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Characterisation of the Reactive Power Capability of Diverse Distributed Generators: Toward an Optimisation Approach

Paul Cuffe, Student Member IEEE, Paul Smith, Andrew Keane, Member IEEE

Abstract—The highly renewable power system cannot assume the ubiquity and constant availability of synchronous plant. For this reason, the provision of ancillary services must shift to renewable generators, concomitant with their rising penetration levels. This work will focus on one aspect of this challenge: the provision of reactive power from distributed generation. As such resources become increasingly important, their incorporation into transmission system operation and planning activities becomes vital. To that end, a capability characterisation appears useful, delineating the range of controllable reactive power available for a given power flow onto a section of distribution network. Work to date has used time series techniques to provide a proxy to this type of capability chart. Distribution system optimisation techniques offer the potential for a more rigorous and general-purpose characterisation methodology. This work will set out how such an optimisation problem may be formulated, and will provide some initial results and validations.

Index Terms—distributed generation, voltage control, transmission planning, distribution planning, optimisation

I. INTRODUCTION

There are growing concerns, articulated in [1] and elsewhere, that the displacement of conventional generators attending the rise of renewables may lead to a paucity of ancillary service providers. While ramping services [2] and synchronous inertia [3] might be mentioned in this regard, this work will address the issue of reactive power provision. The challenge is clear: how can renewable generators play a more integrated role in the provision of reactive power, whether connected at transmission or distribution voltage levels? This theme has been addressed from a regulatory and economic perspective in the literature [4] [5]; this work will proceed on a purely technical footing.

The literature, such as in [6], demonstrates the intuitive conclusion that effective use of reactive power from distributed resources beneficially supports transmission system voltages. If such distributed reactive power sources are to become an essential asset in transmission system operation, their inclusion in transmission planning studies appears essential. Extant work, however, offers little insight on how a widely dispersed controllable reactive power resource may be characterised from the transmission system perspective.

In [7] and [8], the authors have developed a technique which gives an aggregated reactive power capability chart, analogous to the capability diagram familiar from synchronous machine theory, for adjacent wind generators. While offering some initial insights into the problem, the methodology described has two principal limitations. Firstly, it requires historic time series output profiles for each distributed generator, which limits its potential as a planning tool. Secondly, the direct time series simulation approach is unsuitable for networks containing load and generation patterns which are not clearly correlated, thus restricting its applicability to the generation-only distribution networks considered.

Two concepts from the literature suggest paths to extending the scope of the characterisation methodology. Firstly, [9] offers a theoretical approach to establishing the allowable envelope of active and reactive power operating points for a candidate wind farm. This work shows how such operating charts can be useful for discussion of transmission system needs as well as for describing resource availability, and points toward a means to assess any potential shortcomings from a transmission perspective. Another strand in the literature, exemplified here by [10] and [11], develops optimisation techniques to express the permissible operating region for a broad class of networks in the active and reactive power plane.

One question is central to this work: for a given active power flow from a section of distribution network, what is the maximum and minimum level of reactive power that a system operator will be able to dispatch? Not alone does this rely on the basic machine characteristics of each generator and on distribution network impedances, but also on the prevailing dispositions of power injections and load flows on the network. In optimisation parlance, each discrete level of aggregate power flow from the distribution network to the transmission system can be realised by a large feasible region of differing combinations of power injections and loads. Each of these power flow profiles will establish its own set of voltage constraints and loss profiles, which combine to establish how much reactive power import or export is possible.

For this reason, optimisation techniques appear suitable; for each active power flow level, the feasible region can be searched for the combination of power flow injections which establish the most onerous set of restrictions on reactive power provision. This work seeks to validate this approach, as well as offering some initial results and insights.

The details of the proposed methodology are given in Section II. Some initial results, and validating comparisons, are presented in Section III. The conclusions from this initial
assessment are given in Section IV., which also points towards potential future applications of the methodology.

II. METHODOLOGY

A. Voltage Controller Concept

This work adopts an essential assumption underlining the analysis of [7] and [8]: that maximum reactive power will be delivered by the distribution network by operating all distributed generators in voltage control mode with a target voltage equal to the upper voltage limit. In the absence of droop control, this means that each generator exports as much reactive power as possible until it reaches a machine or voltage limit. Regulation to the lower permissible voltage achieves maximum reactive power import. For the sake of clarity of exposition, this work will proceed with characterisations of the distributed export capability, though the methodology is equally applicable to the import case.

This work does not discuss how a distribution network can achieve an optimal reactive power dispatch under an elaborate “smart grid” scheme. Rather, it is assumed that each generator is operating in an autonomous manner to maximise its own reactive power output. While this may be sub-optimal in practice, it represents a realistically implementable control scheme where only voltage control set-points need to be redispatched to achieve a collective reactive power operating point.

B. Optimisation Formulation

The use of optimisation techniques in power systems is not new, and the emergence of distributed generation has suggested many novel applications for AC Optimal Power Flow (OPF) models, which explicitly consider node voltages and reactive power flows. To date, the use of optimisation techniques in distribution system planning and operation has typically taken maximisation of network hosting capacity or minimisation of network losses as an objective function. The review in [12] illustrates the broader range of problems that can be considered from the optimisation perspective.

This work draws on the AC OPF implementation described in [13] which also enumerates the archetypal power flow constraints implicit in the optimisation problem. The AC OPF formulations result in a non-linear programme, for which many generic solvers are available. The present work employs the AC OPF tool to find a combination of generator outputs and loads that minimises total reactive power export, assuming that each constituent generator maximises its own reactive power output. In this respect, it is a search technique, to ascertain the most restrictive network conditions which may align to hinder export of reactive power from distributed generators.

An essential addition to the generic AC OPF tool is a terminal voltage control mode for distributed generators. This is implemented by imposing the following constraint on the reactive power dispatch, \( q_g \), of each generator \( g \) controlling the voltage at bus \( b \):

\[
q_g \propto (V_{b_g} - V_{set_g})
\]

A suitably large constant of proportionality in (1) will enforce a reactive power regime from each generator that brings the voltage error close to zero, ensuring the voltage at each controlled bus, \( V_{b_g} \), is at the stipulated setting of \( V_{set_g} \).

As each generator operates within machine limits, the reactive dispatch is subject to:

\[
q_g^- \leq q_g \leq q_g^+
\]  

(2)

Where the prevailing reactive power limits, \( q_g^- \) and \( q_g^+ \), are related to generator active power output, \( p_g \), by a piece-wise linear machine capability chart.

To capture the effect of voltage regulation by the transformer which couples the distribution network to the transmission system, it was necessary to augment the model with a tap-changing transformer. To avoid mixed integer programming, this took the transformer voltage ratio as a continuous bounded variable.

The objective function is taken as the minimisation of reactive power flow at the defined export bus, using generator active power dispatch as the control variables. The active load to be served at the export bus is raised iteratively, with the solver invoked at each active power flow level and the minimised reactive power flow recorded. This iterative process gives the aggregated resource description to an arbitrary granularity.

C. Restriction of the Feasible Region

Due to fundamental meteorological considerations, it is reasonable to assume that adjacent wind or solar generators will display some correspondence between their instantaneous active power outputs. A similar relationship can be anticipated among neighbouring load customers. This allows the search space for a given active power flow from the distribution network to the transmission system to be restricted to exclude implausible combinations of power flows. This work formulates this as a constraint on the standard deviation of per-unit active power outputs of generators:

\[
\sigma(p_g) \leq \sigma^+
\]  

(3)

By excluding unlikely network conditions, the characterisation of available reactive power delineates the typically dependable resource, rather than the most extreme worst case scenario.

III. INDICATIVE RESULTS

A. Test Networks

The validation of the methodology is performed on two small test networks used in previous work. These networks are free of load, connecting voltage-controlling wind generators to a dedicated transmission node.
The time series characterisation is performed on the networks shown in Fig. 1 by running a year’s worth of time-series power flow simulations, with each generator’s output varying with fifteen minute granularity, based on historical wind power output data. With each generator regulating to a voltage of 1.035 p.u., over the sending voltage at the Dx bus of 1 p.u., we record the large set of resulting active and reactive power flows at the Tx bus.

The horizontal thickness in the cloud of operating points in Fig. 2 shows how one aggregated active power flow can be supplied by a variety of internal network power flow dispositions, each with its own set of voltage constraints and loss profiles. For the sake of a more robust characterisation, the large dataset is reduced by a percentile binning process, so that in Fig. 2 the thick grey line excludes 95% of recorded operated points, thus indicating the dependable reactive power capability. A regression of this line provides a more tractable resource characterisation, though this is omitted here for clarity.

For purposes of exposition, Fig. 3 plots the time series characterisation of Network A for three different levels of output diversity between its constituent generators. The active power output of each constituent generator is normalised to a per-unit figure, and the standard deviation of these outputs is calculated for each period in the time series data set. The data displayed in Fig. 3 is subject to two binning processes – by active power output level and by rounding to the prevailing standard deviation – and the minimum reactive power export achieved for each combination is plotted. It is apparent that the most restrictive network conditions for reactive power export correspond to periods when active power output regimes were widely disparate among generators.

By excluding such atypical network conditions from the optimisation, a more realistic appraisal of likely reactive power capability can be achieved. This is analogous to the binning process used to give the reactive capability expected in 95% of cases in Fig. 2.

A duration curve for prevailing output standard deviations levels for Network A over the test year is given in Fig. 4. This makes it apparent that generator outputs maintain a strong correspondence over nearly all periods, with the standard...
deviation exceeding 0.3 p.u in less than 10% of cases. This suggests that the leftmost line in Fig. 3 provides a pessimistic appraisal of reactive power resource availability.

C. Optimisation Characterisation Validation

The time series data serves as a means to validate the optimisation characterisation approach, as it directly enumerates 35, 040 differing network conditions. Considering just the reactive power export maximisation case, a series of optimisations are run for each network, increasing the implied load to be served at the Tx bus by 0.5 MW in each case.

The results presented in Fig. 5 serve to validate the optimisation methodology, showing broad agreement with the network capability given in Fig. 2. There is a notable resemblance between the middle trace of Fig. 5, for a standard deviation ≤ 0.2 p.u, and the 95% dependable capability shown in Fig. 2. The effect of the standard deviation constraint is made clear: relaxing this constraint allows the solver to find more optimally minimal values for reactive export, though these solutions will correspond to less typical network conditions. This corresponds with the time series analysis in Fig. 3.

To further validate the optimisation methodology, it is applied to the larger Network B, with the resulting characterisation provided in Fig. 6.

Fig. 6 Results of the AC OPF characterisation process for Network B, plotted with a large sample of the time series data

Encouragingly, Fig. 6 shows that even for a larger network with seven voltage controlling generators, the optimisation methodology performs well. The raw time series operating points are plotted alongside the optimisation characterisation, which demonstrates the concurrence between the two techniques. The standard deviation constraint selected, of 0.1 p.u, is seen to exclude the bulk of recorded operating points – this means the capability it describes would be generally available.

IV. CONCLUSIONS

Given the demonstrated effectiveness of the tailored AC OPF tool in this novel characterisation role, many extensions of the work suggest themselves. The most apparent application of the tool is to the characterisation of load-bearing distribution networks. Such descriptions of the distributed resource capability can then be included in transmission planning studies. This can offer prescient insight into the viability of operating the renewables-dominated power system.

APPENDIX A

A. Network A

<table>
<thead>
<tr>
<th>Gen.</th>
<th>Own Feeder</th>
<th>Length (km)</th>
<th>$R_1$ (Ω)</th>
<th>$X_1$ (Ω)</th>
<th>$P_{\text{rated}}$ (MW)</th>
<th>Q class</th>
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<tr>
<td>DG2</td>
<td></td>
<td>14</td>
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<td>5.110</td>
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<td>2</td>
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<tr>
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<td></td>
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<td>0.680</td>
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</tr>
<tr>
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<td></td>
<td>14</td>
<td>7.364</td>
<td>5.628</td>
<td>5</td>
<td>1</td>
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**REFERENCES**


**Paul Cuffe** received the BE (Electrical) in 2009. He is currently pursuing a PhD in the Electricity Research Centre, University College Dublin (UCD), Ireland. His research interests are voltage control techniques for distributed networks with significant renewable penetration.

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**Andrew Keane** received B.E. and Ph.D. degrees in Electrical Engineering from University College Dublin in 2003 and 2007 respectively. He is a lecturer with the School of Electrical, Electronic and Communications Engineering, University College Dublin, Ireland, with research interests in power systems planning and operation, distributed energy resources and distribution networks.