Analysis of an equiaxed dendrite growth model with comparisons to in-situ results of equiaxed dendritic growth in an Al-Ge alloy

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Abstract. The Lipton Glicksman Kurz (LGK) growth model is commonly used to predict growth rates for equiaxed dendrites in solidifying mushy zones. However, the original LGK method treats an isolated dendrite growing in an infinite volume of liquid. In an equiaxed mushy zone, with multiple nucleation events, thermal and solutal interactions take place between the equiaxed dendrites. A modified version of the LGK model was developed that allows for measurement of the solute build-up ahead of the dendrites. To investigate the validity of the model, comparisons are made with results obtained from in-situ synchrotron X-ray videomicroscopy of solidification in a Bridgman furnace of an Al-12wt.%Ge alloy inoculated with Al-Ti-B grain refiner. Comparisons between the original LGK and modified LGK models are presented for discussion. The modified LGK model shows realistic tip temperature trends.

Introduction

The LGK growth model [1] describes tip growth for an isolated equiaxed dendrite growing in an infinite volume of liquid. The LGK model uses the Ivantsov solution [2] to treat both the thermal and solutal diffusion fields around the parabolic solid-liquid interface. The LGK model also uses the marginal stability criterion [3] to determine the operating conditions at the dendrite tip. Ultimately the LGK model gives the total undercooling at the tip as the sum of solutal undercooling, thermal undercooling, and curvature undercooling. The solutal undercooling is calculated based on the nominal composition, which is assumed to exist in the liquid at an infinite distance ahead of the tip. The thermal undercooling is calculated based on the bath temperature at an infinite distance ahead of the tip. The curvature undercooling is calculated using the dendrite tip radius and the Gibbs-Thomson effect at the curved surface. Trivedi and Kurz [4] show that if the temperature profile is assumed \textit{a priori} then the Ivantsov solution is applied to the solutal problem only. In this case, only the solutal and curvature undercoolings are calculated. This approach is adopted in this research; hence, there is no requirement to calculate the thermal undercooling.

Modified LGK Model. A modified version of the LGK model has been proposed [5] whereby the growth conditions at the tip are determined by measuring the solute level at a finite distance ahead of the dendrite. Since the solutal undercooling is calculated using the measured solutal level, solute build up in the liquid is accounted for. A modified Ivantsov solution is

\[ Iv(P, R, Z) = P e^{P \left\{ E_i(P) - E_i(P(1+2Z/R)) \right\}}. \] (1)
R is the tip radius. Z is known as the look-ahead distance and it is the measured length ahead of the tip where the solute level is measured. P is the solutal Peclet number and it is given as

\[ P = \frac{RV}{2D}. \]  

(2)

V is the growth rate of the tip and D is the diffusivity of the solute species in liquid. The modified Ivantsov is used to determine the composition in the liquid at the dendrite tip, \( C_t \),

\[ C_t = \frac{C_f}{\{1 - (1-k)\nu(P,R,Z)\}} \]

(3)

The partition coefficient is given by k. \( C_f \) is the composition in the liquid measured at the finite distance, Z, ahead of the tip. The marginal stability criterion is used to determine the tip radius, R,

\[ R = \frac{\Gamma}{\{\sigma(2mPC_t(k-1) + GR)\}} \]

(4)

\( \Gamma \) is the Gibbs-Thomson parameter, \( \sigma \) is the stability parameter, m is the slope of the liquidus line in the phase diagram, and \( G \) is the temperature gradient at the tip. The overall global undercooling [5], \( \Delta T_T \), at the tip is given by two terms: the solutal undercooling and the curvature undercooling,

\[ \Delta T_T = m\{C_o - C_f/\{1 - (1-k)\nu(P,R,Z)\}\} + 2\Gamma/R \]

(5)

The nominal composition is given as \( C_o \). Alternatively the global undercooling is given as

\[ \Delta T_T = T_{L}(C_o) - T_t \]

(6)

\( T_{L}(C_o) \) is the liquidus temperature given at the nominal composition. \( T_t \) is the tip temperature. The solution to this overall set of equations requires a double numerical iteration scheme. Hence for a measured growth rate, V, and a solute level, \( C_f \), measured at a finite distance, Z, ahead of the dendrite tip, we calculate the Peclet number, P; composition at the tip, \( C_t \); tip radius, R; global undercooling, \( \Delta T_T \); and tip temperature, \( T_t \). For comparison purposes, the original LGK growth model assumes that the nominal composition exists at an infinite distance ahead of the tip. The original LGK is obtained from the modified version by letting the look-ahead distance, Z, tend to infinity (where the second \( E_1 \) term in eq. 1 disappears) and by replacing \( C_t \) with \( C_o \) [5].

**Experimental Measurements.** In-situ X-ray imaging experiments were conducted at BM05 at the European Synchrotron Radiation Facility (ESRF) using aluminum-germanium samples, Al-12wt%Ge, inoculated with an Al-5wt%Ti-1wt%B grain refiner (to give 0.0025wt%Ti). Samples were solidified in a Bridgman furnace and the selected parameters gave a temperature gradient of 0.027 K/µm and a pulling speed of approximately 21μm/s. The incident beam was monochromatised at 15KeV and the employed settings gave a field of view of 1.4 × 1.4 mm² nominal temporal and spatial resolution of 0.5s and 3µm respectively. Mathiesen et al. [6] discuss sample preparation and experiment setup in detail.

**Experimental Results.** During the solidification sequence two neighboring equiaxed crystals nucleated, grew and impinged upon each other. Fig. 1 shows a montage made from individual frames from the solidification sequence – the dendrites in question are clearly marked. The spacing between the epicenters of the two equiaxed dendrites was maintained at a constant 700 µm, i.e., once nucleated the equiaxed dendrites were stationary relative to the moving specimen. The midpoint between the two dendrites is highlighted in fig. 1. The look-ahead distance, Z, and the solute level, \( C_f \), ahead of the impinging dendrite arms were measured throughout the sequence.
Image analysis was used to measure the dendrite tip separation distance and the solute level in the liquid between the impinging dendrite tips. Fig 2(a) shows the look-ahead distances that were measured with time and an interpolation of the look-ahead data. The solute analysis was done based on the contrast in the far liquid above the dendrites where no solute rejection had taken place (i.e., where the composition was C₀) and the region between the two dendrite tips. Fig 2(b) shows the measured solute level at the mid-position between the dendrite tips. An interpolation of the solute levels was attempted using a cosine function. Qualitatively the measured solute level increased; however, due to high noise levels in the experimental data, it was difficult to get a reasonable fit that gave low residual error values. In particular, contrast anomalies were present due to problems with the monochromator used at the BM5 beamline, giving rise to contrast bands in the incident beam cross section with up to 50% contrast variations. Furthermore, the band structure was drifting, preventing complete removal of the incident beam band contrast by flat-field corrections. The growth rates of the impinging dendrite tips were measured frame to frame and fourth-order polynomials were used to interpolate the growth rate data points (shown in fig. 2(c)).

Figure 1. A montage showing two Al-Ge equiaxed dendrites impinging upon each other

Figure 2. Look-ahead distance (a), measured composition (b), and Growth rates versus time (c).

**Modeling Results.** Phase diagram data for the Al-12wt%Ge alloy was obtained from [7] and diffusivity data was obtained from [8]. The data used was C₀=12wt%Ge, m=−2.5 K/wt%Ge, k=0.0208, T₁(C₀)=903 K, Γ =1.9x10⁻⁷Km, and D=2.6x10⁻⁹ m²/s. The stability parameter was taken as 0.0253. The temperature gradient at the tip, G, was assumed to be zero. Using the interpolated data from fig 2 as inputs, the McFadden-Browne (MFB) growth model could predict the global undercooling (ΔT₁ , fig. 3(a)), tip temperature (T, fig. 3(b)), tip composition (Cₜ, fig. 3(c)), and tip radius (R, fig. 3(d)) versus time. For comparison fig. 3 gives the same results using the original formulation of the LGK growth law (that is, assuming the nominal composition at an infinite distance ahead of the tips).
Figure 3. Global undercooling (a), Tip Temperature (b), Tip Composition (c), and Tip Radius (d) versus time with comparisons between LGK and McFadden-Browne (MFB) growth models.

**Conclusion.** Initially there is agreement between the LGK and MFB models, this is because initially there is no solutal interaction between the dendrites [9]. After around a second of growth solutal interaction takes place and model results disagree thereafter. The MFB model determines that the tip temperatures decrease (increase in global undercoolings) and the tip compositions increase. The LGK model predicts a recrudesence at the tip and a decrease in tip composition, which is physically unrealistic in a Bridgman furnace. Interestingly, the models give close agreement on the tip radii, which proves tip radii measurement are necessary but insufficient for determining the growth conditions.

**References**