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The impact of soil autocorrelation on pile load-displacement behaviour

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ABSTRACT

Foundation design is often controlled by the serviceability limit state and the mobilised settlement under operational conditions is often the governing design condition. Accurate predictions of pile displacements are often hampered by the inherent soil variability. This paper describes an analysis which incorporates the uncertainty in soil properties directly into the pile settlement calculations through a monte-carlo simulation. A t-z analysis is performed which assumes the axial load in a pile is resisted by non-linear uncoupled spring elements, which are dependent on the properties of the surrounding soil. The input soil parameters are modelled by log normally distributed variables. The ultimate friction mobilised by the soil springs is calculated using the Cone Penetration Test based Imperial College pile design approach. CPT data from an Irish dense sand test site is used in the analysis. The springs are assumed to be auto-correlated with depth in a similar manner to the CPT profile, with the degree of correlation defined by the scale of fluctuation. In the final section, the results are discussed in light of previous research which assumed uncorrelated soil properties.

RÉSUMÉ

Conception des fondations est souvent contrôlé par l'état limite de service et le règlement mobilisés dans des conditions opérationnelles est souvent la condition régissant la conception. Des prévisions exactes des déplacements des pieux sont souvent entravés par la variabilité inhérente du sol. Ce document décrit une analyse qui intègre l'incertitude dans les propriétés du sol directement dans les calculs de règlement pile par une simulation de Monte-Carlo. Une analyse est effectuée tz qui assume la charge axiale dans une pile est résisté par des non-linéaire des éléments de ressort découplé, qui sont fonction des propriétés du sol environnant. Les paramètres du sol d'entrée sont modélisés par des variables log distribués normalement. Le frottement ultime mobilisés par le sol ressorts est calculé par l'approche Imperial College conception de pieux, qui intègre la résistance à la pénétration cône. CPT données d'un site irlandais sable dense test est utilisé dans l'analyse. Les ressorts sont supposés être autocorrélées avec la profondeur d'une manière similaire au profil de CPT, avec le degré de corrélation définie par l'échelle de fluctuation. Dans la dernière section, les résultats sont discutés à la lumière des recherches antérieures qui a assumé les propriétés du sol non corrélées.

1 INTRODUCTION

For the majority of deep foundation problems, the design is governed by the serviceability limit state, with deflections limited to a design settlement value, which depends on the proposed use and form of the structure. There are many methods available for determining the settlement of a pile under a given load, including the t-z analysis described in the American Petroleum Institute (API, 2007) design guidelines. However, for a given site the inherently variable soil conditions are likely to yield a range of load-displacement values. In determining the pile load –displacement response it is beneficial to consider the soil property variability directly by modelling the soil properties as random variables and quantifying the impact of this variability on the displacement behaviour. In this paper, the pile response is modelled for a driven pile installed in a dense sand site at Blessington in Ireland. By using monte carlo simulations the inherent variability can be considered in the analysis and a probability of failure can be determined.

2 PILE-SOIL INTERACTION AND LOAD TRANSFER

The axial load-displacement behaviour at the pile head is analysed in this paper by considering the load transfer in the pile using the t-z method. This method, proposed by Matlock et al. (1981), is capable of considering both soil non-linearity and inhomogeneous soil layers. The pile is modelled as an elastic column, while the soil reaction is modelled as independent uncoupled springs, as shown in Figure 1. The soil spring reaction is defined by the shear stress acting at the pile interface, t, which is mobilised at a local displacement, z. The pile base behaviour is defined by a q-z transfer curve, which represents the end bearing reaction q mobilised at a tip displacement, z.

The pile-soil behaviour can be analysed by considering the force equilibrium for the pile element. The governing differential equation for the axially loaded pile is therefore given by equation 1, where E and A are the pile Young's modulus and cross sectional area respectively. C is the pile circumference (or for non circular piles is the cross sectional perimeter) and x is the depth from the pile head. By considering the shear stress mobilised at a displacement z, equation 1 can be rewritten as equation 2, where K is the secant stiffness of the t-z curve.

$$EA\frac{d^2z}{dx^2} - tC = 0 \quad [1]$$

$$EA\frac{d^2z}{dx^2} - KCz = 0 \quad [2]$$

Equation 2 can be readily solved by subdividing the pile into n equally spaced intervals, of length Δx and adopting a finite difference approximation. Using the central differencing technique the pile displacement at each node, z_i, can be represented by equation 3.

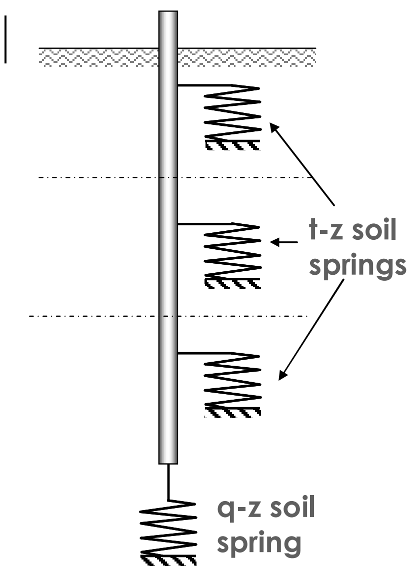


Figure 1: Pile-soil load transfer model

$$z_{i-1} - \left[\frac{CK_i}{EA}(\Delta x^2) + 2\right] z_i + z_{i+1} = 0 \quad [3]$$

The nodes are numbered sequential for each interval starting with node 1 at the pile head and finishing at node n+1 at the pile base. Considering the form of equation 3 it is apparent that two additional displacements have been introduced above the pile head (at n=0) and below the pile base (at n=n+2). Therefore, in order to solve the system of linear equations represented by equation 3, two additional equations are required which are obtained from the boundary conditions at the pile head and the movement of the pile base. This results in n+3 equations, which allows us to solve for the n+3 unknown displacements.

3 SOIL-PILE ULTIMATE RESISTANCE

The Imperial College model (ICP-05) described by Jardine et al. (2005) was used to determine the ultimate shear resistance (t_u) acting at each depth interval and the unit base resistance (q_b) acting on the pile tip. The ICP-05 design method was formulated form the results of field tests using highly instrumented model piles capable of measuring radial effective stresses at the pile-soil interface, which resulted in a more reliable design approach than the traditional vertical effective stress considered in API (2007). This paper considers the impact of soil property variability on the probability of pile failure and the uncertainty in the pile capacity model is not considered in the analysis. The shear stress, t_u, was therefore determined from equation 4 and the unit base resistance, q_b, was determined from equation 5.

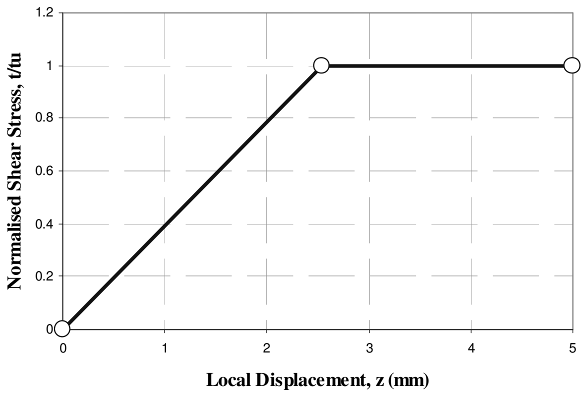
$$t_u = (0.027\, q_c\, (h/R)^{-0.38}\, (\sigma'_{v0} / P_{atm})^{-0.12} + 4G\,(\Delta r / R))\tan\delta \quad [4]$$

$$q_b = q_{c(av)}(1 - 0.5\log(D/0.036)) \quad [5]$$

Where σ'_{v0} is the in-situ vertical effective stress, G is the small strain stiffness, Δr is twice the surface roughness, δ is the interface friction angle at failure and h/R is the distance from the pile tip normalized by the pile radius. The base stress is calculated using the average q_c over a distance 1.5 pile diameters, D above and below the pile tip, termed $q_{c(av)}$.

4 MOBILISATION CURVES

The t-z curves described by the API (2007) guidelines are linear elastic-plastic for piles installed in non-cohesive deposits. A constant stiffness is therefore assumed up to a displacement of 2.54 mm, after which the ultimate resistance is reached, corresponding to the ICP-05 shear stress. The base stress, q-z curves are given by a piecewise curve, which reaches an ultimate resistance at 10% of the pile diameter. These curves are illustrated graphically in Figure 2a and 2b below.



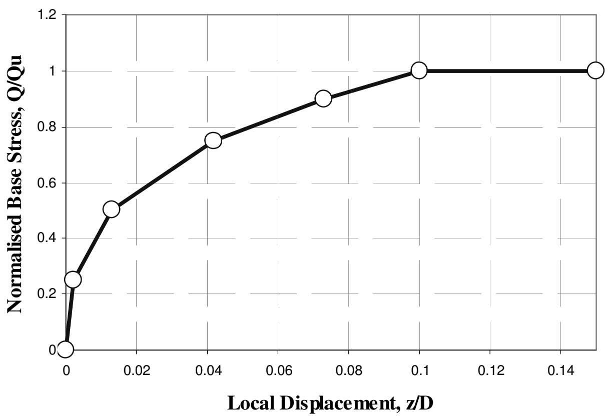


Figure 2a: t-z curves and (b) Q-z curves assumed in the analysis

5 MODELLING PILE RELIABILITY

The pile ultimate shear resistance and mobilisation of that shear response with displacement was modelled statistically using monte-carlo simulations that incorporate the statistical uncertainty of the input soil properties. An arbitrary pile geometry is assumed in this analysis, with a diameter of 750mm. The open-ended steel pile is assumed to be driven to a depth of 10m in the UCD sand research geotechnical research site at Blessington.

5.1 Blessington Site Properties

The Blessington test site contains a deep deposit of over-consolidated, dense uniform sand confirmed by extensive CPT testing and sonic drilling (See Figure 3 and 4) to 20 m below ground level. The geological history of the area has been investigated by Syge (1977). He describes the complex glacial movements that formed the sand deposits. Some layering is evident in excavations with particles grading from silty sand to coarser sand depending on the lake level at the time of deposition. The sand is classified as fine, with D_{50} ranging from 0.1- 0.15 mm. The moisture content measured in a series of boreholes ranged from 10±2%. A total of four deep cone penetration tests, were conducted at the Blessington test site, to depths ranging from 14m to 19.6m. The q_c value of the sand was seen to increase from approximately 10 MPa at ground surface to 30 MPa at 13 m depth. The CPT data plots on a Robertson (1990) classification chart plot as a consistent overconsolidated clean sand to silty sand. Some local pockets of silt are also identified by two of the CPT traces; however the extents of these lenses are less than 0.2m.

Figure 3: Sonic Coring at Blessington Sand Quarry

5.2 Uncertainty in Analysis

The soil properties used to determine the t-z curves at different depth intervals along the pile shaft were assumed to be randomly distributed to account for the inherent spatial variability of the underlying soil strata. The soil property variability was determined directly from statistical analysis of insitu and laboratory test data. The four deep CPTs identified a relatively homogenous layer of dense sand to a depth of 14m. This strata is the primary layer of interest for an ongoing programme of pile test research at the test site. The CPT data contained over 580 data points within the 14m depth of interest and is shown in Figure 4. The average q_c profile was determined by fitting a mean quadratic trend to the data (Doherty and Gavin 2010), which agrees with the recommendations noted by Jaksa, 1995. The standard deviation of the q_c data is seen to increase consistently with depth leading to a coefficient of variation (COV), which varied from 12.1-16.6%. The coefficient of variation is the standard deviation normalised by the mean.

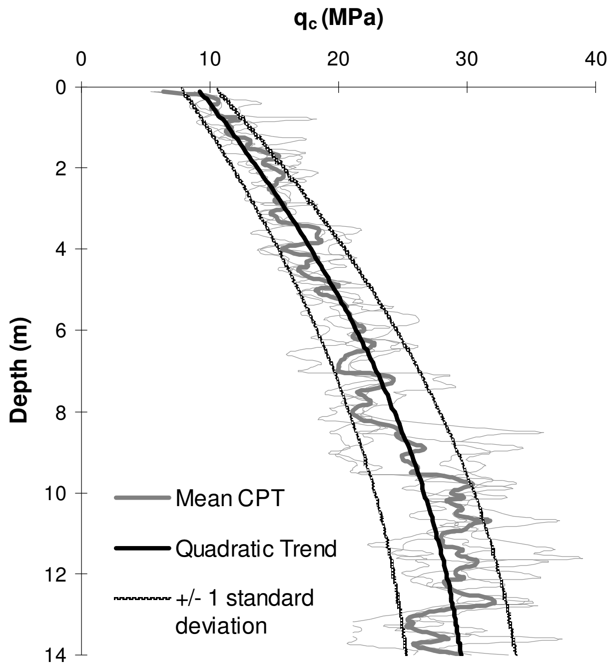


Figure 4: CPT Profiles at Blessington (after Doherty and Gavin, 2010)

The montecarlo simulations were performed using an average COV value of 14% to model the variability of the q_c profile about the quadratic trend shown in Figure 4. The constant COV of 14% led to a depth dependent standard deviation, which is observed to be representative of the measured variability. Further details on the statistical derivation of the q_c variability can be found in Doherty and Gavin (2010). The additional random variables required for the analysis were the unit weight and the interface friction angle, which had measured mean values of 19 kN/m^3 and 27 degrees respectively, which were independent of depth. There was no correlation observed between any of the soil parameters and therefore each soil parameter was considered to be independent of any other parameter in the model.

In addition to the mean soil properties and coefficients of variation, a measure of the autocorrelation with depth is required in order to completely describe the variability of the insitu soil. Any soil property which exhibits random fluctuations due to the inherent spatial variability of the underlying soil will tend to be autocorrelated. This means that the q_c data at any particular depth will be more likely to approximate the q_c data points at adjacent depths. As the distance between two q_c data points increases the correlation between these points decreases, up to a point where they become uncorrelated. The scale of fluctuation, θ, represents a measure of the distance within which a soil property is correlated. The scale of fluctuation was determined by assuming an exponential autocorrelation function, as defined by VanMarcke (1977) using Equation 6.

$$\rho(\tau) = \exp(-\tau / r_0) \qquad [6]$$

Where τ represents the lag distance between two values and r_0 is a constant defined as the distance where the function decays to a value of 1/e and is termed the 'autocorrelation distance'. The scale of fluctuation is then calculated as twice the autocorrelation distance. The range of θ values for the four deep CPT tests conducted at Blessington went from 0.42 to 0.83 m. An average scale of fluctuation of 0.6 was assumed for the q_c data throughout the montecarlo simulations. There was insufficient number of data points to accurately determine the scale of fluctuation for the unit weight and friction angles and therefore a θ value of 0.6 was assumed for all soil properties. It is worth noting that these parameters may have had slightly larger scales of fluctuation, in line with the findings of Phoon and Kulhaway (1999), however there was insufficient data to verify this.

6 MONTE CARLO SIMULATIONS

Multiple realisations of the CPT tip resistance, friction angle, and soil unit weights were generated from underlying lognormal distributions using a simple MATLAB script. The soil profiles were generated by assuming the statistical properties outlined above and adopting an exponential autocorrelation model. The soil data points were generated at 100mm intervals. Note that it is important to simulate soil profiles for depths up to, 1.5 times the diameter of the pile (1.5D) below the pile base so that the qc data can be averaged in the zone of influence effecting the base pressure mobilisation.

The first step in generating the random soil profiles was to generate the autocorrelation matrix from the assumed autocorrelation model, the measured scale of fluctuation and the interval spacing adopted between adjacent points. This matrix is then decomposed into a lower triangular matrix (L) and its transpose, by the process of Cholesky decomposition. The lower triangular matrix is then factored by a normally distributed field, with a zero mean and unit variance, which yields a correlated standard normal field, G. The mean (μ) and standard deviation (σ) are then transformed to the log-normally distributed parameters, μ_{ln} and σ_{ln} using Equation 6.

$$\mu_{\ln} = \ln \mu - 0.5{\sigma_{\ln}}^2 \qquad [6a]$$

$$\sigma_{\ln} = [\ln(1 + COV^2)]^{0.5} \qquad [6b]$$

For each of the soil properties (SP), the log-normally distributed field can then be obtained from equation 7.

$$SP = e^{\mu_{\ln} + \sigma_{\ln} . G} \qquad [7]$$

This process generates soil property profiles with a given mean, COV and scale of fluctuation.

The soil property profiles were subsequently imported into a separate MATLAB programme written to determine the pile load-displacement response using the API-07 code for axial loading. An iterative loop was then used to determine the pile response to a series of applied loads ranging from zero up to the pile ultimate resistance (Q_T), using a series of very small load increments (of 0.01 x Q_T). A global iterative loop was then used to repeat this process for each of the simulated set of soil properties, up to 5000 realisations of the load-displacement behaviour. The first 20 realisations are shown in figure 5.

6.1 Results of Analysis

The first 20 realisations of the load-displacement behaviour is shown in Figure 5. The ultimate capacity is observed to mobilise at 10 percent of the pile diameter or 75mm displacement, in line with the previously described t-z and q-z curves. However, as the controlling shear resistance and base pressure are random variables the resulting ultimate capacity is seen to vary in the range from 3.8 MN to nearly 5MN for the curves shown in Figure 5. This variability becomes particularly evident as the displacement increases, with the standard deviation increasing to a maximum value following 75mm movement. At displacements less than 2.54mm the load is primarily carried by the shaft, which is controlled by the initial linear t-z stiffness. As the plastic yield point is anchored at a fixed deterministic displacement level of 2.54mm, the variability over the initial portion of the load-displacement is limited. This is an assumption of the proposed modelling process, whereas in reality there would be a model error associated with the t-z model that could lead to further uncertainty. This could be explored further through the use of alternative t-z models that are preferably non-linear and therefore would be variable over the initial small loads applied. Further developments would attempt to quantify the model error and obtain a combined pile reliability that could consider the soil variability and the model variability within the same analysis. This paper is concerned only with the likely variability in pile response resulting from the inherent variability of the soil and in this regard the API-07 t-z approach is assumed to be a perfect model for the pile behaviour.

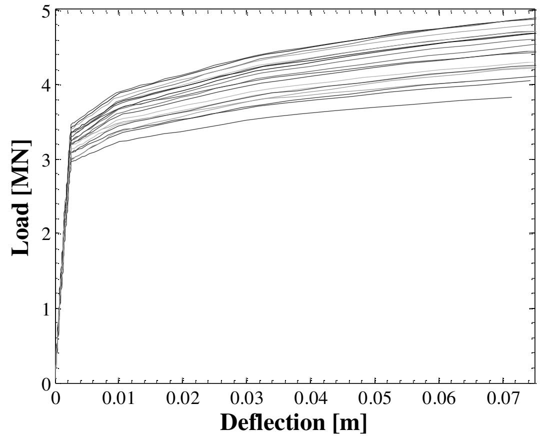


Figure 5: The first 20 Realisations of the Randomly Generated Load-Displacement Response for a pile in Blessington

To analyse the pile reliability under a serviceability limit state (SLS), a displacement tolerance of 25 mm was assumed to act at the pile head. Using this as a SLS failure criterion, the pile capacity histogram shown in Figure 6 was developed. This histogram has a mean value of 3.93 MN for the pile capacity, with a standard deviation of 0.23 MN. By adopting deterministic load values, the number of realisations exceeding each failure load can be summed and the resulting probabilities of failures can be determined by normalising by the total number of realisations. The probability of failures corresponding to a range of applied loads is shown as the solid black line in Figure 7. This essentially represents a complete reliability analysis of a pile installed in dense sand at Blessington.

7 DISCUSSION – IMPACT OF AUTOCORRELATION

The analysis was repeated with the soil properties assumed to be uncorrelated with depth. This is the equivalent of having a scale of fluctuation of zero or saying that the soil property at a given point in the soil mass is independent of the adjacent soil properties. The results of this analysis are shown in Figure 7, where they are compared to the previous correlated results. The shape of the probability of failure curve for the uncorrelated analysis is seen to be slightly more concave than the correlated profile, particularly over the initial load levels up to 3.9MN. The analysis was repeated for piles of different lengths driven in the Blessington test site and similar deviations between the correlated and uncorrelated results were observed for all sets of pile analysis. Considering Figure 7, it can be observed that for a given load, say 3.7MN, the predicted probability of failure will be significantly lower for the uncorrelated profile, at about 0.1, in contrast to the analysis which considered the scale of fluctuation of 0.6m, which yields a higher failure probability of 0.2 for the same load level. This analysis indicates that by ignoring the autocorrelation of the soil properties, the probability of failure is underestimated for a Serviceability Limit State of 25 mm. As the loads and corresponding failure probabilities increase, the deviation between the correlated and uncorrelated failure probabilities also increases considerably. However, the normal target probability of failures assumed by the civil engineering industry would be less than 0.1 and would be most likely in the range 0.01-0.1. As a result, we are concerned mostly with the part of the reliability curve that demonstrated the strongest impact of autocorrelation. Ignoring this component of the statistical variability of the soil could lead to overestimating the pile reliability and ultimately lead to an unconservative design.

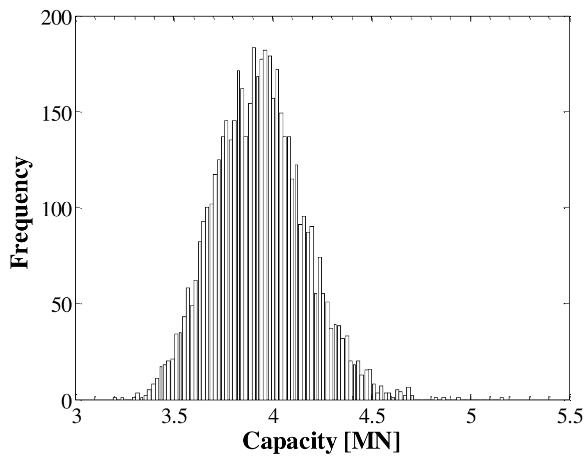


Figure 6: Histogram of the Simulated Capacities at a 25mm displacement limit

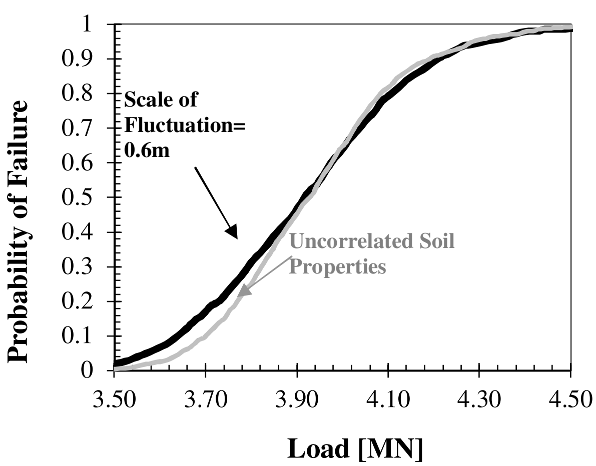


Figure 7: Probabilities of Failures for Different Deterministic Applied Loads

8 CONCLUSIONS

A pile reliability analysis was performed using a monte carlo simulation to account for inherent soil variability. Profiles of soil parameters were generated as a function of depth, which adopted measured mean values, standard deviations and scales of fluctuation. The analysis demonstrated that the reliability based design can easily be performed using a simulation based approach, provided that the soil autocorrelation is also modelled. Comparisons between autocorrelated and uncorrelated simulated soil properties show a tendency for the pile reliability to be over-predicted using an uncorrelated soil profile. The implication is that simpler analysis performed using uncorrelated soil simulations would ultimately lead to an unsafe design.

9 FUTURE RESEARCH

The outline framework described in this paper will be developed to investigate:

- A parametric study to confirm the impact of other factors such as pile diameter, slenderness ratios, different mean soil properties and different degrees of soil variability on the observed impact of soil autocorrelation.
- Using non-linear t-z models, such as those proposed by Fahey and Carter (1993) to replace the linear elastic API models
- Using non-linear base settlement models such as that proposed by Gavin and Lehane (2007).
- Investigating a wider range of soil types to include cohesive materials.

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