Title	Appendix to "Precocious Albion: a new interpretation of the British industrial revolution"
Authors(s)	Kelly, Morgan, Mokyr, Joel, Ó Gráda, Cormac
Publication date	2013-09
Publication information	Kelly, Morgan, Joel Mokyr, and Cormac Ó Gráda. "Appendix to 'Precocious Albion: A New Interpretation of the British Industrial Revolution'" (September, 2013).
Series	UCD Centre for Economic Research Working Paper Series, WP13/12
Publisher	University College Dublin. School of Economics
Item record/more information	http://hdl.handle.net/10197/4797

UCD CENTRE FOR ECONOMIC RESEARCH

WORKING PAPER SERIES

2013

Appendix to "Precocious Albion: a New Interpretation of the British Industrial Revolution"

Morgan Kelly and Cormac Ó Gráda,
University College Dublin and
Joel Mokyr, Northwestern University

WP13/12

September 2013

UCD SCHOOL OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4

Appendix to "Precocious Albion: a New Interpretation of the British Industrial Revolution"

Morgan Kelly, Joel Mokyr, and Cormac Ó Gráda.

This appendix gives a formal derivation of the model described in the paper.

1 A Model of Human Capability and Technological Take-off.

In this section we outline a model of growth where an economy's ability to implement new technology depends on the human capability of its workers, which is determined in turn by the investment of their parents in feeding and educating them. We do not model the act of invention itself but focus on adoption. The adoption of a new technique usually required a fair amount of tweaking and microinvention to adapt a technique to local circumstances. To adopt technology, a country thus needs workers with some minimum level of "competence," which is acquired by investment in human capability through childhood nutrition and education in the form of master-apprentice contact. Output is a function of both the quantity of workers and their "quality" (human capability). The model combines the Nelson and Phelps (1966) model in which productivity growth depends on the difference between the actual techniques in use and the productivity of best-practise techniques with the Ben-Porath (1967) model, which analyses the growth of human capability as a function of parental investment decisions.

The model concerns the diffusion of best-practice knowledge from elite scientists to the level of ordinary artisans where it can be incorporated into everyday production. We therefore define the state of the art level of international scientific knowledge by $\widetilde{A}$ and assume that it grows exogenously. Within a given

country, the level of technology in use is A. This level of technology evolves according to a Nelson-Phelps process, depending on the gap between scientific knowledge and the country's own technology; and on the level of skill of ordinary workers H:

$$\frac{A_t}{A_{t-1}} = \begin{cases} \left(\frac{\widetilde{A}}{A_{t-1}}\right)^{\delta} H_{t-1}^{\eta} & \underline{A} < A_{t-1} < \widetilde{A} \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

where $0 < \delta, \eta < 1$. The technology in use in an economy cannot exceed the frontier value $\widetilde{A}$, and cannot fall below a minimum level $\underline{A}$. To simplify notation in what follows, we assume that the minimum technological level is unity: $\underline{A} = 1$.

The human capability of each artisan evolves according to the Ben-Porath equation

$$\frac{H_t}{H_{t-1}} = I_{t-1}^{\lambda} H_{t-1}^{-\mu} \tag{2}$$

where $0 < \lambda, \mu < 1$. I_{t-1} denotes investment in the young generation of workers in period $t-1$ in the form of nutrition, basic schooling and training as an apprentice, and needs to equal $H_{t-1}^{\mu/\lambda}$ to maintain the existing level of human capability of the workforce.

To close this simple model we need to specify what determines the level of investment in the next generation of workers. We suppose that individuals have two periods in their lives: when young they receive investment from their parents; and when old they receive income as workers that they use to maximize utility which comes from their own consumption C_t and investment in their child (we assume constant population for now)

$$U(C_t, I_t) = C_t^{1-\gamma} I_t^{\gamma} \tag{3}$$

where $0 < \gamma < 1$. Workers supply one unit of labour inelastically, and live hand to mouth, making and receiving no bequests. It follows that parents invest a fraction γ of their income in their children.

Output in this economy comes from the production function

$$Y_t = A_t H_t^{\alpha} N_t^{1-\alpha} \tag{4}$$

where $0 < \alpha < 1$ and N is the number of workers. Every worker receives an income Y_t/N_t.

Each worker pays a share of his income to the government or landlords, receiving nothing in return. In addition, however, the government may tax the

landlord class and redistribute this money to workers in the form of a Poor Law. It follows that the disposable income of workers is $(1-\tau)\,Y/N$ where τ is the net rate of tax and rent after subtracting Poor Law transfers.

To analyse the evolution of useful knowledge A and human capital H it will be simpler, both for intuition and for drawing phase diagrams, to adopt the trick of using the inverse of human capability

$$M \equiv \frac{1}{H} \tag{5}$$

that we will refer to as misery, remembering that it refers to low levels of nutrition, health, schooling and other outcomes of childhood deprivation.

It follows that useful knowledge and misery evolve according to the log-linear system of difference equations

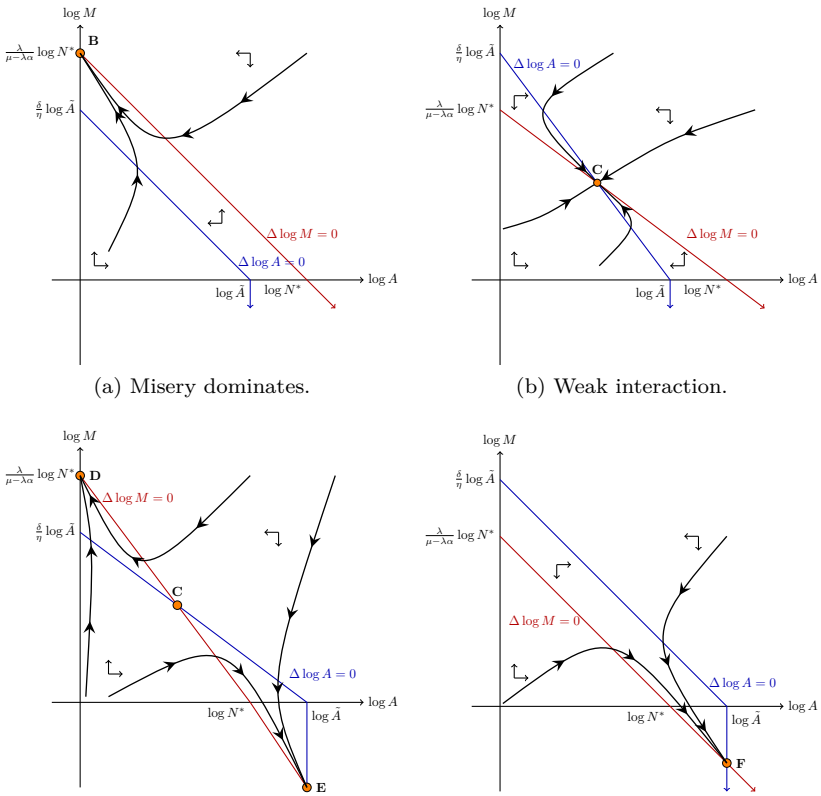
$$\begin{aligned} \Delta \log A_t =& \delta \log \widetilde{A} - \delta \log A_{t-1} - \eta \log M_{t-1} \\ \Delta \log M_t =& \lambda \log \frac{N_t^{\alpha}}{\gamma\,(1-\tau)} - \lambda \log A_{t-1} - (\mu - \lambda\alpha) \log M_{t-1} \end{aligned} \tag{6}$$

Assuming that population is more than a handful of people so that $N_t^{\alpha} > \gamma\,(1-\tau)$, only one coefficient of this system has a sign that is not immediately obvious: the $(\mu - \lambda\alpha)$ term multiplying $\log M$ in the misery equation. If $\mu < \lambda\alpha$, the real wage at the technological minimum rises as population N or net taxes τ increase as (9) below shows. In addition, the misery process is unstable: a rise in misery lowers output and human capital investment, increasing misery next period in a self-reinforcing process so long as population remains stable. We therefore assume that $(\mu - \lambda\alpha)$ is positive.

In this case the system resembles an ecosystem where two biological species, in our case technology and misery, compete against each other for resources so that the growth of one is retarded by the population size of the other: see Hofbauer and Sigmund (1998, 22–30)

The isoclines of the system, between the minimum and maximum levels of technology, are

$$\begin{aligned} \Delta \log A = 0 \qquad & \log M = \frac{\delta}{\eta} \log \widetilde{A} - \frac{\delta}{\eta} \log A \\ \Delta \log M = 0 \qquad & \log M = \frac{\lambda}{\mu - \lambda\alpha} \log \frac{N_t^{\alpha}}{\gamma\,(1-\tau)} - \frac{\lambda}{\mu - \lambda\alpha} \log A \end{aligned} \tag{7}$$



(a) Misery dominates.

(b) Weak interaction.

(c) Strong interaction.

(d) Knowledge dominates.

Figure 1: The four equilibria of the knowledge-misery system.

The knowledge-misery system has four possible steady states depending on the relative position and slope of these isoclines.[1] In the phase diagrams we denote $N_t^{\alpha}/\gamma\,(1-\tau)$ by N^*.

In the first panel of Figure 1, the misery isocline lies everywhere above the knowledge isocline so that misery dominates. The only equilibrium is at point B, with the log of useful knowledge equal to its lower bound, which we have set at zero.

[1] There is also the case where the isoclines exactly coincide, making every point along them a steady state.

In the second and third panels the isoclines intersect at $C = \left(\overline{\log A}, \overline{\log M}\right)$ where

$$\begin{aligned}\overline{\log A} &= \frac{1}{\lambda\eta - \delta(\mu - \lambda\alpha)}\left[\lambda\eta \log \frac{N_t^\alpha}{\gamma(1-\tau)} - \delta(\mu - \lambda\alpha)\log \widetilde{A}\right] \\ \overline{\log M} &= \frac{\delta\lambda}{\delta(\mu - \lambda\alpha) - \lambda\eta}\left[\log \frac{N_t^\alpha}{\gamma(1-\tau)} - \log \widetilde{A}\right] \end{aligned} \tag{8}$$

In the second panel, the own effect terms in (6) dominate the cross effect terms $\delta(\mu - \lambda\alpha) > \lambda\eta$ so the technology isocline is steeper. As a result, the intersection point C is globally stable.

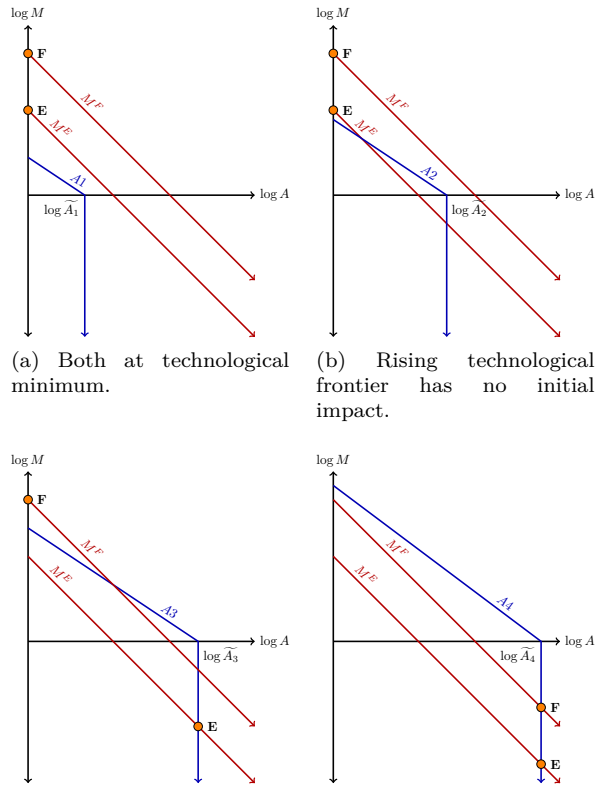
In the third panel, cross effects dominate own effects $\delta(\mu - \lambda\alpha) < \lambda\eta$ so the misery curve is steeper. As a result, the intersection point $C = \left(\overline{\log A}, \overline{\log M}\right)$ is a saddle dividing the space into two basins of attraction, one converging on point D with technology at its lower bound, the other at point E where the misery isocline cuts the upper bound of technology $\log \widetilde{A}$.

Finally, in the last panel of Figure 1, the knowledge isocline lies everywhere above the misery isocline, so the system converges to a steady state at point F where the misery isocline cuts the upper bound of technology $\log \widetilde{A}$.

Intuitively, the evolution of useful knowledge and misery in (6) resembles an eco-system with two competing species: the growth of each species is retarded by the presence of the other. In the first panel of Figure 1, conditions are so favourable to the first "species", misery in our case, that its "population" will be high regardless of the second species which it always drives to its minimum level; with the converse holding in the fourth panel where knowledge dominates. In the second panel, the species have little impact on each other and both co-exist at positive levels, while in the third panel they have a strong impact on each other but the outcome depends on which species initially has a sufficiently large population to dominate the system. We now show how this simple interaction of knowledge and misery leads to a sudden take-off in knowledge: an Industrial Revolution.

1.1 France and England.

There are two economies that we shall call France and England. Each faces the same technological frontier $\tilde{A}$ that rises through time, reflecting the progress of Enlightenment scientific knowledge.



(a) Both at technological minimum.

(b) Rising technological frontier has no initial impact.

(c) England moves to technological frontier.

(d) France moves to technological frontier.

Figure 2: Impact of rising technological frontier $\widetilde{A}$ on equilibrium in two economies with different misery isoclines.

France and England differ in only one way: because of a poor law, disposable income is higher in England than in France: $\tau_E < \tau_F$ where subscripts E and F denote England and France respectively.[2]

This implies that, at the technological minimum where the system begins, workers in each economy $i = E, F$ have log disposable income

$$\log(1-\tau_i)\, w_i = \frac{1}{\mu - \lambda\alpha}\left[\log\gamma + \mu\log(1-\tau_i) - \alpha\mu\log N_i\right] \tag{9}$$

In this pre-industrial world, both economies start in Malthusian equilibrium where births equal deaths. For this to be possible with English living standards

[2] We can also allow England to have a higher value of λ in the Ben-Porath equation (2), to reflect the greater efficiency of its apprenticeship system in creating useful skills; but we will focus here solely on the impact of greater transfers to workers.

and, as we will see below, life expectancies above French ones, it must be that England has a lower marriage rate and fertility of marriage. This can stem from the incentive created by a poor law for landlords to minimize the number of mouths they have to subsidize as the disposable income equation (9) shows. If landlords are required to provide workers with a higher level of disposable income, they can increase poor law transfers by reducing net taxes and rents τ_i, or they can reduce N_i, by discouraging early marriage and so increase the marginal product of labour directly. We therefore expect that, in otherwise identical regions of France and England, the English requirement to provide a higher level of subsistence to workers implies that English regions would support lower populations: $N_E < N_F$. This means that the English misery isocline $\Delta \log M = 0$ will lie below the French one.

In Figure 2 we denote the English and French misery isoclines by M^E and M^F respectively, and the equilibrium of each economy by E and F. We want to see how these change as the technological frontier $\tilde{A}$ gradually rises through time.

Our starting point, in panel (a), is a stark Malthusian world with little knowledge: $\log \widetilde{A}$ is arbitrarily small and the knowledge isocline $A1$ lies completely below the two misery isoclines. As a result, both economies are at an equilibrium at the lower bound of knowledge. As time passes scientific knowledge $\tilde{A}$ will rise exogenously, reflecting the progress of the Enlightenment, and this will be the driving force behind the model.

In panel (b) the knowledge frontier has risen so that the knowledge isocline $A2$ now intersects the English misery isocline. We suppose that misery and useful knowledge strongly affect each other $\delta(\mu - \lambda\alpha) < \lambda\eta$ so that the misery isocline is steeper: when the opposite holds the evolution of the system is broadly similar as we will see below. While a steady state exists at the knowledge frontier, as in the third panel of Figure 1, because the English economy is starting in the basin of attraction of the low knowledge equilibrium, it stays at this point. The rising technological frontier has no impact on production technology because the level of human capability is too low to absorb it: the technological enlightenment has no impact down on the farm.

In the third panel of Figure 2, the continued gradual rise in scientific knowledge causes the technology isocline to move above the English misery isocline, but still to cut the French one from above. As a result, while France stays at the minimum technology steady state at F, England jumps to the technological frontier at E. A small rise in the knowledge frontier causes a sudden divergence

between the economies to occur. Because English human capability is slightly higher than French, thanks to a poor law that transfers resources to workers and gives landlords an incentive to minimise population, England can start to apply technological knowledge to production, giving rise to a cumulative process of rising living standards, rising human capital, and rising production technology. A gradual rise in knowledge above a critical level causes England to experience an industrial revolution, while France appears mired in age-old backwardness.

This divergence is not permanent however. As the knowledge frontier $\log \widetilde{A}$ continues to rise in the last panel of Figure 2, the technology isocline $A4$ moves above the French misery isocline, causing France to converge to the same technological frontier as England. While technology in both economies is the same, living standards in England will be higher so long as it continues to enjoy higher transfers and a lower population. However, to the extent that rising living standards erode political support for a generous poor law, while urbanization removes social restrictions on early marriage, English and French living standards will converge.

If, on the other hand, misery and technology interact weakly $\delta(\mu - \lambda\alpha) > \lambda\eta$ so that the knowledge isocline is steeper, the evolution of the system is slightly different. When the technology isocline first rises above the English misery isocline, England moves to a steady state where the two intersect, and France will follow some time afterward. Both economies move steadily down along their misery isoclines as the knowledge isocline rises, until they reach the technological frontier.

References

Ben-Porath, Y. 1967. "The production of human capital and the life cycle of earnings." *Journal of Political Economy* 75(4):352–365.

Hofbauer, Josef and Karl Sigmund. 1998. *Evolutionary Games and Population Dynamics*. Cambridge: Cambridge University Press.

Nelson, Richard R. and Edmund D. Phelps. 1966. "Investment in Humans, Technological Diffusion, and Economic Growth." *American Economic Review* 56:69–75.

UCD CENTRE FOR ECONOMIC RESEARCH – RECENT WORKING PAPERS

WP12/19 Adrian Corcoran and Robert Gillanders: 'Foreign Direct Investment and The Ease of Doing Business' July 2012
WP12/20 Wen Fan and Yuanyuan Ma: 'Estimating the External Returns to Education: Evidence from China' August 2012
WP12/21 David Madden: 'The Socioeconomic Determinants of Mental Stress in Ireland' August 2012
WP12/22 Sinéad Kelleher: 'Playing by the Rules? The Development of an Amended Index to Measure the Impact of Rules of Origin on Intra-PTA Trade Flows' September 2012
WP12/23 Christopher Jepsen, Kenneth Troske and Paul Coomes: 'The Labor-Market Returns to Community College Degrees, Diplomas, and Certificates' September 2012
WP12/24 Ronald B Davies: 'CCCTB 4 EU? SA vs. FA w/ FTA' October 2012
WP12/25 Ronald B Davies and Tine Jeppesen: 'Export mode, Trade Costs, and Productivity Sorting' October 2012
WP12/26 Vincent Hogan, Patrick Massey and Shane Massey: 'Competitive Balance and Match Attendance in European Rugby Union Leagues' October 2012
WP12/27 Mark E McGovern: 'Don't Stress: Early Life Conditions, Hypertension, and Selection into Associated Risk Factors' October 2012
WP12/28 Vincent Hogan, Patrick Massey and Shane Massey: 'Analysing Determinants of Match Attendance in the European Rugby Cup' October 2012
WP12/29 Karl Whelan: 'TARGET2 and Central Bank Balance Sheets' November 2012
WP12/30 Wasiu Adekunle Are: 'Poverty-Reducing Directions of Indirect Marginal Tax Reforms in Ireland' December 2012
WP12/31 Wasiu Adekunle Are: 'Growth and Income Redistribution Components of Changes in Poverty: A Decomposition Analysis for Ireland, 1987-2005' December 2012
WP13/01 David Madden: 'Self-Reported and Measured BMI in Ireland: Should We Adjust the Obesity Thresholds?' February 2013
WP13/02 Cormac Ó Gráda: 'Eating People is Wrong: Famine's Darkest Secret?' March 2013
WP13/03 David Madden: 'The Poverty Effects of a "Fat-Tax" in Ireland' March 2013
WP13/04 Cormac Ó Gráda: 'Great Leap, Great Famine' March 2013
WP13/05 David Madden: 'Health and Wealth on the Roller-Coaster: Ireland, 2003-2011' May 2013
WP13/06 Karl Whelan: 'Ireland's Economic Crisis - The Good, the Bad and the Ugly' July 2013
WP13/07 Neil Cummins, Morgan Kelly, and Cormac Ó Gráda: 'Living Standards and Plague in London, 1560–1665' July 2013
WP13/08 Neil Cummins, Morgan Kelly and Cormac Ó Gráda: 'Living Standards and Plague in London, 1560–1665' July 2013
WP13/09 Karl Whelan: 'Sovereign Default and the Euro' July 2013
WP13/10 Orla Doyle, Colm Harmon, James J Heckman, Caitriona Logue and Seong Hyeok Moon: 'Measuring Investment in Human Capital Formation: An Experimental Analysis of Early Life Outcomes' August 2013
WP13/11 Morgan Kelly, Joel Mokyr and Cormac Ó Gráda: 'Precocious Albion: a New Interpretation of the British Industrial Revolution' September 2013