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<td><strong>Authors(s)</strong></td>
<td>Doherty, Paul; Gavin, Kenneth</td>
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<tr>
<td><strong>Publication date</strong></td>
<td>2010-05-09</td>
</tr>
<tr>
<td><strong>Publication information</strong></td>
<td>2nd International Symposium on Cone Penetration Testing : volume 3</td>
</tr>
<tr>
<td><strong>Conference details</strong></td>
<td>2nd International Symposium on Cone Penetration Testing (CPT10), Huntington Beach, California, USA, May 9-11, 2010</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>CPT 10</td>
</tr>
<tr>
<td><strong>Link to online version</strong></td>
<td><a href="http://cpt10.com/PDF_Files/3-05dohasr.pdf">http://cpt10.com/PDF_Files/3-05dohasr.pdf</a></td>
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<td><strong>Item record/more information</strong></td>
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Statistical review of CPT data and implications for pile design

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ABSTRACT: Natural soil variability can result in a range of pile capacities at a specific test site. Soil variability is ideally determined using in-situ testing techniques such as the CPT. This paper presents a series of CPT profiles from a dense sand site in Wicklow, Ireland. The q_c value is detrended using a quadratic profile with depth and the scatter in the remaining data is quantified using the coefficient of variation (COV). In addition, the autocorrelation of each CPT trace is quantified using the scale of fluctuation. The measured values from the site, including the trend, COV and scale of fluctuation are subsequently incorporated into a Monte Carlo simulation that generates multiple realisations of the CPT data from an underlying lognormal distribution. These realisations are applied to a simple pile design model to calculate the variability in capacity that arises from the site specific variability. Spatial averaging is shown to be highly influential in reducing the uncertainty in pile resistance, with long piles and small scale of fluctuations demonstrating the lowest COV in the calculated capacity.

1 INTRODUCTION

An ongoing program of research at University College Dublin (UCD) is currently investigating the influence of pile type and installation procedure on the ultimate axial capacity of pile foundations. As part of this research a suitable test site was required to compare the resistance developed by bored, driven and jacked piles, and also to assess the impact of end condition by installing both low displacement open ended steel pipe piles and closed ended displacement piles. The ideal site selection would exhibit uniform ground conditions across the site, with homogenous material yielding consistent strength properties. This would allow direct comparisons between different pile tests, without having to consider the potential impact of the underlying soil conditions.

A potentially suitable sand site was identified at Blessington, Co.Wicklow which was anecdotally described as consistent. Cone penetration tests (CPT) were used to both characterize the underlying deposits and also quantify the in-situ variability. The cone tip resistance q_c soil variability was then applied to the pile capacity problem, which quantified the expected variability in pile resistance. This paper describes the
test site, the statistical properties of the $q_c$ data and the process which allowed the expected variance in pile capacity to be determined.

2 BLESSINGTON TEST SITE

The CPTs presented in this paper were performed as part of the characterization of a geotechnical test site at Blessington, a small village located 25km to the southwest of Dublin, Ireland. The site is within an active quarry, with the underlying deposits of uniform sand confirmed by extensive excavations, as illustrated in Figure 1.

The geological history of the area has been investigated by a number of researchers (eg. Syge 1977) who describe the complex glacial movements that formed the underlying sand deposits. The CPTs were performed in horizontally bedded uniformly graded sand deposited at the bed of a glacial lake, with the stratification clearly visible in Figure 1. The interbedded layers have a particle grading from silty sand to coarser sand depending on the lake level at the time of deposition. The site is heavily overconsolidated from a combination of post depositional glacial processes and more recent excavations at the quarry, resulting in a maximum preconsolidation pressure of 800kPa at ground surface. The sand is classed as fine with $D_{50}$ ranging from 0.1-0.15mm. The moisture content measured in a series of boreholes has a range of 10±2%.

3 CPT DATA AND STATISTICS

A total of four deep cone penetration tests, termed CPT1 to 4, were conducted at the Blessington test site, to depths ranging from 14m to 19.6m. For uniformity, all CPTs
will be analysed to the minimum depth of 14m, which equates to 560 data points. The qc data was observed to increase from approximately 10MPa at ground surface to 30MPa at 13m depth, as illustrated in Figure 2. The CPT data is shown to follow a second order polynomial trend function, \( \mu \), which was fitted through non linear regression analysis, in line with the recommendations described by Jaksa (1995). The cone penetration tests are plotted on a Robertson (1990) classification chart in Figure 3, which identifies the material as overconsolidated clean sand to silty sand, with the close grouping of the data highlighting the uniformity of the material. Some local pockets of silt are also identified by two of the CPT traces; however the extents of these lenses are typically less than 0.2m suggesting minimal influence on subsequent pile capacities.

![Robertson (1990) CPT Chart with data from the Blessington site](image)

The residual random fluctuating component of the data is determined by subtracting the mean trend from each q_c trace, such that \( r = q_c - \mu \). The inherent variability of the CPT data can then be quantified through the standard deviation (\( \sigma \)) of the residual component (r), which is defined using Equation 1 for a set of N q_c values.

\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (r^2)}
\]  

The standard deviation of the individual CPT traces yields average values ranging from 2.6 - 3.7MPa. However closer inspection of Figure 2 highlights an increasing variability with depth. The magnitude of the residuals typically increases with depth in line with the increasing mean values, such that higher scatter about the mean trend is observed for the higher values of q_c. Considering this, the coefficient of variation (COV), which is defined as the standard deviation normalized by the mean, is shown to better represent the q_c uncertainty. The COV ranges from 12.1-16.6% for the four q_c traces, with 14% forming a representative average. Adopting the assumption of a
constant COV, results in the depth dependent standard deviation which is included in Figure 2.

In order to completely describe the statistical properties of the underlying CPT data, the correlation structure of the residuals must also be considered. Any soil property which exhibits random fluctuations due to the inherent spatial variability of the underlying soil will tend to be autocorrelated. This means that the $q_c$ data at any particular depth will be more likely to approximate the $q_c$ data points at adjacent depths. As the distance between two $q_c$ data points increases the correlation between these points decreases, up to a point where they become uncorrelated. The scale of fluctuation, $\theta$, represents a measure of the distance within which a soil property is correlated. The scale of fluctuation was determined by assuming an exponential autocorrelation function, as defined by VanMarcke (1977) using Eqn 2.

$$\rho(\tau) = \exp\left(-\tau / r_0\right) \quad (2)$$

Where $\tau$ represents the lag distance between two values and $r_0$ is a constant defined as the distance where the function decays to a value of $1/e$ and is termed the ‘autocorrelation distance’. The scale of fluctuation is then calculated as twice the autocorrelation distance. The measured autocorrelation, $\rho$, for CPT3 is shown to provide a reasonable fit to a theoretical exponential function in Figure 4. The corresponding range of $\theta$ values for CPT1 to 4 is 0.42 to 0.83m.

![Figure 4: Autocorrelation function for CPT3](image)

4 MONTE CARLO SIMULATION OF CPT PROFILES

Considering the statistical properties of the measured CPT profiles, appropriate base-line data can be used for simulation of multiple realizations of the CPT data, within a simple MATLAB code. The global mean trend, $\mu$, of the CPT data is adopted for simulation purposes, with a constant COV, of 14% and a scale of fluctuation of 0.6m. The $q_c$ data are assumed to follow a log-normally distributed random field, with an assumed exponential autocorrelation model. The autocorrelation matrix is first decomposed into a lower triangular matrix (L) and its transpose, by the process of Cholesky decomposition, and then multiplied by a normally distributed field, with a zero mean and unit variance. This results in a correlated standard normal field, G. The mean and standard deviation are then transformed to the log-normally distributed parameters, $\mu_{ln}$ and $\sigma_{ln}$ using Equation 3.
\[ \mu_{ln} = \ln \mu - 0.5\sigma_{ln}^2 \quad (3a) \]
\[ \sigma_{ln} = [\ln(1 + COV^2)]^{0.5} \quad (3b) \]

The log-normally distributed field \( q_c \) is then determined by equation 4.
\[ q_c = e^{\mu_{ln} + \sigma_{ln} G} \quad (4) \]

This process generates \( q_c \) profiles with a given mean, COV and scale of fluctuation.

5 APPLICATION TO PILE DESIGN

The multiple \( q_c \) profiles generated can then be used to determine multiple realizations of the pile capacity at the site by invoking a suitable pile design model. In this case, the Imperial College model described by Jardine et al. (2005) was used to determine the pile shaft (\( Q_s \)), base (\( Q_b \)) and total capacity (\( Q_T \)) for each realization of the \( q_c \) data. The shear stress, \( q_s \), was therefore determined from equation 5 and the unit base resistance, \( q_b \), was determined from equation 6. For calculating the capacity, a 6m long pile with 0.2m diameter, \( D \), was used, resulting in a slenderness ratio, \( L/D \) of 30.

\[ q_s = (0.027 \ q_c \ (h/R)^{-0.38} \ (\sigma'_{v0} / \ P_{atm})^{-0.12} + 4G \ (\Delta r / R)) \tan \delta \quad (5) \]
\[ q_b = q_{c(\text{av})} (1 - 0.5 \log(D / 0.036)) \quad (6) \]

Where \( \sigma'_{v0} \) is the in-situ vertical effective stress, \( G \) is the small strain stiffness, \( \Delta r \) is twice the surface roughness, \( \delta \) is the interface friction angle at failure and \( h/R \) is the distance from the pile tip normalized by the pile radius. The base stress is calculated using the average \( q_c \) over a distance 1.5D above and below the pile tip, termed \( q_{c(\text{av})} \). The results of this analysis, illustrated in Figure 5, yielded a total capacity of 765kN and a COV of 6.55%, which is significantly lower than the 14% COV of the input \( q_c \) profile. The distribution of the base and shaft capacity can also be obtained from the analysis, with the COV of \( Q_b \) shown to be four times larger than the COV of \( Q_s \) at 10.2% in comparison to 3.73%.

![Figure 5: Example of Pile Variability arising from the inherent scatter in CPT results](image-url)
The process of spatial averaging is responsible for the low COV of pile capacity in comparison to the point variability of the \( q_c \) data. Essentially as larger lengths of soil are required to fail, high points tend to balance low points and the COV of the soil property reduces as the failure zone increases in size. The pile shaft has a significantly large zone of influence, equal to the shaft length, in comparison to the pile base (a zone of 3D) and therefore the shaft undergoes significantly more spatial averaging. To further illustrate this point, the pile length was varied with the resulting L/D ratios between 10 and 70 and the variation in pile capacity shown in Figure 6. The base capacity is independent of the slenderness ratio, with an approximately constant COV of 10.5%, which is unsurprising since the zone of influence is only diameter dependent and therefore unaffected by the pile length. In contrast, the pile shaft variability reduces with increasing slenderness ratio as the zone of influence increases. However, the most dramatic reductions occur at low L/D ratios with the rate of reduction decreasing with increasing slenderness. The total capacity has a variability which lies between the shaft and base capacity and also decreases consistently with L/D. The rate of reduction in the COV of total capacity is steeper than the shaft component, reflecting (i) the decreasing shaft variability and (ii) the lower proportion of base capacity in comparison to the total capacity.

![Figure 6: Impact of slenderness ratio on pile uncertainty](image.png)

6 SENSITIVITY ANALYSIS

The final part of this study involved a sensitivity analysis, which was conducted to assess the impact of the input parameters on the pile variability. A standard pile with dimensions of 0.2m diameter and 6m length was used throughout the sensitivity analysis. The statistical parameters of the CPT data were then varied, within a reasonable range while maintaining the same mean value. Specifically the COV of \( q_c \) was varied between 10 and 81\%, and the scale of fluctuation was varied from 0.1 to 2.2m, in line with the ranges reported by Phoon and Kulhaway (1999). This provides some insight into the cross sensitivity between the soil variability, as determined from CPT data, and the pile variability. This can then be used to identify other appropriate research.
sites which would lead to acceptably low levels of pile variability, based on the CPT statistics.

Figures 7 illustrates the sensitivity of $Q$ to both variations in $q_c$ and the scale of fluctuation. A near linear dependence is observed between the COV of the pile capacity components and the COV of the input, with the base, shaft and total variability approximately 70%, 45% and 25% of the variability in $q_c$. In contrast the scale of fluctuation exhibits a strongly non linear relationship with the pile variability, with the COV of $Q$ increasing with $\theta$. This suggests that sites which exhibit highly correlated $q_c$ profiles will tend to yield more variable pile capacities. However, the rate of increase is largest for small scale of fluctuations less than 1m.

![Graph](image1.png)

Figure 7: Sensitivity to $q_c$ variability and scale of fluctuation

7 CONCLUSION

This paper used CPT profiles to determine the in-situ soil variability at a dense sand site. This was subsequently used to predict the likely scatter in pile capacity at the site. The COV of the $q_c$ data was determined to be 14%, however spatial averaging was shown to play a vital role in reducing the expected COV of the pile resistance to 6.5%. Pile capacity variability was also shown to exhibit an inverse relationship with the slenderness ratio. The final section involved a brief sensitivity analysis which highlighted the interrelationship between pile capacity variability, $q_c$ variability and the scale of fluctuation. In conclusion, the Blessington test site is suitable for conducting piling research due to the low variability of the expected capacities, particularly for the case of long tension piles which will have a capacity nearly unaffected by soil variability.

8 ACKNOWLEDGEMENTS

The first author is funded by Sustainable Energy Ireland (SEI). In addition the first author would like to acknowledge financial awards received from the Geotechnical Society of Ireland. The authors would like to thank Trinity College Dublin for carrying out the CPT tests and in particular Dr. Eric Farrell and Dr. Brendan O’Kelly.
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