Optimal IPO Design with Informed Trading

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Abstract

We characterize optimal IPO design in the presence of distinct adverse selection problems: one affecting the IPO stage and one arising in the after-market. Allocating shares to an investor with superior information in the after-market depresses the share’s value to less informed investors. However, because it facilitates truthful interest report at the IPO stage it increases the expected offer price provided disadvantaged investors are sufficiently unlikely to flip their share. We compare the book-building’s outcome to that of uniform price auction. The auction can enhance the expected offer price only if it systematically allocates a share to the strategic trader.

JEL Classification: G24, G32.

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1 Introduction

Most of the literature considering the optimal allocation and pricing of shares in an initial public offering (hereafter IPO) ignores the outcome of after-market trading and its impact on the offer price. Yet, if an investor has access to superior information in the after-market about the future value of the stock he can use it strategically and make a profit at the expense of less informed investors. The prospect of winning or losing money in the secondary market necessarily affects the investors’ interest for the shares and consequently the offer price.

Incorporating after-market trading in IPO analysis is critical when there is residual uncertainty about the value of the stock which some traders can learn and use. Empirical evidence of strategic trading in the aftermarket is documented in Krigman et al. (1999), Minnigoulov (2001), and more recently in Boehmer et al. (2006). These articles show that flipping is a significant predictor of future stock performance.\(^1\) They establish evidence of after-market trading based on information that has not been revealed during the IPO stage. Allocating shares to investors with access to superior information in the after-market depresses the willingness to pay of investors with no such knowledge and thus the offer price. A priori one would think that strategic traders, when identified, should not be allocated any shares. We prove otherwise.

To our knowledge only Ellul and Pagano (2006) and Busaba and Chang

\(^1\)Flipping is defined as selling in the after-market on the first day of trading the acquired shares during the IPO.
(2005) analyze IPO modeling both the pre-market and the secondary market. Ellul and Pagano (2006) considers a setting with two types of private information, one affecting the primary market and the other arising in the secondary market. They rationalize IPO underpricing as resulting from after-market illiquidity. More precisely, underpricing is required to compensate investors who buy shares in an IPO and potentially liquidate these in the after-market where some traders have superior information. We consider a similar (informational) setting but focus on optimal IPO design. In that respect Busaba and Chang (2005) is closer to this paper. Considering investors who possess private information prior to the IPO, they analyze price discovery under book-building and fixed price offering. With book-building private information is (strategically) revealed during the road show and used for the pricing and allocation of shares. By opposition price discovery takes place in the after-market under fixed price offering. Interestingly they show that unless entry is restricted under book-building, fixed price offering targeting uninformed investors minimizes underpricing. Extracting truthful information during the road show (book-building) is expensive since investors can extract the usual informational rents at the pre-market stage as well as rents in the after-market. As a main contrast, we consider that investors who possess private information at the IPO stage are at disadvantage in the after-market. While their information has been revealed during the road show, some residual uncertainty remains and is privately known and used by other traders.

Our paper characterizes the optimal pricing and allocation of shares in the
presence of distinct adverse selection problems under a book-building mecha-
nism. We then compare this outcome with that of a uniform price auction.

We rely on a simple model where investors taking part in an IPO differ in their
information and thus motives. As in Benveniste and Spindt (1989), some in-
vestors have private information about the company going public at the time of
the IPO. They only participate in the secondary market if they face liquidity
needs. Others access private information about the future value of the stock
once shares are allocated and strategically trade the asset in the after-market.
The after-market is modelled as a competitive dealer market (similar to Glosten
and Milgrom (1985)). Non strategic traders are sufficiently numerous to exhaust
the issue and the underwriter knows the investors’ identity.

We abstract from agency problem between the issuer and underwriter and
assume that the underwriter seeks to maximize the expected offer price sub-
ject to inducing truthful information reports from initially informed investors.
Misreporting strong interest allows these investors to gather informational rents
but depreciates the after-market profits.\(^2\) Indeed the market maker infers in-
formation from the offer price which reflects the expressions of interest. Thus
down-playing interest results in a lower bid price leading the investors to incur a
self-inflected punishment. As liquidity needs become more likely, the incentive
to misreport strong interest weakens. Therefore two solutions emerge depending
on the probability with which investors face liquidity needs. When it is high
enough, the incentive constraint is less of a burden and the underwriter never

\(^2\)This contrasts sharply with Busaba and Chang (2005) where investors benefit from lying
as they transfer their informational advantage to the after-market.
serves the strategic trader. Serving this investor would depreciate the willingness to pay of other investors and thus the offer price. When liquidity needs are sufficiently unlikely allocating a share to the strategic trader helps to punish dishonest investors. Not only does it diminish their odds of getting a share, it also lowers their rents from after-market trading.

Under a uniform price auction the underwriter commits to sell the shares to the highest bidders and keeps no discretion over pricing and allocation. Because investors care about the aftermarket outcome and because bids are observable they can be used to signal information to dealers. In particular the bid price in the after-market increases as good information is inferred from the bids. Two results stand out. First, we prove that a situation where the strategic trader never wins is not an equilibrium. Second, we show that an auction can raise more revenue than book-building provided the strategic trader outbids other investors and thus always gets a share.

Finally, this paper also relates to the literature on auctions with externalities. Indeed, due to aftermarket activity, a bidder’s valuation of the asset is determined by the allocation of shares. The following two features separate our model from this literature. First, all winners bear an externality due to fact that we have a multi-unit setting. Second, externalities are endogenously determined via the allocation rule.

The paper unfolds as follows. The model is presented in the next section. Section 3 and 4 provide the results. We conclude in section 5. Unless in the

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3See Jehiel and Moldovanu (1996) and (2000) and Jehiel et al. (1996).
2 The Model

We consider a setting where the firm going public and its underwriter have the same objective. Their goal is to allocate $Q$ shares (where $Q = 2$) at the highest possible offer price. There are three investors who differ in their information. Two are initially informed (at the IPO stage) and are willing to keep the share unless they face liquidity needs. We refer to these as informed investors. In addition there is one investor who has no relevant information at the IPO stage but has access to information in the aftermarket. He is then in a position to trade his share strategically and we refer to him as the strategic trader. The underwriter knows which investor can trade strategically. Each investor is willing to buy at most one share thus there are enough informed investors to exhaust the issue.

The value of a share is determined as follows. As in Benveniste and Spindt (1989) it reflects the valuation or interest of informed investors. However there is residual uncertainty that these investors cannot learn. As in Pagano and Ellul (2006) this represents news that will eventually become common knowledge but is privately observed by the strategic trader after the IPO. This residual uncertainty is denoted by $\tilde{\epsilon}$, with $\tilde{\epsilon} \in \{-\varepsilon, +\varepsilon\}$ each arising with equal probability. More precisely each informed investor can have a strong or a weak interest in the issue. Let $\tilde{\eta} \in \{-\eta, +\eta\}$ represent an informed investor’s interest (or in-
formation) where $+\eta$ occurs with probability $q$. There are 3 relevant states of nature ($k = 0, 1, 2$). Each refers to the number of informed investors with strong interest $(+\eta)$. In state $k$, the final value of the share ($\bar{V}_k$) is

$$\bar{V}_k = v + \eta_k + \bar{\varepsilon},$$

where $v > 0$, and $\eta_k$ reflects the aggregate interest for the asset ($\eta_2 = 2\eta$, $\eta_1 = 0$ and $\eta_0 = -2\eta$). We assume that $v$ is sufficiently large to ensure that the expected value of the asset to any investor is always positive so that they wish to buy it.

We now describe the IPO stage and highlight the differences between book-building and auction. Under book-building the timing is as follows. First informed investors privately learn their information about the company going public: each draws a value for $\bar{\eta}$ independently. Second the underwriter announces a mechanism specifying the offer prices and the allocation rules for any possible set of messages. Third, given the mechanism the informed investors send a message and the underwriter sets the offer price and allocates the shares. Finally, all investors update their information (about $\bar{\eta}$) based on the price observed and the allocation rule and accept or reject the underwriter’s offer.

Under a uniform price auction the underwriter commits to sell the shares to the highest bidders at a price equal to the highest losing bid. The timing is as follows. First each informed investor learns his information. Second all investors bid. Third shares are allocated and priced. We assume that bidding is
a commitment to buy at a price at most equal to the bid.

Basically the two mechanisms differ in the amount of discretion the underwriter has in allocating the shares. With an auction the underwriter has less discretion.

Once shares are allocated the after-market trading stage begins. It is modelled as a dealer market. Prices are set by competitive risk neutral dealers. At this stage the strategic trader privately observes $\varepsilon$ and, if he has a share, decides to sell it or keep it based on his information. The informed investors are non-strategic players and sell their share with probability $z$, where $z$ is the probability of facing liquidity needs (as in Pagano and Ellul (2006)).

3 IPO under Book-building

We first analyze the outcome of the after-market to evaluate the investors' willingness to pay. Then we solve for the optimal IPO.

3.1 The aftermarket and the investors' valuations

Let $p^b_k$ refer to the bid price when the dealer infers state $k$ has occurred given the offer price and allocation. This is in sharp contrast with Pagano and Ellul (2006) where the state of Nature becomes public information. Given this approach, an informed investor will confuse both the underwriter and the dealer when misreporting interest.

Assume the strategic trader got a share in state $k$ with probability $\alpha_k$ ($k = 0, 1, 2$) and $\alpha_k \in [0,1]$. Consequently the conditional probability of facing an
informed trader for a sell order is given by \( \frac{\alpha_k}{\alpha_k + 2z} \) and thus

\[
p_k^b = \frac{\alpha_k}{\alpha_k + 2z} [v + \eta_k - \varepsilon] + \frac{2z}{\alpha_k + 2z} [v + \eta_k].
\]

Let \( R^I_k(\alpha_k) \) (respectively \( R^S_k(\alpha_k) \)) represents the value of a share to an informed investor (respectively strategic trader) in state \( k \). Subject to truthful reports we have:

\[
R^I_k(\alpha_k) = zp_k^b + (1 - z) (v + \eta_k)
\]

(2) since \( E(\varepsilon) = 0 \). With probability \( z \) the investor has liquidity needs and sells his share at \( p_k^b \), and with probability \( (1 - z) \) he keeps the share. Simplifying we get

\[
R^I_k(\alpha_k) = v + \eta_k - z\varepsilon \frac{\alpha_k}{\alpha_k + 2z}.
\]

(3) Conditional on getting a share in state \( k \) the strategic trader values it

\[
R^S_k(\alpha_k) = \frac{1}{2}p_k^b + \frac{1}{2} (v + \eta_k + \varepsilon).
\]

(4) Indeed, given \( p_k^b \), the strategic trader is better off selling if and only if he gets a bad news (\( \tilde{\varepsilon} = -\varepsilon \)). The above simplifies to

\[
R^S_k(\alpha_k) = v + \eta_k + \varepsilon \frac{z}{\alpha_k + 2z}.
\]

\[4\]We implicitly assume that the strategic trader is able to perfectly learn the state of nature. This assumption will be verified in equilibrium.
Two features appear. First the strategic trader has a higher willingness to pay: \( R_k^I (\alpha_k) < R_k^S (\alpha_k) \). Second, if \( z > 0 \), allocating a share to the strategic trader decreases the informed investors' willingness to pay.

### 3.2 Optimal IPO design

The underwriter wants to maximize the revenue from the IPO which is equivalent, in this situation, to maximizing the expected offer price per share given by

\[
\left[ (1 - q)^2 p_0^o + 2q (1 - q) p_1^o + q^2 p_2^o \right],
\]

where \( p_k^o \) refers to the offer price in state \( k \). The allocation and offer prices must satisfy the voluntary participation and the incentive compatibility constraints.

- Voluntary participation constraints.

We follow Benveniste and Spindt (1989) and assume that the underwriter’s offer is accepted (or rejected) ex-post, that is after information about \( \eta_k \) can be updated. Since the strategic trader has a higher willingness to pay than the informed investor, all voluntary participation constraints are satisfied if and only if

\[
p_k^o \leq R_k^I (\alpha_k) \quad k \in \{0, 1, 2\}.
\]
• Incentive compatibility constraints.

Let \( x_{k'}(m) \) denote the probability that an informed investor gets a share when he sends message \( m \in \{-\eta, +\eta\} \) and when the underwriter infers state \( k' \) with \( k' = 0, 1, 2 \) given the 2 messages received. We have \( x_{k'}(m) \in [0, 1] \) for all \( k' \) and \( m \). An investor with strong interest \((+\eta)\) reveals it provided

\[
qx_2(+\eta) \left[R_2^I(\alpha_2) - p_2^o\right] + (1 - q)x_1(+\eta) \left[R_1^I(\alpha_1) - p_1^o\right] \\
\geq qx_1(-\eta) \left[R_1^I(\alpha_1) + 2\eta(1 - z) - p_1^o\right] \\
+ (1 - q)x_0(-\eta) \left[R_0^I(\alpha_0) + 2\eta(1 - z) - p_0^o\right].
\]

(8)

It is important to mention that, in our setting, when the informed investor lies he misleads the dealer who bases her information on the observed offer price and allocation. As in most asymmetric information problem the constraint ensuring that an investor with low interest reports it truthfully is automatically satisfied and we therefore omit it.

**Proposition 1**: Under symmetric information it is optimal to give all shares to informed investors. Formally we have \( \alpha_k = 0 \) for \( k = 0, 1, 2 \), and offer prices reflect the willingness to pay: \( p_0^o = v - 2\eta, p_1^o = v, \) and \( p_2^o = v + 2\eta \). The expected offer price is given by

\[
P^* = \left[v + 2\eta(2q - 1)\right].
\]

**Proof.** Allocating a share to the strategic trader systematically decreases the willingness to pay of the informed investors and has no benefits. Since the
format of the mechanism imposes that all pay the same offer price, there is no possibility to increase the offer price to the strategic trader’s valuation as informed investors must get at least a unit to exhaust the issue. ■

In the particular case where \( z = 0 \), i.e. when informed investors systematically keep the share, any allocation rules lead to \( P^* \).

We now consider asymmetric information at the IPO stage. The underwriter’s maximization problem is given by:

\[
\max_{\{p_k^0, x_k(\cdot)\}_{k=0,1,2}} \left[ (1-q)^2 p_0^o + 2q (1-q) p_1^o + q^2 p_2^o \right]
\]

subject to (8) and (7). Besides, to exhaust the issue we must have

\[
\begin{align*}
2x_2(\pm \eta) + \alpha_2 &= 2, \\
2x_0(-\eta) + \alpha_0 &= 2, \\
x_1(\pm \eta) + x_1(-\eta) + \alpha_1 &= 2.
\end{align*}
\]

Given that the strategic trader buys at most one unit we also have:

\[
\begin{align*}
2x_2(\pm \eta) &\geq 1, \\
2x_0(-\eta) &\geq 1, \\
x_1(\pm \eta) + x_1(-\eta) &\geq 1.
\end{align*}
\]
Proposition 2: The optimal solution under asymmetric information is such that
- any investor reporting +η gets a share (α₂ = 0 and x₁(η) = 1);
- the strategic trader is allocated one share in states k = 0 and k = 1 provided liquidity needs are sufficiently unlikely.

Formally we have α₀ = α₁ = α* with

\[ \alpha^* = \begin{cases} 
1 & \text{if } z \leq \hat{z} \\
0 & \text{if } z > \hat{z} 
\end{cases} \]

where the value \( \hat{z} \in [0,1] \) is unique and solves

\[ \frac{\varepsilon \hat{z}}{1 + 2\hat{z}} (1 - q) - \eta (1 - \hat{z})q = 0. \]

Offer prices are such that \(^5\)

\[ p^o_2 = R^I_2(0) - \frac{\eta (1 - z) (2 - \alpha^* (1 + q))}{q}, \quad (12) \]
\[ p^o_0 = R^I_0(\alpha^*), \quad p^o_1 = R^I_1(\alpha^*). \quad (13) \]

The expected offer price is then given by

\[ P(\alpha^*, \alpha^*, 0) = q^2 R^I_2(0) + 2q(1 - q) \left[ R^I_1(\alpha^*) - \frac{q}{1 - \eta} \eta (1 - z) (1 - \alpha^*) \right] \]
\[ +(1 - q)^2 \left[ R^I_0(\alpha^*) - \frac{q}{1 - \eta} \eta (1 - z) (2 - \alpha^*) \right]. \quad (14) \]

\(^5\) We consider \( q > 0. \)
Proof. See Appendix.

It is straightforward to understand why $\alpha_2 = 0$ is optimal. A greater $\alpha_2$ forces $p_2^o$ down without facilitating honest interest reports. The values $\alpha_0$ and $\alpha_1$ are determined trading off lower informational rents with higher offer prices.

It is important to remember that the underwriter cannot allocate a share to the strategic trader to take advantage of his higher valuation. Instead this investor is used strategically to lower the cost of truthful information revelation. To satisfy (8) the underwriter can raise the benefit of revealing good news (option 1) or the cost of lying (option 2). More precisely, under option 1 the underwriter sets $\alpha_k = 0 \ \forall k$ and lowers the offer price in state 2. Under option 2 he serves the strategic trader in states 0 and 1. This lowers the informed investors’ willingness to pay in those states (and thus decreases the offer prices) but allows to set a higher $p_2^o$. Which option is best depends on $z$.

As said earlier, serving the strategic trader in state $k = 0, 1$ decreases the offer price in those states and the depreciation increases with $z$. However the offer price in state 2 (given by (12)) increases with $z$ because a greater concern for the aftermarket’s outcome makes misreporting strong interest less attractive. Indeed, by reporting low interest the informed investor who received $+\eta$ misleads the dealer who sets a lower bid price. Thus as $z$ increases option 1 becomes less costly and dominates option 2.

Finally, let us analyze how $\widehat{z}$ varies with exogenous information parameters $\eta$ and $\varepsilon$. We have

$$\frac{\partial \widehat{z}}{\partial \eta} > 0 \text{ and } \frac{\partial \widehat{z}}{\partial \varepsilon} < 0.$$
Informational rents, as shown in (12), are proportional to \( \eta \). The greater \( \eta \) the more \( p^2_0 \) must be lowered to guarantee incentive compatibility. Thus, as \( \eta \) increases allocating a share to the strategic trader (option 2) which permits to balance the increase in rents, is best for a wider range of values of \( z \). The parameter \( \varepsilon \) only affects the offer prices in states 0 and 1 provided \( \alpha^* = 1 \). As \( \varepsilon \) increases, the valuations of the informed investors decrease and so do \( p^0_0 \) and \( p^1_0 \). Thus the greater \( \varepsilon \) the less attractive it is to allocate a share to the strategic trader.

4 Uniform Price Auction

There are several factors that differentiate a uniform price auction from book-building.\(^6\) One important difference has to do with the discretion the underwriter has in allocating and pricing the shares. Assume that the underwriter sells the shares using a uniform price auction (which is the format used on the website openIPO.com). Doing so he commits to sell the shares to the highest bidders and sets the offer price equal to highest losing bid. We inquire whether the underwriter can achieve a higher revenue with such an auction. We maintain the assumption that each investor wants at most 1 share.

Investors must bid possessing only their own information. They do not observe any offer price that would allow them to revise their valuations. Let \( b(y) \) denote a bid from an informed investor with \( y \in \{+\eta, -\eta\} \) and \( b^S \) denote

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\(^6\)For details see Jenkinson and Ljungqvist (2001).
the strategic trader’s bid. We assume that bids are observable at the start of the after-market. This means that investors have the possibility to use their bids to signal their interest. In particular a strongly interested investor can increase the bid price in the aftermarket if he successfully signals his valuation. We search for a separating equilibrium where \( b(\eta) > b(-\eta) \). Besides we are interested in a situation where the strategic trader does not pool with the informed investors, that is \( b^S \neq b(y) \) with \( y \in \{\eta, -\eta\} \).

There are 3 candidates for the type of equilibrium we are looking for:

(i) \( b(\eta) > b(-\eta) > b^S \) where \( \alpha_k = 0 \) \( \forall k = 0, 1, 2 \),

(ii) \( b^S > b(\eta) > b(-\eta) \) where \( \alpha_k = 1 \) \( \forall k = 0, 1, 2 \),

(iii) \( b(\eta) > b^S > b(-\eta) \) where \( \alpha_2 = 0, \alpha_1 = \alpha_0 = 1 \),

We consider that the dealer sets the share’s bid price according to the following beliefs. Let \( \beta(b_1, b_2, b_3) \in \mathbb{R}^3 \) denote a probability vector \( (\beta_0, \beta_1, \beta_2) \) where \( \beta_k \) is the probability that the dealer assigns to state \( k \) given that she observes bids \( b_1, b_2 \) and \( b_3 \). For any given bids, we have

\[
\sum_{k=0,1,2} \beta_k = 1.
\]
Consistent with the equilibrium we must have:

\[
\beta(b(\eta), b(\eta), b^S) = (0, 0, 1), \\
\beta(b(\eta), b(-\eta), b^S) = (0, 1, 0), \\
\beta(b(-\eta), b(-\eta), b^S) = (1, 0, 0).
\]

Given these equilibrium beliefs, and for any out-of-equilibrium beliefs, the following result holds.

**Lemma:** There does not exist a separating equilibrium such that

\[b(\eta) > b(-\eta) > b^S.\]

**Proof.** See Appendix. ■

If an informed investor is guaranteed a share whatever his interest then he is better off bidding \(b(\eta)\). Doing so does not affect the offer price but changes the beliefs of the dealer who mistakenly eliminates state \(k = 0\).

Equilibria (ii) and (iii) require that we define out-of-equilibrium beliefs. We use the above lemma and consider stable out-of-equilibrium beliefs.\(^7\)

Assume the strategic trader deviates and bids any \(b \neq b^S\). When the market maker does not observe \(b^S\) she knows that the strategic trader deviated from the equilibrium path. She sets the price of a sell order considering the following

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\(^7\)See Cho and Kreps (1987) for a definition of stability.
beliefs:

\[
\beta(b(\eta), b(\eta), b) = (0, 0, 1), \forall b \neq b(-\eta),
\]
\[
\beta(b(-\eta), b(-\eta), b) = (1, 0, 0), \forall b \neq b(+\eta),
\]
\[
\beta(b(\eta), b(-\eta), b) = (0, 1, 0), \forall b \neq b(\eta) \text{ and } b \neq b(-\eta).
\]

The value for \(\alpha_k\) is set rationally according to both the above beliefs and \(b\).

Finally

\[
\beta(b(-\eta), b(-\eta), b(\eta)) = (0, 1, 0) \text{ and } \alpha_1 = 1,
\]
\[
\beta(b(\eta), b(\eta), b(-\eta)) = (1, 0, 0) \text{ and } \alpha_0 = 1.
\]

These beliefs are refined considering that the strategic trader has no incentive to lower his bid to \(b(+\eta)\) which in cases (ii) and (iii) is the lowest bid.

Assume now that an informed investor deviates. By observing at least one \(b^S\) the market maker knows that an informed investor deviated. We set

\[
\beta(b(\eta), b^S, b) = (0, 1 - \beta_2, \beta_2),
\]

with \(\beta_2 = 0\) if \(b < b(\eta)\). Finally we have

\[
\beta(b(-\eta), b^S, b) = (1 - \beta_1, \beta_1, 0),
\]

with \(\beta_1 = 0\) if \(b < b(\eta)\). These beliefs are refined considering that an informed
investor with a high valuation has no interest in lowering his bid. The value for 
\( \alpha_k \) is set consistently by comparing \( b^S \) to the remaining bids.

**Proposition 3** Given the beliefs defined above equilibria (ii) and (iii) exist.

We have

\[
p^2_k = v - 2\eta - \mu, \text{ for } k = 0, 1,
\]

where \( \mu = \frac{\varepsilon z}{1 + 2z} \).

Under bids ranking (ii) we have

\[
p^2_2 \in \left[ b, v - \mu + 2\eta \frac{2 - q}{q} \right]
\]

with

\[
b = v + 2\eta - \mu - \min \left\{ \mu + 4 \frac{\eta z}{q} [q(1 - \beta_2) + (1 - q)(1 - \beta_1)], 2\eta \left[ 1 + z - \frac{2z}{q} \right] \right\}.
\]

Under bids ranking (iii) we have

\[
p^2_2 \in \left[ v + \frac{2\eta z}{q}, v + \frac{2\eta}{q} \right].
\]

**Proof.** See Appendix.

These are signaling equilibria in which all but the informed investor with 
a bad signal bid aggressively. Indeed, since they do not systematically pay 
their bids and because they are only concerned about expected profits (not 
knowing the information possessed by other bidders) the strategic trader and
the informed investor with strong interest bid more than their expected value of the asset.

We finally compare revenues. Keep in mind that the revenue from book-building depends on whether \( z \) is greater or less than \( \bar{z} \). Taking this into account we establish the following result. Let \( P^A \) and \( P^B \) refer to the expected offer price from the auction and book-building respectively.

**Proposition 4**

Any equilibrium with bids ranked according to (iii) raises less revenue than book-building.

Under ranking (ii) the auction equilibrium such that

\[
b(+\eta) = v - \mu + 2\eta \frac{2 - q}{q}
\]

can raise a higher revenue than book-building.

When \( q < \frac{1}{2} \)

\[
P^A > P^B \iff z < z',
\]

where \( z' \in \bar{z}, 1 \) and solves

\[
\frac{2 \bar{z}}{1 + 2 \bar{z}} - 2\eta(1 - \bar{z})q = 0.
\]

When \( q > \frac{1}{2} \)

\[
P^A > P^B \iff z < z'',
\]
where $z'' \in ]0, \hat{z}[ \) and solves

$$q \frac{\varepsilon z}{1 + 2z} - \eta(1 - z)(1 - q) = 0.$$

For the particular case $q = 1/2$ we have

$$p^A > p^B \iff z < \hat{z}.$$

**Proof.** See Appendix. ■

In an auction the underwriter gets the lowest possible valuation in states

$k = 0, 1$ where $p^o_k = R^1_0(1)$. The only possibility to outperform the book-

building offer price occurs in state $k = 2$ when both informed investors are

strongly interested. This is so because investors bid more than their valuations

to signal their interest to the dealer. Interestingly the auction can perform

better only if it leads to a radically different allocation of shares.

5 Conclusion

This paper solves for optimal IPO design in the presence of two distinct adverse

selection problems. One, as in the traditional literature, affecting the IPO stage

and the other arising in the after-market. It adds to the traditional literature

by incorporating secondary market trading and its effect on the investors’ will-
ingness to pay for the asset. We can therefore highlight the cost and benefits of

having strategic traders participating in an IPO.
Strategic traders are systematically excluded under symmetric information because they depress the offer price. Indeed, informed investors anticipate that they will be at disadvantage in the secondary market and therefore reduce their willingness to pay for the asset. Strategic traders are beneficial under asymmetric information at the IPO stage. Indeed, provided informed investors are sufficiently unlikely to face liquidity needs, allocating shares to informed traders can increase the expected offer price as it lowers the cost of inducing truthful revelation of information.

As we consider the allocation of shares using a uniform price auction, three features emerge. First, the auction offers the possibility for informed investors to signal their interest to the market maker. As they do not pay their bid and consider only expected revenue, it leads them to bid more than their expected value of the asset. Second, the strategic trader cannot be discarded. And third, the auction can enhance the expected offer price only if it systematically allocates a share to the strategic trader.

The paper also provides a rationale for allocating shares to the so-called flippers and more particularly to informed flippers. Within this strand, Fishe (2002) and Fishe and Boehmer (2000) also consider this issue. However they do not consider asymmetric information and provide a very different rationale.
6 Appendix

Proof of proposition 2.

Consider the maximization problem (9). By adding and subtracting $R_I^1(\alpha_1)$ from $p_0^1$ and $R_I^2(\alpha_2)$ from $p_0^2$ and using the full allocation of the shares as given by (10) we can rewrite the underwriter’s objective as

$$\max 2(1 - q)\pi_0^2 + 4q(1 - q)R_I^1(\alpha_1) + 2q^2R_I^2(\alpha_2) - 2q\Pi(\eta, \eta)$$

(15)

$$-2q(1 - q)(x_1(\eta) + \alpha_1)\left[R_I^1(\alpha_1) - p_0^1\right] - q^2\alpha_2\left[R_I^2(\alpha_2) - p_0^2\right],$$

where $\Pi(x, y)$ is the expected payoff to an informed investor with information $x$ when reporting $y$. Using the fact that (8) binds in equilibrium, we can further simplify the above and get

$$\max 2(1 - q)\pi_0^2 + 4q(1 - q)R_I^1(\alpha_1) + 2q^2R_I^2(\alpha_2)$$

$$-q^2\alpha_2\left[R_I^2(\alpha_2) - p_0^2\right] - 2q(1 - q)(x_1(\eta) + \alpha_1)\left[R_I^1(\alpha_1) - p_0^1\right]$$

(16)

$$-4q\eta(1 - \eta)[qx_1(\eta) + (1 - q)x_0(\eta)] + \Pi(-\eta, -\eta).$$

From this expression it is obvious that $\alpha_2 = 0$ is optimal. Since $(x_1(\eta) + \alpha_1) > 0$ it is optimal to set $(R_I^1(\alpha_1) - p_0^1) = 0$. Moreover to minimize $\pi(-\eta, -\eta)$ and set it to zero we let $p_0^2 = R_I^1(\alpha_0)$. Finally, to minimize

$$[qx_1(\eta) + (1 - q)x_0(\eta)]$$
we set $x_1(+\eta) = 1$ given (11). Using these results the objective function can be rewritten as

$$W(\alpha_0, \alpha_1, 0) = 2q^2R^I_2(0) + 4q(1 - q) \left[ R^I_1(\alpha_1) - \frac{q}{1 - q}(1 - z)(1 - \alpha_1(0)) \right] + 2(1 - q)^2 \left[ R^I_0(\alpha_0) - \frac{q}{1 - q}(1 - z)(2 - \alpha_0) \right].$$

$W(.)$ is convex in $\alpha_0$ and in $\alpha_1$. Thus we necessarily have a corner solution. Evaluating the above expression at $(0, 0, 0), (1, 0, 0), (0, 1, 0)$ and $(1, 1, 0)$ leads to the result in proposition 2.

**Proof of the Lemma.**

Consider a ranking of bids such that

$$b(+\eta) > b(-\eta) > b^S$$

and assume it forms an equilibrium where bidders get non-negative payoffs (or else losing would be best). In equilibrium an informed investor with bad signal receives an expected payoff (denoted $\pi^{-\eta}(b(-\eta))$) equal to:

$$\pi^{-\eta}(b(-\eta)) = qR^I_1(0) + (1 - q)R^I_0(0) - b^S = v - 2\eta(1 - q) - b^S.$$
payoff from deviation is

$$\pi^{-\eta}(b(\eta)) = q [v + 2\eta z] + (1 - q) [v - 2\eta (1 - z)] - b^S$$

$$= v - 2\eta (1 - q) - b^S + 2\eta z > \pi^{-\eta}(b(-\eta)) .$$

**Proof of Proposition 3.**

To ease the exposition let

$$\mu^I(\alpha) = \frac{2\varepsilon \alpha}{\alpha + 2z},$$

and

$$\mu^S(\alpha) = \frac{2\varepsilon}{\alpha + 2z},$$

and

$$\mu = \mu^I(1) = \mu^S(1).$$

Let $$\pi^I(b)$$ denote the expected profit to an informed investor when bidding $$b$$ and let $$\pi^S(b)$$ denote the expected profit to a strategic trader when bidding $$b$$.

Finally let $$p^\beta(\beta(.))$$ denote the expected bid price set by a market maker with beliefs given by $$\beta(b_1, b_2, b_3)$$ and the corresponding rational $$\alpha_k$$:

$$p^\beta(\beta(.)) = \beta_2 \left( v + 2\eta - \frac{\alpha_2 \varepsilon}{\alpha_2 + 2z} \right) + \beta_1 \left( v - \frac{\alpha_1 \varepsilon}{\alpha_1 + 2z} \right)$$

$$+ \beta_0 \left( v - 2\eta - \frac{\alpha_0 \varepsilon}{\alpha_0 + 2z} \right).$$
1- Assume there exists an equilibrium such that $b^S > b(\eta) > b(\omega)$.

In equilibrium we must have

$$\pi^{-\eta}(b(\omega)) = 0.$$ 

Indeed if $b(\omega)$ led to a positive expected payoff then an profitable deviation would be to slightly increase the bid when receiving a bad signal. This would allow him to win with probability 1 instead of 0.5 when $k = 0$ and still pay the same price. Thus, the only possibility is to have

$$b(\omega) = R^I_0(1) = v - 2\eta - \mu.$$ 

Furthermore in equilibrium the expected profits to an informed with strong interest are

$$\pi^{+\eta}(b(\omega)) = {1 \over 2} q \left[ R^I_1(1) - b(\omega) \right] + (1-q) \left[ R^I_1(1) - b(\omega) \right] = {1 \over 2} q(v - \mu) + \eta(2 - q) - {1 \over 2} q b(\omega).$$

To guarantee that $\pi^{+\eta}(b(\omega)) \geq 0$ we must have

$$b(\omega) \leq v - \mu + 2\eta {2 - q \over q}.$$
Finally, for the strategic trader we have

\[ \pi^S(b^S) = q^2(v - b(\eta)) + 2\eta q(2 - q) + \mu(2 - q^2), \]

under (19) \( \pi^S(b^S) > 0 \).

We analyze deviations considering each investor separately.

- **Strategic trader:**

  Provided (19) holds, he has no interest in deviating from his bid. Indeed doing so would potentially lower his probability to win, or else worsen the beliefs of the market maker.

- **Informed investor with strong interest.**

  Clearly he has no incentive to bid below \( b(\eta) \) as it would lower his probability to win and send a bad signal to the market maker. For any \( b > b(\eta) \) he gets

  \[ \pi^{+\eta}(b)\big|_{b>b(\eta)} = q \left[ v - \mu + 2\eta \left( 1 - z(1 - \beta_2) \right) - b(\eta) \right] + 2\eta(1 - q) \left( 1 - z(1 - \beta_1) \right). \]

  To guarantee that

  \[ \pi^{+\eta}(b)\big|_{b>b(\eta)} \leq \pi^{+\eta}(b(\eta)) \]

  \[ \iff b(\eta) \geq v + 2\eta - 2\mu - \frac{4\eta z}{q} [q(1 - \beta_2) + (1 - q)(1 - \beta_1)]. \quad (20) \]
• Informed investor with bad signal.

His expected profit from deviating is given by the following expressions:

\[ \pi^{-\eta}(b) = \pi^{-\eta}(b(-\eta)) = 0 \text{ for all } b \in [b(-\eta), b(+\eta)], \]

\[ \pi^{-\eta}(b(+\eta)) = \frac{q}{2} (v - \mu - b(+\eta)) + \eta z(2 - q), \]

\[ \pi^{-\eta}(b(+\eta)) \leq 0 \iff b(+\eta) \geq v - \mu + 2\eta z \frac{2 - q}{q}. \quad (21) \]

Finally for any \( b > b(+\eta) \) we have

\[ \pi^{-\eta}(b)\big|_{b>b(+\eta)} = 2\eta z \beta_1 (1 - q) + q \left[ v - \mu + 2\eta z \beta_2 - b(+\eta) \right]. \]

Thus

\[ \pi^{-\eta}(b)\big|_{b>b(+\eta)} \leq 0 \]

\[ \iff b(+\eta) \geq v - \mu + \frac{2\eta z}{q} \left[ q \beta_2 + (1 - q) \beta_1 \right] \]

Clearly the best deviation is to set \( b = b(+\eta) \). The lower bound for \( b(+\eta) \) to achieve an equilibrium depends on which prevails between (20) and (21). Nonetheless, whichever prevails is below the upper bound given by (19).
The following bids form an equilibrium

\[
\begin{align*}
b(\eta^-) &= v - 2\eta - \mu, \\
b(\eta^+) &= v - \mu + 2\eta \frac{2 - q}{q}, \\
b^S &= b(\eta^+) + \zeta \text{ with } \zeta > 0.
\end{align*}
\]

2. Assume there exists an equilibrium such that \(b(\eta^+) > b^S > b(\eta^-)\).

We provide a sketch of proof as it is similar to the proof above. As before in equilibrium we must have

\[
\pi^{-\eta}(b(\eta^-)) = 0
\]

which implies

\[
b(\eta^-) = R_0^\epsilon(1) = v - 2\eta - \mu.
\]

Furthermore in equilibrium the expected profits are given by

\[
\pi^{+\eta}(b(\eta^+)) = 2\eta - q b^S + q v.
\]

To guarantee that \(\pi^{+\eta}(b(\eta^+)) \geq 0\) we must have

\[
b^S \leq \frac{2\eta}{q} + v. \quad (22)
\]
Finally
\[ \pi^S(b^S) = 2\mu(1 - q^2) + 4\eta q(1 - q) \geq 0. \] (23)

Under (22) an informed investor with a good signal has no interest in deviating from \( b(+\eta) \). Indeed doing so would potentially lower his probability to win, or else worsen the beliefs of the market maker.

Consider an informed investor with a bad signal. In equilibrium he gets a 0 expected payoff. If he deviates his expected profit becomes

\[ \pi^{-\eta}(b) = 0, \forall b \in [b(-\eta), b^S[. \]

\[ \pi^{-\eta}(b^S) = \frac{q}{2} [v - \mu I (1/2) - b^S], \]

For \( b \in ]b^S, b(+\eta)[ \), we have

\[ \pi^{-\eta}(b) = q [v - b^S]. \]

For any \( b = b(+\eta) \)

\[ \pi^{-\eta}(b(+\eta)) = qv + 2\eta z - qb^S. \]

Finally

\[ \pi^{-\eta}(b) \big|_{b>b(+\eta)} = qv + 2\eta z [\beta_2 q + \beta_4 (1 - q)] - qb^S. \]

Clearly the best deviation is to set \( b = b(+\eta) \). To deter it we need

\[ b^S \geq v + \frac{1}{q} 2\eta z. \] (24)
Comparing the above with (22) the interval for possible $b^S$ is well defined.

To complete the proof we need to guarantee that the strategic trader is not tempted to deviate. His expected profits in equilibrium are given by (23). We then have

$$
\pi^S(b)|_{b \in [b^S, b(\mu + \eta)]} = \pi^S(b^S)
$$

$$
\pi^S(b(\mu + \eta)) \leq \pi^S(b^S) \Leftrightarrow b(\mu + \eta) \geq v + 2\eta + \mu^S(1/3),
$$

$$
\pi^S(b)|_{b > b(\mu + \eta)} \leq \pi^S(b^S) \Leftrightarrow b(\mu + \eta) \geq v + 2\eta + \mu^S(1).
$$

It is trivial to show that any bid below $b^S$ either lead to the same payoff as $b^S$ or worse if he matches $b(-\eta)$.

The following bids form an equilibrium

$$
b(-\eta) = v - 2\eta - \mu^T(1)
$$

$$
b^S = v + \frac{2\eta}{q}
$$

$$
b(\mu + \eta) = \max \left\{ v + 2\eta + \mu^S(1/3), v + \frac{2\eta}{q} + \kappa \right\}, \kappa > 0.
$$

Proof of proposition 4.

- Type (ii) equilibrium.

The expected offer price from the auction is given by

$$
P^A = v - \mu + 2\eta(2q - 1).
$$
Consider any $z < \bar{z}$ for which, by definition:

\[ \mu(1 - q) - \eta(1 - z)q < 0. \]  \hspace{1cm} (25)

We have $P^A > P^B$ when the allocation is $\alpha_{0,1} = 1$ and $\alpha_2 = 0$ if and only if

\[ q \mu - \eta(1 - z)(1 - q) < 0. \]

If $q < 1/2$ then the above inequality holds for all $z \in [0, \bar{z}]$. If $q > 1/2$ then there exists a unique $z''$ such that the above inequality holds for $z \in [0, z'']$.

Consider any $z > \bar{z}$ for which, by definition:

\[ \mu(1 - q) - \eta(1 - z)q > 0. \]  \hspace{1cm} (26)

We have $P^A > P^B$ when $\alpha_k = 0 \ \forall k$, if and only if

\[ \mu - 2\eta(1 - z)q < 0. \]

If $q > 1/2$ then the above inequality never holds. If $q < 1/2$ then there exists a unique $z'$ such that the above inequality holds for $z \in [\bar{z}, z']$. The case $q = 1/2$ is straightforward.
• Type (iii) equilibrium.

The highest expected offer price from the auction is given by

\[ P_A = v + 2\eta (q(1 + q) - 1) - \mu(1 - q^2). \]

It is straightforward to show that when (25) holds so that \( \alpha_{0,1} = 1 \) and \( \alpha_2 = 0 \), then we have \( P^A < P^B \). Similarly when (26) holds so that \( \alpha_k = 0 \; \forall k \) then we have \( P^A < P^B \).
7 References


