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MATHEMATICAL MODELLING OF A LOW APPROACH EVAPORATIVE COOLING PROCESS FOR SPACE COOLING IN BUILDINGS

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ABSTRACT

This paper describes a mathematical model of a low approach open evaporative cooling tower for the production of high temperature indirect cooling water (14-16°C) for use in building radiant cooling and displacement ventilation systems. There are several potential approaches to model evaporative cooling, including: the Poppe method, the Merkel method and the effectiveness-NTU (ε-NTU) method. A common assumption, applied to the Merkel and ε-NTU methods, is that the effect of change in tower water mass flow rate due to evaporation is ignored, which results in a simpler model with reduced computational requirements, but with somewhat decreased accuracy. In this paper, a new improved method, called the corrected ε-NTU approach is proposed, where the water loss due to evaporation is taken into account. It is expected by this correction the results of improved ε-NTU in the category of heat transfer will be more close to the results of more rigorous Poppe method. The current mathematical model is evaluated against experimental data reported for a number of open tower configurations, subject to different water temperature and ambient boundary conditions. It is shown that the discrepancies between the calculated and experimental tower outlet temperatures are to within ±0.35°C for a low temperature cooling water process (14-16°C), subject to temperate climate ambient conditions and ±0.85°C for a high temperature cooling water process (29-36°C), subject to continental climate ambient conditions. Considering the associated tower cooling loads, predicted results were found to be within a 6% root-mean-square difference compared to experimental data.

1. INTRODUCTION

Recent interest in radiant cooling and displacement ventilation systems, as a means of building cooling in temperate climates, has prompted renewed interest in evaporative cooling for generating cooling water (Dieckmann 2009, Olesen 2008, Harvey 2006, Strand 2003). Evaporative cooling has the potential to offer an efficient approach for producing chilled water for such systems, particularly in temperate climates, where conventional mechanical air-conditioning systems are, for certain buildings, sometimes considered to be an over-engineered solution and where passive cooling is insufficient to offset cooling loads (Alexander and O’Rourke 2008). The viability of this concept, however, largely depends on achieving low approach conditions, at acceptable levels of energy performance. Previous experimental work for a full scale evaporative cooling system, based on an indirect open cooling tower which incorporated a primary and secondary circuit (see Fig. 1), has shown that it is possible, for temperature climates, to produce cooling water at low primary approach conditions (1-3 K), at the higher temperatures utilised by radiant cooling systems (14-16°C) and with varying levels of annual availability (Costelloe and Finn 2003). Costelloe reports that for water, supplied for an indirect radiant cooling system by means of an open evaporative cooling tower, at temperatures of 14°C or 16°C, availability levels of 72% and 88% respectively, are possible for Dublin, Ireland, as a representative northern European city, when the difference between the building supply temperature (secondary approach) and wet bulb temperature is 3 K (Costelloe and Finn 2003). It was similarly shown for Milan, Italy, as a representative southern European city, availability of 40% and 50%, for water at 14°C and 16°C, respectively, was also possible (Costelloe and Finn 2003).
2. LITERATURE REVIEW

A typical cooling tower, such as that shown in Fig. 1, contains of three separate zones, namely: the spray zone, the packing or fill zone and the rain zone. There are several well-known methods for analysis of heat and mass transfer processes associated with wet cooling towers including: the Merkel method, the Poppe method and the effectiveness-NTU method (Jaber and Webb 1989, Kloppers and Kroger 2005).

Merkel was one of the earliest researchers to develop a methodology for describing the heat and mass transfer associated with cooling towers (Kloppers and Kroger 2005). The Merkel method makes use of some critical assumptions (Kloppers and Kroger 2005, Khan and Zubair 2003) such as:

- The Lewis factor, relating heat and mass transfer is assumed equal to 1.
- The air exiting the tower is assumed to be saturated.
- The reduction of water flow rate due to evaporation is neglected in the tower water mass balance.
- The specific heat capacity of air-stream mixture at constant pressure is the same as that of the dry air.

Using the Merkel method, the cooling tower exit water temperature can be relatively simply and correctly calculated, but on the other hand this model does not consider the effect of water loss due to evaporation and has limitations for the calculation of the tower air exit state (Kloppers and Kroger 2005). The Poppe model, developed by Poppe and Rogener (1991), does not make the simplifying assumptions of Merkel (Kloppers and Kroger 2005). This method more accurately predicts the temperature of the tower outlet air compared to the Merkel or ε-NTU methods. The governing equations in this method can be solved by a fourth order Runge-Kutta method (Mathews 1992). Kloppers (2003 & 2005) gives a detailed derivation of the Poppe method. A comparison of the differences between Poppe and Merkel approaches for heat transfer estimation was examined by Kloppers and Kroger, who found that the predicted heat rejection by the Poppe method is better than that estimated by the Merkel method, for all ambient conditions (Kloppers and Kroger 2003). For example, it is shown that the heat rejection as predicted by Poppe approach for mechanical draft cooling tower is approximately 3.2% higher at 290K (16.85°C) than the heat rejection predicted by Merkel approach (Kloppers and Kroger 2003).

Jaber and Webb (1989) applied the effectiveness-NTU method for evaluation of the performance of counter flow or cross flow cooling towers. This method relied on the same simplifying assumptions as outlined in the Merkel method. Kloppers and Kroger note that the heat transfer, as predicted for a mechanical draft cooling tower, by the Merkel and ε-NTU methods are very similar and that the discrepancy between these two methods is less than 1% (Kloppers and Kroger 2003). Moreover, it is noted the heat transfer predicted by ε-NTU is higher than heat transfer estimated by the Merkel method. Kloppers notes that the Poppe method gives better results compared to the Merkel or ε-NTU method for high temperature heat rejection applications typical of Rankine cycles (30-35°C), albeit at an increased complexity and computational overhead compared to the Merkel and ε-NTU methods (Kloppers and Kroger 2003 & 2005). Based on the fact that the Poppe method does not consider the simplifying assumptions used by the Merkel method, the Lewis factor should be estimated according to the Bosnjakovic relation (Kloppers and Kroger 2003) and then solved using a fourth order Runge-Kutta approach.

A number of researchers have developed other mathematical models for open type cooling towers, notably Khan and Zubair (2001 & 2003), Naphon (2005), and Li et al. (2010). Khan and Zubair (2003) describes a mathematical model of a wet counter flow cooling tower to analyze heat and mass transfer when the Lewis factor is equal to 0.9 and NTU is estimated by means of an empirical equation based on the measurement of Simpson and Sherwood (1946). It is noted that the majority of heat transfer (more than 60%) is due to evaporation in this kind of cooling tower, when the entering dry bulb and wet bulb temperatures were 29°C and 21.1°C, respectively and the inlet water temperature was 28.7°C. The experimental values of NTU were compared with Khan and Zubair model. It was shown that the tolerance in calculation of the NTU could be as much as ±7% (Khan and Zubair 2001). Naphon (2005) presented a mathematical model of a direct contact counter flow wet cooling tower using an iterative solution method based on an approach described by Khan and Zubir (2001). The predicted results were validated against experimental data. It was shown that ±10% difference existed between the modeled and measured data for outlet temperatures of water and air, when the inlet air is 23°C and the tower inlet water temperature were between 30°C and 40°C. Li et al. (2010) proposed a dynamic model of mechanical draft counter flow wet cooling tower based on finite volume method. In this model, the Lewis factor is estimated based on the Bosnjakovic relation and also the water loss is considered by the mass transfer coefficient.

Costelloe and Finn (2003) designed and built a full scale evaporative cooling system including: an open counter flow cooling tower, a primary and secondary circuit with an intermediate heat exchanger (Fig. 10b).
coupled with a variable system load. It is shown that this system was capable of delivering cooling water at between 14 and 16°C for climatic conditions typical of Dublin, Ireland, which can be used for radiant cooling. Costelloe and Finn (2009) published correlations for estimating the cooling tower coefficient or the Merkel number for the cooling tower. It was estimated that the average uncertainty associated with the correlated relation was within ±5% in comparison of the associated experimental data.

Fig 1. Schematic of an indirect evaporative cooling system and the associated cooling tower (Costelloe and Finn 2009)

The objective of the current paper is to develop a mathematical model of the cooling tower shown in Fig. 1 and to validate the model against existing experimental data for temperature climate conditions (Costelloe and Finn 2009). The approach used is based on a corrected $\varepsilon$-NTU method, which accounts for water evaporation from the tower, during the cooling process. The cooling tower model will, at a later stage, be integrated into a building energy simulation model, thereby allowing the evaporative/radiant cooling system to be evaluated for different building types and climates, as well as allowing system optimisation studies to be carried out.

3. MATHEMATICAL MODEL

A mathematical model of the mechanical draft counter-flow cooling tower, as shown in Fig. 1, using the proposed corrected effectiveness-NTU method is described in this section. In this method, the Lewis factor is assumed to 1.0 and the exiting air is considered saturated, but in contrast to standard effectiveness-NTU method (Kloppers and Kroger 2003) or the Merkel method (Kloppers and Kroger 2003), the change in tower water mass flow rate due to evaporation is calculated and the specific heat of air-stream mixture is not constant and it is estimated in each step of the numerical calculation based on the equations given in Kroger (2004).

A control volume of a differential element of the counter flow wet cooling tower is shown in Fig. 2. Referring to Fig. 2, the heat transfer between the water and the air can be written as follows:

$$dQ = m_w d_i_w = m_w c_{pw} dT_w = m_a d i_{ma}$$

It is shown (Jaber and Webb 1989) that the energy equation for a counter flow cooling tower can be approximated by the following relation, based on the air enthalpy driving potential:

$$dQ \approx K ((i_{masw} - i_{ma}) dA$$

where $K$ is the mass transfer coefficient, $i_{masw}$ is enthalpy of saturated air at the interface condition and $i_{ma}$ is enthalpy of moist air at the bulk condition.
Equation (1) can be rewritten as follows:

\[ dQ = m_w c_{pw} dT_w = \frac{m_w c_{pw}}{d i_{masw}} d i_{masw} \] (3)

Based on Equation (1) \( d i_{ma} = dQ / m_a \) and from the relation \( d i_{masw} - d i_{ma} = d (i_{masw} - i_{ma}) \), it can be shown that:

\[ d (i_{masw} - i_{ma}) = dQ \left( \frac{d i_{masw}}{m_w c_{pw}} - \frac{1}{m_a} \right) \] (4)

When \( dQ \) from Equation (2) is substituted to Equation (4), it gives:

\[ \frac{d(i_{masw} - i_{ma})}{(i_{masw} - i_{ma})} = K \left( \frac{d i_{masw}}{m_w c_{pw}} - \frac{1}{m_a} \right) dA \] (5)

This equation is developed in conjunction with heat exchanger effectiveness-NTU equation (Incropera and DeWitt 2002), as given here, which is integrated over the entire tower:

\[ \frac{d(T_h - T_c)}{(T_h - T_c)} = -U \left( \frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}} \right) dA \] (6)

There are two possible cases for equation (5), where \( m_a \) is greater or less than \( m_w c_{pw} / \left( \frac{d i_{masw}}{d T_w} \right) \).

According to these parameters, the maximum is defined as \( C_{\text{max}} \) and the minimum as \( C_{\text{min}} \), thus the gradient of the saturated air enthalpy-temperature curve is given as:

\[ \frac{d i_{masw}}{d T_w} = \frac{i_{masw1} - i_{masw0}}{T_{w1} - T_{w0}} \] (7)

The maximum theoretical amount of heat rate that can be transferred is estimated by:

\[ Q_{\text{max}} = C_{\text{min}} (i_{masw1} - f - i_{mai}) \] (8)

where \( i_{masw1} \) is the saturated air enthalpy at the water inlet condition, \( i_{mai} \) denotes air inlet enthalpy and \( f \) is a correction factor, to improve the approximation of the enthalpy of saturated air versus water temperature. \( f \) is defined by:

\[ f = (i_{masw0} + i_{masw1} - 2i_{maswm}) / 4 \] (9)

where \( i_{maswm} \) indicates the enthalpy of saturated air at the mean water temperature.
The current equation that is used to determine the heat transfer rate in the Merkel and ε-NTU methods is;

\[ Q = m_w c_{ pwm} (T_{wi} - T_{wo}) \] (10)

As can be seen in equation (10), the loss of water mass flow rate is ignored, so in contrast to Poppe method, the Merkel and ε-NTU methods predict lower heat rejection rate than the Poppe method. In order to correct for the Merkel assumption that ignores the evaporation of cooling tower water, the mass flow rate due to evaporation can be calculated as follows;

\[ m_{evap} = m_a \times (\omega_o - \omega_i) \] (11)

where \( \omega_o \) denotes the humidity ratio of saturated air that can be estimated by Equation (12)

\[ \omega_o = 0.622 \frac{P_a}{P_s} \] (12)

where \( P_a \) is atmospheric pressure and \( P_s \) indicates saturation pressure of water.

According to Equation (11), a new improved equation for energy equation can be incorporated in the Merkel approach that includes the loss of mass water in the cooling tower, as follows:

\[ Q = m_{wi} c_{ pwm} T_{wi} - (m_{wi} - m_{evap}) c_{ pwm} T_{wo} \] (13)

When Equation (13) is used for the calculation of heat transfer, estimates by the improved Merkel method and Poppe method are quite similar (Kloppers and Kroger 2003).

The wet cooling tower coefficient or the Merkel number \( (M_e) \) is an important factor for calculation of the number of transfer units in cooling towers. Various investigations have examined this issue, such as Khan and Zubair (2001). Khan presented that the cooling tower coefficient can be calculated by:

\[ M_e = \frac{K_a V}{L} = c \times \left( \frac{m_w}{m_a} \right)^n \] (14)

where \( c \) and \( n \) are empirical constants.

Costelloe and Finn (2009) developed an empirical relation for the wet cooling tower coefficient or the Merkel number, for the open cooling tower described in Fig. 1 and it was found to be:

\[ M_e = 1.3 \times \left( \frac{m_w}{m_a} \right)^{-0.77} \] (15)

The number of transfer units (NTU) for a counter flow cooling tower is calculated using Equation (15) along with the parameters \( m_a \) and \( m_w c_{ pw} / (d_i_{ masw}/dT_w) \). If \( m_w c_{ pw} / (d_i_{ masw}/dT_w) \) is greater than \( m_a \), then:

\[ NTU = M_e \times m_w / m_a \] (16)

Alternatively if \( m_w c_{ pw} / (d_i_{ masw}/dT_w) \) is less than \( m_a \), then:

\[ NTU = M_e \times (d_i_{ masw}/dT_w) / c_{ pw} \] (17)

When the inlet conditions, such as mass flow rate of water and air, dry bulb and wet bulb temperature of inlet air and inlet water temperature for the wet counter flow cooling tower are known, the outlet water temperature and the outlet air temperatures, can be calculated, thereby allowing the effectiveness and heat transfer for the cooling tower to be determined. The properties of the air-water vapor mixture and moist air which are needed at each step of the numerical calculation are obtained from the property equations given in Kroger (2004).

4. RESULTS

Using the approach described in Section 3, where the tower air and water inlet conditions are known, the tower outlet water temperature is calculated using the mathematical model. Table 1 shows the results using the current method. The experimental data are taken from the mechanical draft towers shown in Figure 1 and is described elsewhere by Costelloe and Finn (2009). Using the data from Table 1, a comparison between the calculated outlet water temperatures and the experimental data are presented in Fig. 3. The largest difference in calculation of water temperatures can be seen to be 0.28°C (Table 1), equivalent to an error of less than ±2.5% (Fig. 3) referenced with respect to 0°C.
Table 1: Comparison between the results of model and experimental data (Costelloe and Finn 2009)

<table>
<thead>
<tr>
<th>Test No</th>
<th>Load kW</th>
<th>DBT (°C)</th>
<th>RH (%)</th>
<th>Water Inlet Temp. (°C)</th>
<th>Water Outlet Temp. (°C)</th>
<th>Water Outlet Temp. Difference</th>
<th>Heat Transfer (kW)</th>
<th>Heat Transfer Difference (kW)</th>
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<tr>
<td>1</td>
<td>18.192</td>
<td>14.09</td>
<td>57.69</td>
<td>15.06</td>
<td>12.89</td>
<td>-0.07</td>
<td>19.115</td>
<td>0.923</td>
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<tr>
<td>2</td>
<td>19.042</td>
<td>15.36</td>
<td>51.61</td>
<td>16.21</td>
<td>13.37</td>
<td>-0.13</td>
<td>20.313</td>
<td>1.271</td>
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<tr>
<td>3</td>
<td>18.512</td>
<td>14.99</td>
<td>60.18</td>
<td>14.61</td>
<td>12.69</td>
<td>0.20</td>
<td>16.981</td>
<td>-1.531</td>
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<tr>
<td>4</td>
<td>18.695</td>
<td>14.98</td>
<td>60.95</td>
<td>14.93</td>
<td>12.70</td>
<td>0.16</td>
<td>17.752</td>
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<td>0.28</td>
<td>17.380</td>
<td>-1.9</td>
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<tr>
<td>6</td>
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<td>66.67</td>
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<td>-0.12</td>
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<td>-1.153</td>
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<td>19.953</td>
<td>14.26</td>
<td>64.31</td>
<td>15.16</td>
<td>12.36</td>
<td>0.06</td>
<td>19.898</td>
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<tr>
<td>8</td>
<td>17.437</td>
<td>14.76</td>
<td>62.93</td>
<td>15.55</td>
<td>12.35</td>
<td>0.02</td>
<td>17.694</td>
<td>0.257</td>
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<tr>
<td>9</td>
<td>17.018</td>
<td>14.91</td>
<td>59.80</td>
<td>15.97</td>
<td>11.91</td>
<td>0.01</td>
<td>17.343</td>
<td>0.325</td>
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</table>

Fig. 3. Comparison of tower water outlet temperature and data measured by Costelloe and Finn (2009).

Fig. 4. Comparison between the calculated heat transfer rate and the experimental data (Costelloe and Finn 2009).

It can be seen from Table 1 that the maximum error in calculation of heat transfer is 1.9 kW, which shows the tolerance in calculation of heat transfer to be within ±10%. Fig. 4 shows the comparison between current...
model and experimental for heat rejection of the cooling tower. Considering all the data, the predicted results were found to be within 6% root-mean-square difference compared to experimental data.

Table 2 compares prediction of the current model with experimental and modeled data presented by Li et al. (2010) for a higher temperature (33 to 44°C tower water) heat rejection application. The experimental data used by Li et al. (2010) to benchmark their work was originally collected by Simpson and Sherwood (1946). It can be seen that the largest differences in calculation of water outlet temperature and air outlet temperature is 0.54°C and 0.83°C (Table 2), respectively, which means an error is less than 2% for water outlet temperature and it is less than 2.5% for air outlet temperature. It is clear that the results for water outlet temperature are in acceptable accuracy in comparison to Li model.

Table 2: Comparison between current model, Li model and experimental data (Li et al. 2010, Simpson and Sherwood 1946)

<table>
<thead>
<tr>
<th>Case</th>
<th>m_a,in (kg/s)</th>
<th>m_w,in (kg/s)</th>
<th>T_{db,in} (°C)</th>
<th>T_{wb,in} (°C)</th>
<th>T_{w,in} (°C)</th>
<th>T_{w, out} (°C)</th>
<th>T_{db, out} (°C)</th>
<th>Temp. Diff.</th>
<th>Temp. Diff.</th>
<th>Temp. Diff.</th>
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</thead>
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<tr>
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<td>1.1871</td>
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<tr>
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<td>1.0088</td>
<td>31.78</td>
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<td>34.39</td>
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<td>28.95</td>
</tr>
<tr>
<td>3</td>
<td>1.1584</td>
<td>0.7548</td>
<td>35</td>
<td>23.89</td>
<td>43.61</td>
<td>27.89</td>
<td>32.78</td>
<td>28.12</td>
<td>0.23</td>
<td>28.24</td>
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<td>4</td>
<td>1.2653</td>
<td>1.0088</td>
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<td>0.7548</td>
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<td>26.67</td>
<td>43.06</td>
<td>29.72</td>
<td>33.89</td>
<td>29.94</td>
<td>0.22</td>
<td>29.93</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

A mathematical model of a low approach open evaporative cooling tower for the production of high temperature indirect cooling water (14-16°C) for use in building radiant cooling systems is described. The model, called the corrected ε-NTU approach is described, where the water loss due to evaporation is taken into account. The current mathematical model is evaluated against experimental data reported for a number of different open tower configurations subject to different water temperature and ambient boundary conditions. It is shown that the discrepancies between the calculated and experimental tower calculated outlet temperatures are to within ±0.3°C for a low temperature cooling water process (14-16°C) subject to temperate climate ambient conditions and ±0.85°C for a high temperature cooling water process (29-36°C). Future work will focus on integrating the cooling tower model into a building energy simulation model, thereby allowing the evaporative/radiant cooling system to be evaluated for different building types and climates, as well as facilitating system optimisation studies to be carried out.

NOMENCLATURE

\( A \) area (m²) \( a \) surface area per unit volume, m⁻¹
\( c_p \) specific heat at constant pressure (J·kg⁻¹·K⁻¹) \( h \) heat capacity rate, \( mc_p \) (W · K⁻¹)
\( f \) enthalpy correction factor (dimensionless) \( i \) specific enthalpy (J·kg⁻¹)
\( K \) mass transfer coefficient (kg·m⁻²·s⁻¹) \( L \) water mass flow rate (kg·s⁻¹) \( M_e \) Merkel number
\( m \) mass flow rate, (kg·s⁻¹) \( NTU \) number of transfer units \( P \) pressure
\( Q \) heat transfer rate (W) \( T \) temperature (°C or K)
\( U \) overall heat transfer coefficient (W·m⁻²·K⁻¹) \( V \) volume of tower m³
\( \omega \) humidity ratio (kg water vapor per kg dry air)
SUBSCRIPTS

cold
hot
mean
maximum
minimum
saturated air at the local bulk water
saturation
water

REFERENCES