Extrusion Process by Finite Volume Method Using OpenFoam Software

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Abstract. The computational codes are very important tools to solve engineering problems. In the analysis of metal forming process, such as extrusion, this is not different because the computational codes allow analyzing the process with reduced cost. Traditionally, the Finite Element Method is used to solve solid mechanic problems, however, the Finite Volume Method (FVM) have been gaining force in this field of applications. This paper presents the velocity field and friction coefficient variation results, obtained by numerical simulation using the OpenFoam Software and the FVM to solve an aluminum alloy direct cold extrusion process, and comparisons with the experimental results obtained by Thomsen and Frisch.

Keywords: Finite Volume Method, Extrusion, Friction, Aluminium.

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INTRODUCTION

Nowadays, in the engineering practice, the industrial process of fabrication of parts and products are increasingly being modeled mathematically by employing the computational codes. The metal forming has a huge importance inside of the fabrication techniques of metallic parts, where the extrusion process has an important role because is widely utilized to produce bars of complex cross section profile.

In an extrusion process, the knowledge of the metal behavior aspect such as: velocity field, deformations, stresses, friction and temperature can help to predict failure in components or products produced by extrusion. One way to determine such information is the numerical method, that are fast and accurate and by this manner, helping to reduce the cost that could appear in the experimental process.

Traditionally, the numerical method used to solve solids mechanic problems has been the Finite Element Method. However, the Finite Volume Method (FVM) has been gaining space in the resolution of solids mechanic problems in the last decade.

In 90’s at Imperial College of London, a free code package written in C++ and using the FVM to make discretization of the mathematical equations, was created by Professor Hrvoje Jasak’s research group. This free code package was used to
manipulate and operate tensor field and it was in the beginning and for a long time a CFD (Computational Fluid Dynamic) package [5,6,7]. This free code package was called OpenFoam (Open Field Operation and Manipulation) [8,9] and nowadays is also being used to solve solids mechanic problems.

Following the tendency to apply the FVM in solid mechanic problems and using the OpenFoam tool, it was written a computational program to predict the velocity field and the friction variation in an aluminum alloy direct cold extrusion process.

**MATHEMATICAL MODEL FOR COLD METAL EXTRUSION**

**Governing Equations**

In a cold extrusion process, the governing equations for metal flow are given by the principles of conservation of mass and momentum that are described in the integral form and Eurelian approach [1] by:

\[
\frac{d}{dt} \int_V \rho \, dV + \int_S \rho \mathbf{v} \cdot dS = 0
\]  

(1)

\[
\frac{d}{dt} \int_V \rho \mathbf{v} \, dV + \int_S \rho \mathbf{v} \mathbf{v} \cdot dS = \int_S \mathbf{\sigma} \cdot dS + \int_V \rho \mathbf{b} \, dV
\]  

(2)

where, \( \rho \) is the specific mass, \( t \) is the time, \( \mathbf{v} \) is the velocity vector, \( V \) is the volume, \( S \) is the surface area, \( \mathbf{b} \) are body forces and \( \mathbf{\sigma} \) are the surface forces.

**Constitutive Equations**

Usually, the constitutive equations used in an extrusion process are for a material with rigid-visco-plastic behavior and the elastic properties are neglected.

In this model, the equation that make the relation between stresses with strain rate is given by generalized Stocks Law presented by Eq. 3, the strain rate tensor is given by Eq. 4, the dynamic viscosity is calculated using the Levy-Mises flow Law, that for perfect-plastic case, i.e., material with no strain hardening, is presented by Eq. 5.

\[
\mathbf{\sigma} = -p \mathbf{I} + 2\eta \dot{\mathbf{\varepsilon}}
\]  

(3)

\[
\dot{\mathbf{\varepsilon}} = \frac{1}{2} \left[ \text{grad} \mathbf{v} + (\text{grad} \mathbf{v})^T \right]
\]  

(4)

\[
\eta = \frac{1}{3} \frac{\sigma_y}{\dot{\varepsilon}}
\]  

(5)

Where, \( \mathbf{\sigma} \) is the stress tensor, \( \dot{\mathbf{\varepsilon}} \) is the strain rate tensor, \( p \) is the pressure, \( \mathbf{I} \) is identity tensor, \( \eta \) is the dynamic viscosity, \( \mathbf{v} \) is the velocity vector, \( \sigma_y \) is the yield stress, \( \dot{\varepsilon} \) is the effective strain rate. Eq. 5 is valid for \( \dot{\varepsilon} \neq 0 \), otherwise, the material
would behave as rigid. Because of numerical stability in the simulation process, a minimum value must be defined and this value used was about $\dot{\varepsilon} \leq 10^{-3} \, s^{-1}$.

**Numerical Method**

The OpenFoam software uses a numerical structure described in the numerical method presented with details in [1-2]. In the numerical method presented, the governing equations jointly with the Eq. 3 can be represented by the general transport equation that, after discretized, it can be written in the following manner:

$$ rac{d}{dt} \int_V \rho \mathbf{B} \, dV + \sum_{j=1}^{n_v} \sum_{S_j} \int_{S_j} \rho \mathbf{F} \cdot dS - \sum_{j=1}^{n_v} \sum_{S_j} \int_{S_j} \Gamma \phi \mathbf{G} \cdot dS = \sum_{j=1}^{n_v} \sum_{S_j} \int_{S_j} q_{\phi S} \cdot dS + \int_V q_{\phi V} \, dV \quad (6) $$

where, the generic variable $\phi$ represents the velocity components $v_i$ and the meaning of the others terms can be found in the reference [1,2].

In the discretization process of the Eq. 6, the time was divided in a finite number of the subinterval $\Delta t$ and the spatial domain was divided in a finite number of contiguous controls volumes of arbitrary shape and considering the computational point situated in the control volume center.

Assembling the terms of the Eq. 6 generates a set of coupled non-linear algebraic equations that are solved employing segregated iterative procedure. This procedure decoupled the set of equations, generating a linear algebraic equations system. The system is solved using the conjugate gradient method with incomplete Cholesky preconditioning. Others details of the procedure are found in [1,2].

The coupling of velocity-pressure was made using the PISO method that is presented in details in the reference [3].

To complete the mathematical model, for the cold extrusion process, it should be specified the initial conditions and the boundary conditions. In the initial conditions must be imposed a value of velocity to all domain points. The OpenFoam allow the imposed or Dirichlet boundary condition type where the velocity is specified in the boundary either the Neumann kind, where the velocity gradient can be imposed in the boundary.

**RESULTS**

**Case Description**

The investigated case with the OpenFoam – FVM was the direct cold extrusion process of aluminum alloy which was performed experimentally by Thomsen and Frisch [4]. The simulation employed an unstructured and three-dimensional mesh with 7238 hexahedral elements. The density of aluminum was $2850 \, \text{g/m}^3$ and yield stress was $80 \, \text{MPa}$. The extrusion velocity was $v_0 = 1.9 \cdot 10^{-2} \, \text{m/s}$ and minimum effective strain rate was $\dot{\varepsilon}_0 = 7 \cdot 10^{-3} \, \text{s}^{-1}$. Figure 1 shows the extruded geometry and the mesh,
where it can be observed that the reduction was from a square profile to a square bar. It was simulated two kinds of cases: in the first case, it was imposed no friction condition in the die wall surface contact that was given by a zero velocity gradient. In the second case, it was imposed a different friction condition by applying different velocity gradient on the die wall contact surface, imposing a different friction factor.

Figure 2 shows the comparison between OpenFoam-FVM results for the half left side with the right side experimental results presented in [4]. The experimental test was carried out ensuring zero friction. To simulate this condition on the OpenFoam – FVM, it was imposed zero gradient onto the tangential velocity component on the die wall contact surface and it was fixed a value zero onto the normal velocity component. The results presented in the Figure 2 represent the profile of constant velocity contours calculated by $|v|/v_0$. In Figure 2, it can be observed that there is a good agreement between the OpenFoam – FVM results with the experimental results presented in [4]. However, the profile of constant velocity contours in the billet center part show better agreement than the profile velocity contours near the die wall contact surface.

**FIGURE 1.** Geometry of extrusion and the unstructured mesh for the simulated case.

**Discussions Results**

**FIGURE 2.** Comparison between profile of constant velocity contours, $|v|/v_0$, for no friction extrusion process of aluminum alloy. (a) OpenFoam – FVM results, (b) experimental results by Thomsen [4].
Figure 3 shows the friction factor $m$ variation in function of the velocity gradient $\nabla \text{grad}U_{yz}$ variation imposed on the die wall contact surface. The friction factor was calculated by the following equation:

$$m = \frac{2 \dot{\gamma}_{ij}}{3 \bar{e}}$$  \hspace{1cm} (7)

where, $m$ is the friction factor and $\dot{\gamma}_{ij}$ is the plastic shear strain rate. In the graphic showed in the Figure 3, it can be observed that a variation of one hundred times in the velocity gradient produced only a variation of ten times in the friction factor.

Figure 4 presents the comparison between the profiles of constant velocity contours for OpenFoam – FVM results with the no friction experimental results. The simulation results was calculated imposing a gradient velocity of $1.4 \times 10^{-3} \text{s}^{-1}$ on the die walls contact surfaces and this value produced a friction factor equal to $m = 0.112$. From this, it can be observed that both profiles are much different, this happened because the level of friction is very different.

**CONCLUSIONS**

From the analysis of the numerical simulation results for cold extrusion of aluminum alloy by OpenFoam, the following conclusions can be obtained:

- For the extrusion process case with no friction, the profile of constant velocity contours, the comparison between OpenFoam – FVM and the experimental results has good agreement;
- If we change the velocity gradient imposed on the die walls contact surfaces we can see the friction factor modification and influence;
- OpenFoam is a powerful computational tool to operate tensors field that appear in solids mechanic problems;
- FVM is a numerical tool which is able to generate good results in solids mechanic problems.
FIGURE 4. Comparison between profiles of constant velocity contours. (a) OpenFoam – FVM results for $m = 0.1$, (b) experimental results by Thomsen and Frisch for $m = 0$ [4].

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