this is significant. Equation performance is confirmed by comparison of predicted and measured stage-discharge relationships for flow through artificial cylindrical stems, and by comparison of calibrated and measured drag coefficient values for natural vegetation.

Introduction

Emergent vegetation is a common feature in wetlands and rivers, and has a strong influence on physical and biological processes. Management of vegetated waterways requires the ability to relate these processes to the discharge regime. This entails establishing the relationship between discharge and hydraulic variables, such as velocity and flow depth, through resistance equations.

Conventional resistance equations, such as those of Manning, Chézy and Darcy-Weisbach account for resistance arising from boundary shear, and the resistance coefficients can be related
to the size of roughness elements. In natural rivers, resistance arises also from a variety of other energy loss mechanisms associated with channel characteristics, including vegetation. The Manning equation has become the most popular for natural rivers, with the resistance coefficient, \( n \), constituting a lumped parameter to account for all energy loss influences and estimated largely on the basis of qualitative descriptions and “judgement” (e.g. Soil Conservation Service, 1963). More fundamental approaches for estimating overall resistance coefficient values that account for the drag force on vegetation stems have been proposed (e.g. by Petryk and Bosmajian, 1975).

Use of the conventional equations for vegetated channels has been criticized because they really apply to situations where flow is resisted by boundary shear and not by drag exerted through the flow depth (e.g. Kadlec, 1990). In the latter situation velocity is essentially uniform over the flow depth (Lindner, 1982), rather than being depth-dependent as in the former. The consequent dependence on depth of average velocity implied by these equations requires compensation through depth-dependent resistance coefficients in vegetated channels. Manning’s \( n \), for example, has been shown to vary with the product of average velocity (\( V \)) and hydraulic radius (\( R \)) (as discussed by Smith et al, 1990). Although relationships between \( n \) and \( VR \) have been established, this approach is not satisfactory because \( VR \) does not uniquely specify a flow condition, and the \( n-VR \) relationship is not independent of slope (Smith et al, 1990). Kadlec (1990) also points out that flow through vegetation is often transitional between laminar and turbulent, for which Manning’s equation does not apply, even in unvegetated channels.

To address these difficulties, a more general form of resistance equation has been suggested (Turner and Chanmeesri, 1984; Smith et al, 1990; Kadlec, 1990), viz

\[
q = aS^b y^c
\]  
(1)

in which \( q \) is the unit width discharge, \( y \) is the flow depth, \( S \) is the bed slope, and \( a, b \) and \( c \) are empirical parameters that depend on vegetation type and flow condition. Kadlec (1990) proposed some rationale for the exponent and coefficient values, attributing variation of \( c \) to vertical variation in vegetation density and bed topography. He suggested that \( b \) should be 1.0 for stem Reynolds numbers in the laminar range and 0.5 for the turbulent range.
More fundamental analyses of flow through emergent vegetation have been presented by Thompson and Roberson (1976) and James et al (2001). These elucidate the resistance phenomena quite rigorously, but require considerable computational effort and are not suitable for direct practical application.

In this paper we propose a simple equation for flow through emergent vegetation that has an appropriate form and a rational basis. It is developed from a simple force balance analysis similar to that of Petryk and Bosmajian (1975), and is confirmed through experimentation with both idealized and reasonably realistic conditions.

**Experimental Investigation**

An experimental program was carried out to investigate the influence of vegetation characteristics on resistance under emergent conditions. Tests were carried out with artificial stems in two different flumes to assess the influences of stem density and flow depth, and tests on real reed stems were carried out in one of the flumes. Resistance through vegetation is dominated by stem drag, which depends strongly on the morphology of the stems. Drag coefficient values were therefore measured for single stems with different foliage conditions. The experimental conditions are summarized in Tables 1 and 2, and the procedures and results are described in the following paragraphs. Further details are presented by James et al (2001).

**Series A Experiments**

Experimental Series A was conducted to establish the effects of stem density on flow resistance under different hydraulic conditions determined by bed slope and discharge. The experiments were done in a 0.10 m wide, 3.0 m long, glass-sided tilting flume, with the bed roughened with a layer of angular, 2.4 mm to 4.8 mm diameter sand. Uniform flow was ensured by adjustment of a downstream weir. Discharge was measured volumetrically and water surface levels were measured with a pointer gauge. Vegetation stems were simulated using round, 5 mm diameter, rigid steel rods arranged in a staggered grid pattern with equal longitudinal and transverse
spacings. Three different stem spacings and two different bed slopes were used, and a range of discharges tested for each condition (Table 1). The effective roughness of the bed was established without stems (Tests A1). Darcy-Weisbach $f$ values of 0.105 and 0.110 were determined for the greater and lesser discharges respectively, using the side-wall correction procedure of Vanoni and Brooks (1957). A representative value of effective bed roughness ($k_s$) was determined as 0.0125 m from the Colebrook-White equation.

Tests A3, A4 and A5 were conducted with the same bed slope but different stem spacings. Flow resistance (represented by Manning’s $n$) increased significantly with stem density, and also varied significantly with flow depth (Fig.1). (In this analysis $n$ was calculated using the flow depth in place of the hydraulic radius, implying an assumption that the resistance afforded by the glass side walls was relatively insignificant). Tests A2 and A3 were conducted with the same stem density (25 mm spacing), but different bed slopes, showing that slope has a small effect on resistance, with $n$ being slightly lower for the steeper slope (Fig. 1). This could reflect a response to a change in velocity, Reynolds number, or stem drag coefficient. (The $n$ value for the lowest measured depth for Test A2 is clearly erroneous, and is omitted from further consideration).

**Series B Experiments**

Further flow resistance data were obtained from an associated study of sediment movement through stems (James et al, 2001) (Series B). The experiments were conducted in a 0.38 m wide tilting flume with a mobile bed of sand with a mean diameter of 0.45 mm. Bed slope was a dependent variable and experiments were run until equilibrium slopes were established. Uniform flow was ensured by a downstream weir, and water levels were measured on scales fixed to the glass sides of the flume. Discharge was measured using a V-notch weir below the flume outlet and controlled by valves in the supply line. The stems were as used in Series A, but with only the 25 mm spacing. Bed roughness could not be determined because mobile bed experiments were only conducted with the stems in place. The full experimental procedure is described by James et al (2001) and the tests relevant to resistance are listed in Table 1. It is not possible to compile stage-discharge relationships from the Series B data because of the variable slope.
(ranging from 0.0118 to 0.0184 with an average of 0.0145). The resistance can, however, be represented by Manning’s $n$, which shows variation with flow depth similar to the Series A results (Fig.1), and consistency with the results of Test A2 for which the slope was similar.

**Series C Experiments**

A series of experiments were carried out using harvested reed (*Phragmites australis*) stems in the 0.38 m wide flume. The experimental set-up was similar to that for the Series B experiments, but flow depth was measured using 6 stilling pots connected to tapping points at 1.0 m spacing along the centre of the flume. The reeds were secured in a 0.05 m thick layer of 6 mm to 7 mm angular stones, by a wire mesh stretched between wooden battens spanning the flume width at regular intervals. The stems were arranged in a staggered pattern with centre spacings of 60 mm longitudinally and 90 mm transversely. The first tests (C1) were conducted with fully foliated stems, as harvested. For Tests C2 the leaves were stripped off, and for Tests C3 the stems were removed completely, to obtain an estimate of the resistance of the bed on its own. The variations of Manning’s $n$ with flow condition are presented in Fig. 2.

**Stem Drag Measurements**

Drag force tests were carried out in a 24 m long, 0.915 m wide, horizontal flume. A length of the stem to be tested was secured at the lower end of a rectangular frame mounted in a pivoting support above the water level. The force required to balance rotation of the frame under the influence of drag on the stem was used to calculate the drag force by moment equilibrium. Flow velocities were measured with an electromagnetic flow meter at three locations at the stem level, and the average value used in the calculation of drag coefficient. The stems tested included the 5 mm round rod used in test Series A and B, as well as two reed (*Phragmites australis*) and one bulrush (*Typha capensis*) stems (Table 2). In Tests D6 to D9, reed 2 was progressively stripped of foliage. First, the stem was tested with all leaves and branches (Test D6), then with just 6 leaves (Test D7), then with 3 leaves (Test D8) and finally with only the bare stem (Test D9). The foliage areas of the stems were measured by tracing the outlines on to squared paper.
The drag force \( F_D \) of a stem is related to local flow velocity \( V \) by

\[
F_D = C_D A \frac{1}{2} \rho V^2
\]  

(2)
in which \( A \) is the stem area projected in the flow direction, \( \rho \) is the water density, and \( C_D \) is the drag coefficient. \( C_D \) depends on the stem size and shape and the Reynolds number \( Re = Vd/\nu \) where \( d \) is the stem diameter and \( \nu \) is the kinematic viscosity of the water, equal to \( 1.14 \times 10^{-6} \) m\(^2\)/s for the measured temperature of about 20°C).

Values of \( C_D \) are commonly presented graphically as functions of \( Re \) (e.g. Albertson et al, 1960). Similar relationships were established for the round rod, and the reed and bulrush stems, by calculating \( C_D \) from equation (2) and \( Re \). For the real stems the projected area was defined by the main stem length and diameter only. The derived values of \( C_D \) and corresponding \( Re \) are plotted in Figs 3 and 4, together with the standard relationship for infinitely long circular cylinders presented by Albertson et al (1960). The measured values for the round rod coincide closely with the standard curve for \( Re > 200 \) (Fig. 3), confirming the reliability of the experiments in this range. The amount of foliage increases the value of \( C_D \) considerably (Fig. 4). For the natural stems, \( C_D \) shows dependence on \( Re \) at higher values than for the cylinders presented in the standard relationship; this could be at least partially attributable to variation of the projected area with velocity, as the leaves deflect. These results are limited, and development of resistance equations based on drag quantification should be accompanied by extensive experimental determination of \( C_D \) for relevant vegetation types.

**Resistance Equation**

In the interpretation of the experimental results, flow resistance was expressed and presented in terms of Manning’s \( n \), because it is the most familiar and widely used representation. This enabled the influence of stem density on flow resistance to be clearly shown. However, the results also show that Manning’s \( n \) for vegetated channels varies very significantly with flow depth, confirming the unsuitability of the equation. A more theoretically justifiable equation form is therefore presented, incorporating a resistance coefficient that can be related to
measurable plant characteristics and does not require full stage-discharge measurements for its determination.

The conventional free surface flow resistance equations can be developed from the balance of forces driving and resisting the water movement. The driving force originates in the downslope weight component of the water, and the resisting force in the shear stress imposed by the boundary. The balance of these forces leads to a dependence of the boundary shear stress on flow depth, and combination of this with an assumed relationship between boundary shear stress and velocity leads to an equation for velocity as a function of flow depth. If the resistance to flow is exerted by stem drag rather than boundary shear, however, the velocity is independent of flow depth. This can be shown by considering steady, uniform flow of a unit width element in stem-dominated flow, and equating the driving force (the downflow weight component of the element) to the resisting force (the sum of drag forces from all the stems within the element), i.e.

\[
g \gamma y \left( L - \frac{m \pi d^2}{4} \right) S = C_D \frac{1}{2} \rho (m \gamma d) V^2
\]  

in which \( \gamma \) is the unit weight of water, \( m \) is the total number of stems within the element, \( L \) is the length of the element, and \( S \) is the slope of the channel.

Expressing the number of stems in terms of density (the number of stems per unit area), \( N \), i.e. \( m = NL \) for unit width, and rearranging, gives an equation for the flow velocity:

\[
V = \frac{1}{F} \sqrt{S}
\]

with

\[
\frac{1}{F} = \sqrt{\frac{2g}{C_D N d} \left( 1 - \frac{N \pi d^2}{4} \right)}
\]

Equation (4) suggests that the exponent of \( S \) in equation (1) should be 0.5, and that flow velocity is independent of depth if the resistance is caused exclusively by stem drag. Flow depth therefore appears in equation (1) for continuity reasons only, and its exponent should be 1.0. The
resistance coefficient defined by equation (5) depends on stem density, stem diameter and drag coefficient. It is expressed in inverse form in order to preserve proportionality of its value with resistance, as with Manning’s $n$ and the Darcy-Weisbach $f$, but unlike the Chézy $C$.

Nuding (1994) used an equation similar to equations (4) and (5), excluding the stem volume term $(1 - \frac{NBd^2}{4})$, based on substitution into the Darcy-Weisbach equation of an expression for $f$ given by Lindner (1982). Tsujimoto and Kitamura (1994) used the Chézy equation with $C$ given by a relationship similar to $1/F$, again ignoring the stem volume influence.

Values of $F$ have been calculated from the Series A experimental results using equation (4), and corresponding theoretical values from the stem characteristics using equation (5) with values of $C_D$ from the standard curve for infinitely long circular cylinders presented by Albertson et al (1960). The measured (points) and theoretical (solid line) values are plotted as functions of flow depth in Fig. 5. The measured resistance coefficient $F$ is clearly much less dependent on flow depth than Manning’s $n$ (cf. Fig. 1), becoming constant as flow depth increases. The flow depth at which it becomes constant increases with stem spacing, indicating that equations (4) and (5) become more reliable as stem density increases and the contribution of bed shear to resistance becomes small. The theoretical values of $F$ agree remarkably well with the measured values once they become constant. Equation (4) with $F$ given by equation (5) therefore works well with the ideal stems and arrangements used in the experiments for high stem densities and relatively deep flows.

For low flow depths, sparse stem densities and very rough boundaries the influence of bed shear on overall resistance can be expected to be important, as reflected by the increasing values of measured $F$ with decreasing flow depth in Fig. 5. Under these conditions an equation that accounts for bed shear is therefore necessary. In the absence of stems, the shear stress increases linearly with depth to a maximum value at the bed, where it is balanced by the shear stress imposed by the bed. For unit width within a wide uniform flow the bed shear is given by

$$\tau_0 = \gamma y S \quad (6)$$

In the presence of stems, some of the downslope weight component of the flow is carried by the
stems, and the force resisted by bed shear is reduced. If it is assumed that this reduction can be represented by the total stem drag force (right hand side of equation (3)) divided by the plan area of flow, the actual bed shear can be expressed as

$$\tau_0 = \left(1 - \frac{N\pi d^2}{4}\right)\rho S - C_D \frac{1}{2} \rho N y d V^2$$  \hspace{1cm} (7)$$

The conventional free surface flow resistance equations are based on an assumed proportionality between boundary shear and average flow velocity (e.g. Henderson, 1966), i.e.

$$\tau_0 = a\rho V^2$$  \hspace{1cm} (8)$$

If it is assumed that this proportionality holds in the presence of stems, in terms of the reduced bed shear given by equation (7), an equation for velocity can be derived which accounts for both bed shear and stem drag. Combining equations (7) and (8) gives

$$a\rho V^2 = \left(1 - \frac{N\pi d^2}{4}\right)\rho S - C_D \frac{1}{2} \rho N y d V^2$$  \hspace{1cm} (9)$$

Equation (8) can be recast as the Darcy-Weisbach equation, so that the parameter $a$ can be represented by $f/8$. Incorporating this in equation (9) and rearranging, gives the velocity under the influence of bed shear and stem drag, i.e.

$$V = \frac{1}{F_f} \sqrt{S}$$  \hspace{1cm} (10)$$

with

$$\frac{1}{F_f} = \sqrt{\left(1 - \frac{N\pi d^2}{4}\right)\rho S \left(\frac{f}{8} + C_D \frac{1}{2} N y d\right)}$$  \hspace{1cm} (11)$$

In practical applications there may be a preference for describing bed resistance in terms of Manning’s $n$ rather than $f$. In this case the bed resistance term in equation (11) can be replaced
by the corresponding Manning formulation, i.e.

\[
\frac{f}{8} = \frac{gn^2}{y^{3/2}}
\]  

(12)

The analysis leading to equations (10) to (12) is essentially the same as that presented by Petryk and Bosmajian (1975), which they extended to formulate an expression for Manning’s \( n \) to account for the influence of stems. We believe that the form of equations (4) and (10) is preferable to Manning’s because it does not require the resistance coefficient to vary with flow depth to compensate for the unrealistic variation of velocity with flow depth implied by Manning’s equation.

Values of \( F_f \) have been calculated (by equation (11) for the Series A experimental conditions, and are plotted (as the broken lines) together with the measured and calculated values of \( F \) on Fig. 5. In these calculations, \( f \) was determined using the Colebrook-White equation with \( k_s = 0.0125 \) m, as determined from Test A1. Again, \( C_D \) was estimated using the standard relationship for infinitely long cylinders presented by Albertson et al (1960). It can be seen that equation (11) describes the combined resistance of stems and bed roughness realistically. The increase of resistance coefficient with decreasing flow depth is reproduced well in trend, although not as well in magnitude. The resistance for the most dense stem arrangement is not predicted accurately, particularly at low flow depths. This may be a reflection of underestimation of \( C_D \) using the standard relationship with \( Re \) for infinitely long, single stems. The local approach velocity associated with drag is significantly different from the average velocity at high stem densities (Li and Shen, 1973), and additional drag associated with surface distortion would be expected to contribute significantly at low flow depths.

**Equation Confirmation**

The performance of the proposed equations and resistance coefficients has been assessed by comparison of measured and predicted stage-discharge relationships for the Series A, B and C experiments, and some results for real bulrush (\textit{Scirpus validus}) plants obtained by Hall and Freeman (Waterways Experiment Station, 1994).
For the Series A and B conditions the drag coefficient was estimated from the standard curve for circular cylinders of Albertson et al (1960). Predicted stage-discharge curves for the Series A experiments with $S = 0.002$ are compared with the measured values in Fig. 6. Although the errors are fairly large (averaging 23.8% using $F$ and 13.5% using $F_f$), with discharge almost invariably overpredicted, the slopes of the curves are accurately reproduced, implying that the forms of the equations are sound. As expected, the predictions of equations (4) and (10) are very similar where stem density is high and the bed shear contribution relatively small (Test A3), but diverge as the stem density decreases (Tests A4 and A5) with equation (10) performing better.

Discharges have also been predicted for the Series B experiments. Only equation (4) could be applied in this case, because the roughness of the bed was not measured. The average absolute error for all the experiments was 9.64%, with a standard deviation of 4.26%, confirming reasonable performance. As in most of the Series A applications, the discharge was always overpredicted by equation (4). This is unlikely to be because the bed resistance component was neglected, because of the close similarity of equation (4) and (10) predictions for the same stem density in Test A3. It appears, therefore, that the stem drag is underestimated by using the standard $C_D(Re)$ relationship, with $Re$ in terms of the average velocity. It should be noted that the bed slopes in the Series B tests were much higher than for Tests A3 to A5, and the performance of equation (4) for both series confirms its reliability over a wide range of slopes.

Assessment of the equation performance with natural stems is not as easy, because the drag coefficient values are not as well defined. The approach followed is to fit the equations to measured stage-discharge values by adjusting the value of $C_D$ in the resistance coefficient. These $C_D$ values are then compared with single stem values measured in test Series D.

This approach was followed using data obtained by Hall and Freeman (Waterways Experiment Station, 1994) for a dense stand of bulrushes ($Scirpus validus$) grown over a length of approximately 15 m in a 1.2 m wide concrete channel. Tests were conducted at two different growth stages (with stem densities of 403/m$^2$ and 807/m$^2$ and stem diameters of 7.0 mm and 7.6 mm) and two controlled tailwater conditions. No direct measurements of stem drag were made. The stage-discharge data presented in Table 3 were derived from their reported results. The values of $C_D$ required to reproduce these results with equations (4) and (5) are also listed in Table
3, and are plotted together with the values measured for single reed stems with different foliage states in Fig. 7. The fitted values conform reasonably well with the measured values in terms of both magnitude and trend with $Re$. The differences in values for the three test conditions are not clear, but could reflect a vertical variation of foliage density. The estimation of representative gradient for the nonuniform flow in the experiments could also account for some uncertainty.

The same approach was followed using the test Series C stage-discharge data. In this case, the stem density was low enough for the bed shear to contribute to overall resistance for the lower flow depths ($Ny < 50$; see criterion to follow). Equations (10), (11) and (12) were therefore used, with an average value of $n = 0.0223$ as determined from the results of Test C3. The calibrated values of $C_D$ for foliated (Test C1) and defoliated (Test C2) stems are compared with the values measured for single reed stems with different foliage states in Fig. 7, again showing reasonable agreement. The spread of the values across the single stem curves is related to the variation of foliage density. For the stems used in Test C1, the lowest leaf was attached to the stem about 50 mm above the bed and the subsequent spacing between attachment points was between 150 and 200 mm. The number of leaves attached below the water surface would therefore have increased from 0 at the lowest flow depth tested up to between 3 and 4 for the highest.

These applications suggest that the form of the proposed equations is appropriate, i.e. the average flow velocity does not depend on depth, and the resistance coefficient can be determined from the stem density, diameter and drag coefficient. Variations of the resistance coefficient are associated with changes in the value of drag coefficient with foliage density and deflection, and Reynolds number. Variation of the resistance coefficient with depth to compensate for its appearance in the resistance equation is avoided.

**Equation Application**

Application of the proposed resistance equations requires estimation of the vegetation density ($N$), stem diameter ($d$), drag coefficient ($C_D$), and - where appropriate - the bed resistance in terms of $f$ or $n$. There are many recommendations for estimating $f$ or $n$ through descriptions of surface roughness, either qualitatively or more objectively in terms of the effective roughness, $k_s$ (Chow,
1959; Henderson, 1966). James et al (2001) showed that discharge predictions are relatively insensitive to estimation of stem diameter, but very sensitive to estimates of drag coefficient and stem density. Sensitivity of depth prediction for a given discharge is considerably less, by virtue of the typical form of the stage-discharge relationship. Stem density can be measured quite reliably in the field. Determination of appropriate values of $C_D$ is onerous, but can be done using sample stems in the laboratory; this is preferable to collecting stage-discharge data in the field, which is required for reliable determination of Manning’s $n$.

Repetition of the stage-discharge predictions for the Waterways Experiment Station (1994) November tests excluding the stem volume term $(1 - N \frac{Bd^2}{4})$ in equations (5) and (11) resulted in an average absolute error over the full range of flows of 0.85%, indicating that this term can be neglected to simplify the equation with no significant loss in accuracy.

The deviations between $F$ and $F_f$ shown in Fig. 6 imply that ignoring bed shear resistance is acceptable for some conditions but not for others. The conditions where bed resistance becomes important is dependent on (at least) the flow depth and the stem density. The error in specifying the resistance coefficient as $F$ rather than the more complete $F_f$ increases rapidly with the product $Ny$ below a fairly well defined threshold (Fig. 8). (The plot is in terms of $-(F - F_f)F_f$, as the actual error will always be an underestimate). As a rough guide, it would appear that bed resistance should be accounted for (i.e. through equation (11)) if the value of $Ny$ is less than about 50. The influence of bed resistance might also be expected to depend on $f$, $C_D$, and $d$, but their inclusion led to less satisfactory criteria in the range of conditions represented by the Series A experiments.

**Conclusions**

Conventional resistance equations, such as Manning’s, are inappropriate where the dominant resistant force arises from stem drag rather than bed friction. Equations (4) and (5) provide a rational alternative form, in which average flow velocity is (correctly) independent of flow depth and proportional to the square root of channel gradient. The resistance coefficient ($F$) depends on the diameter, density and drag coefficient of the stems. Sensitivity analysis has shown that accurate determination of stem density and drag coefficient are essential, but that stem diameter
is a relatively insensitive parameter. Equations (10) and (11) allow the influence of bed friction to be included, through the Darcy-Weisbach $f$ or Manning’s $n$, which is appropriate under certain conditions (provisionally represented by the product of stem density and flow depth ($N_y$) being below a threshold of approximately 50). Deviations of flow conditions from hydraulically rough (associated with normal use of the Manning equation) are accounted for by the dependence of drag coefficient on Reynolds number.

The derived resistance formulations perform well for simple cylindrical stems in regular arrangements, and realistically for natural conditions. The formulations therefore constitute a sound basis for development for practical application. In practice, the resistance coefficient ($F$ or $F_f$) may be determined directly from field stage-discharge data (as is done for Manning’s $n$), in which case it will exhibit significantly less variation with flow depth than Manning’s $n$. It can also be determined from measurable substrate and vegetation characteristics: $f$ or $n$ from conventional sources, stem density and diameter from field data, and drag coefficient from laboratory tests. At present, few measurements of drag coefficient are available for natural vegetation, but these are easier to determine than lumped resistance coefficients.

**Acknowledgements**

Most of the work presented was funded by the Water Research Commission, whose permission for its publication is gratefully acknowledged. The flume tests with reed stems were carried out during a study visit by J O’Sullivan, funded by the University of Ulster and the Centre for Water in the Environment, University of the Witwatersrand. The stem drag measurements were taken by Matthew Wolstenholme.

**Notations**

- $A$ projected area of stem
- $a$ coefficient in general form of resistance equation; coefficient in bed shear equation
- $b$ exponent on slope in general form of resistance equation
C  Chézy resistance coefficient
$C_D$  drag coefficient
$c$  exponent on flow depth in general form of resistance equation
$d$  stem diameter
$F$  resistance coefficient accounting for stem drag in proposed resistance equation
$F_D$  drag force on stem
$F_f$  resistance coefficient accounting for stem drag and bed shear in proposed resistance equation
$f$  Darcy-Weisbach friction factor
$k_s$  effective bed roughness height
$m$  number of stems in flow element
$N$  number of stems per unit area
$n$  Manning’s resistance coefficient
$q$  unit width discharge
$R$  hydraulic radius
$Re$  Reynolds number
$S$  channel slope
$V$  average flow velocity
$y$  flow depth
$(\rho)$  unit weight of water
$\nu$  kinematic viscosity of water
$D$  density of water
$J_o$  bed shear stress

References


VANONI, V.A. and BROOKS, N.H. (1957), Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams, Sedimentation Laboratory Report No. E68, California Institute of Technology, Pasadena, California, USA.

Table 1  Experimental conditions

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<th>Test</th>
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</tr>
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<td>0.00421</td>
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<td>0.0015</td>
<td>1.424 - 6.448</td>
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<td>3.023 - 15.178</td>
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</table>

Table 2  Stem drag experiments

<table>
<thead>
<tr>
<th>Test</th>
<th>Stem Type</th>
<th>Stem Length (m)</th>
<th>Stem Diameter (mm)</th>
<th>Foliage Area (m²)</th>
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<tbody>
<tr>
<td>D1</td>
<td>round</td>
<td>0.895</td>
<td>5</td>
<td>-</td>
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<tr>
<td>D5</td>
<td>reed 1</td>
<td>0.880</td>
<td>10.8</td>
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<td>D6</td>
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<td>0.0340</td>
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<tr>
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<td>0.860</td>
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<td>0.0318</td>
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<tr>
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<tr>
<td>D9</td>
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Table 3  Stage-discharge data from the Waterways Experiment Station (1994) experiments

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<th>Tests</th>
<th>Depth (m)</th>
<th>Discharge (m³/s)</th>
<th>Slope</th>
<th>$C_D$</th>
<th>$Re$</th>
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<td>0.0105</td>
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<td>0.057</td>
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<tr>
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<td>0.403</td>
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<tr>
<td>WES III: November 1992 Tests</td>
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<td>0.0028</td>
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</table>
Figure 1  Effect of flow depth, stem density and channel slope on Manning’s $n$

Figure 2  Variation of Manning’s $n$ with flow depth for real stem (Series C) experiments

Figure 3  Drag coefficients for single stems
Figure 4  Effect of foliage on drag coefficient for reed stems

Figure 5  Measured and predicted values of $F$ and $F_f$ for round stems

Figure 6  Measured and predicted stage-discharge relationships for round stems
Figure 7  Comparison of calibrated $C_D$ values for Waterways Experiment Station (1994) and Series C experiments with measured single stem values (Fig. 4)

Figure 8  Error introduced to resistance coefficient by excluding bed shear contribution