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Development of a Finite Volume Based Structural Solver for Large Rotation of Non-Orthogonal Meshes

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Presentation Outline

- Background & Motivation
- Mathematical Model
- Implementation in OpenFOAM
- Numerical Test Cases
- Mesh Movement
- Summary & Conclusions
Background & Motivation
Stance with Active Hill Muscle

*Gluteus Medius*
Half Gait cycle  heel-strike to push-off

Ground Reaction (N)

- 949.4014
- 800
- 600
- 400
- 200
- 65.912

Time: 0.000 s
Mathematical Model
Implementation in OpenFOAM
Alternatively, the St. Venant-Kirchhoff momentum equation. The rate of engineering stress, where the rate of engineering strain, is given by:

\[
\dot{\epsilon} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T \cdot \nabla \mathbf{u})
\]

Structural Solver & Contact Methodology 1.3. Constitutive Relations

1.3.2 Large Strain & Large Rotation

When adopting the incremental form of the momentum equation, the engineering stress, is the second Piola-Kirchhoff stress tensor at time

\[
\dot{\mathbf{S}} = 2\mu \dot{\mathbf{E}} + \lambda \text{tr}(\dot{\mathbf{E}}) \mathbf{I}
\]

Conservation of Linear Momentum

Finite Volume Stress Analysis

Rate of Momentum Surface Forces Body Forces

\[
\frac{\delta}{\delta t} \int_{\Omega} \rho \mathbf{v} \, d\Omega = \int_{\Gamma} \mathbf{\sigma} \cdot d\Gamma + \int_{\Omega} \rho \mathbf{b} \, d\Omega
\]

Constitutive Relation

St. Venant-Kirchhoff Hyperelastic Relation

2nd Piola-Kirchhoff Stress Tensor

Strain Measure

Lagrangian Green Strain Tensor

\[
\mathbf{S} = 2\mu \mathbf{E} + \lambda \text{tr}(\mathbf{E}) \mathbf{I}
\]

Initially a methodology has been developed for contacting small strain linear elastic reversible and time independent. Thermal, plastic and viscous effects are neglected, as there is no mass flow across the surface of the volume of interest. Alternatively, it is equivalent to the three-dimensional Lagrangian approach, the convection term is zero.

Finite Volume Stress Analysis 1.3. Governing Equation

Constitutive Relation

\[
\mathbf{S} = 2\mu \mathbf{E} + \lambda \text{tr}(\mathbf{E}) \mathbf{I}
\]

Strain Measure

\[
\mathbf{E} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T \cdot \nabla \mathbf{u})
\]

Lagrangian Green Strain Tensor
The presented mathematical models can be arranged in the general form:

1.1, with the computational node overlap and fill the space completely. A typical control volume cell is shown in Figure

ber of convex polyhedral cells bounded by convex polygonal faces. The cells do not solved in a time-marching manner. The solution domain space is split into a finite num-

Discretisation of the solution domain comprises the discretisation of time and the dis-

procedure.

The mathematical models of the governing equations presented in the preceding section

presented exact integral relations. The discretisation procedure is separated into two

are now discretised using the cell-centred finite volume method. It is important to note

The mathematical models of the governing equations presented in the preceding section

Decompose into Implicit & Explicit Terms

Updated Lagrangian Approach

\[
\frac{\delta}{\delta t} \int_{\Omega_u} \rho_u \frac{d\mathbf{u}}{dt} \, d\Omega_u = \frac{1}{\delta t} \oint_{\Gamma_u} [2\mu \delta \mathbf{E}_u + \lambda tr(\delta \mathbf{E}_u) \mathbf{I}] \cdot d\mathbf{\Gamma}_u + \\
\frac{1}{\delta t} \oint_{\Gamma_u} \left[ \mathbf{S}_u \cdot \delta \mathbf{F}_u^T + \delta \mathbf{S}_u \cdot \delta \mathbf{F}_u^T \right] \cdot d\mathbf{\Gamma}_u + \int_{\Omega_u} \rho_u \frac{\delta \mathbf{b}}{\delta t} \, d\Omega_u
\]

Implicit Term

Explicit Term

\[
\frac{d}{dt} \int_{\Omega_\phi} \rho_\phi \frac{d\phi}{dt} \, d\Omega_\phi = \oint_{\Gamma_\phi} (2\mu + \lambda) \nabla \phi \cdot d\Gamma_\phi + \oint_{\Gamma_\phi} \mathbf{Q}_\Gamma \cdot d\Gamma_\phi + \int_{\Omega_\phi} \mathbf{Q}_\Omega \, d\Omega_\phi
\]
Implementation in OpenFOAM-1.6-ext

```
elasticNonLinSolidFoam

fvVectorMatrix DUEqn
{
    fvm::d2dt2(rho,DU)
    ==
    fvm::laplacian(2*mu + lambda, DU)
    + fvc::div
    {
        mu*gradDU.T() + lambda*(I*tr(gradDU))
        - (mu + lambda)*gradDU
        + mu*(gradDU & gradDU.T())
        + 0.5*lambda*(gradDU && gradDU)*I
        + ((sigma + DSigma) & DF.T())
    }
};
```
Discretisation & Non-Orthogonal Correction

$$\int_{\Gamma_\phi} D \left( \nabla \phi \right) \cdot d\Gamma_\phi = \sum_{f=1}^{F} \int_{\Gamma_f} D_f \left( \nabla \phi \right)_f \cdot d\Gamma_f$$

$$\approx \sum_{f=1}^{F} D_f \left[ |\Delta_f| \frac{\phi_N - \phi_P}{|d_f|} + k_f \cdot \left( \nabla \phi \right)_f \right] |\Gamma_f|$$

Implementation of Custom Boundary Conditions with Non-Orthogonal Correction

- fixedDisplacement
- solidTraction
- solidSymmetryPlane
- solidDirectionMixed
Numerical Test Cases
Steady-State Rotation of a Sphere
Rotation of a Sphere

During quasi-static rotation, a body experiences zero stress. To examine that the current updated Lagrangian procedure correctly predicts zero stress, a sphere, radius = 0.1 m, subjected to a quasi-static rotation is considered. The mesh of the sphere, created in commercial meshing software ANSYS ICEM CFD \[47\], contains 1,664 hexahedral cells, some of which are moderately non-orthogonal, as shown in Figure 1.6. A Young's modulus of 200 GPa, and a Poisson's ratio of 0.3 has been employed.

A rotational displacement of 90° about the z-axis is applied to the boundary surface of the sphere, in increments of 1°. The displacement increment for boundary face \(f\) is given by:

\[
\delta u_f = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot C_f - C_f
\]

where \(\theta\) is the increment of rotation, and \(C_f\) is the positional vector of the boundary face centre.

As the small strain approach neglects the higher order strain terms, unphysical stresses are predicted when there are large rotations \[31\]. To check this, the current rotation problem has additionally been simulated using the developed small strain total Lagrangian solver. For 1° of rotation, the small strain solver predicts the uniform stress.
Rotation of a Sphere

Initial Attempt with fixedValue Boundary (no boundary non-orthogonal correction)
Rotation of a Sphere

After 1° Rotation

fixedValue

fixedDisplacement
Rotation of a Sphere

Tight solution tolerance required to ensure correct zero stresses

Degrees Rotation: 1
Rotation of a Sphere

Mesh movement becomes skewed

Standard inverse-distance interpolation

Time: 0 s
Mesh Movement

Interpolation from cell centres to vertices
Interpolation

Determine value at P from values at N

Consider an internal vertex, \( P \), in an unstructured mesh, as shown in Figure 1.4. The value of variable \( \phi \) is known at the \( N \) neighbour cell centres, \( N_i \). It is required to calculate a good approximation of \( \phi \) at the vertex \( P \).

We want to find the value of \( \phi \) at point \( P \). We know the value of \( \phi \) at all neighbours \( N \). \( P \) is at a vertex, \( N \) are at cell centres. The cells are of arbitrary shape and orientation, and \( P \) is connected to an arbitrary number of cells but at least one.

We fit a plane to the values of \( \phi \) at the neighbours \( N \) surrounding \( P \). A plane has the general equation:

\[
LS(x, y, z) = ax + by + cz + d
\]

We use the least squares method to determine the best fit values of \( a, b, c \) and \( d \). The error at \( N \), \( e_N \), between \( N \) and the plane \( LS \) is:

\[
e_N = LS(x, y, z) - (ax_N + by_N + cz_N + d)
\]

Where \( x_N, y_N \) and \( z_N \) are the 3D spatial coordinates of the point \( N \).
Inverse Distance Interpolation

\[ \phi_P = \frac{\sum_{i=1}^{N} \omega_{PN_i} \phi_{N_i}}{\sum_{i=1}^{N} \omega_{PN_i}} \]

\[ \omega_{PN_i} = \frac{1}{|r_P - r_{N_i}|} \]
Least Squares Interpolation

The method of least squares is used to determine the coefficients of the linear equation $\phi(x, y, z) = ax + by + cz + d$.

The inverse distance interpolation method essentially calculates the value of a property at a point by finding a weighted average of the values at surrounding cell centres, where the weighting factor is the inverse of the distance from the point to the cell centre.

The error at each neighbour, $e_{Ni}$, is given by:

$$e_{Ni} = \phi(x_{Ni}, y_{Ni}, z_{Ni}) - \phi_{Ni} = ax_{Ni} + by_{Ni} + cz_{Ni} + d - \phi_{Ni}$$

The sum of the errors squared, $J$, is given by:

$$J = \sum_{i=1}^{N} \left[ ax_{Ni} + by_{Ni} + cz_{Ni} + d - \phi_{Ni} \right]^2$$
Least Squares Interpolation

leastSquaresVolPointInterpolation class

\[
\begin{bmatrix}
\sum_{i=1}^{N} x_{Ni}^2 & \sum_{i=1}^{N} x_{Ni} y_{Ni} & \sum_{i=1}^{N} x_{Ni} z_{Ni} & \sum_{i=1}^{N} x_{Ni} \\
\sum_{i=1}^{N} x_{Ni} y_{Ni} & \sum_{i=1}^{N} y_{Ni}^2 & \sum_{i=1}^{N} y_{Ni} z_{Ni} & \sum_{i=1}^{N} y_{Ni} \\
\sum_{i=1}^{N} x_{Ni} z_{Ni} & \sum_{i=1}^{N} y_{Ni} z_{Ni} & \sum_{i=1}^{N} z_{Ni}^2 & \sum_{i=1}^{N} z_{Ni} \\
\sum_{i=1}^{N} x_{Ni} & \sum_{i=1}^{N} y_{Ni} & \sum_{i=1}^{N} z_{Ni} & N
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
= \begin{bmatrix}
\sum_{i=1}^{N} \phi_{Ni} x_{Ni} \\
\sum_{i=1}^{N} \phi_{Ni} y_{Ni} \\
\sum_{i=1}^{N} \phi_{Ni} z_{Ni} \\
\sum_{i=1}^{N} \phi_{Ni}
\end{bmatrix}
\]

Solve this linear system using Gaussian elimination implemented in simpleMatrix class
Special treatment of boundaries

Coupled boundaries uses neighbour cell centre values, and empty patch values are included so A is not singular.
Comparison of Inverse Distance & Least Squares Methods
Comparison of Inverse Distance & Least Squares Methods

Inverse Distance

Least Squares
Comparison of Inverse-Distance & Least Squares Methods

Inverse Distance

Least Squares
Comparison of Inverse-Distance & Least Squares Methods

Rotation: 0

Inverse Distance

Rotation: 0

Least Squares
An updated Lagrangian finite volume structural solver suitable for large rotations has been developed.

Boundary non-orthogonal correction is imperative.

Standard inverse distance interpolation shows poor performance on non-orthogonal grids.

Therefore, a least squares approach has been developed.
Development of a Finite Volume Based Structural Solver for Large Rotation of Non-Orthogonal Meshes

Thanks

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