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The Conditional Pricing of Systematic and Idiosyncratic Risk in the U.K. Equity Market

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Abstract
We test whether firm idiosyncratic risk is priced in a large cross-section of U.K. stocks. A distinguishing feature of our paper is that our tests allow for a conditional relationship between systematic risk (beta) and returns in our tests, i.e., conditional on whether the excess market return is positive or negative. We find strong evidence in support of a conditional beta/return relationship which in turn reveals conditionality in the pricing of idiosyncratic risk. We find that idiosyncratic risk is significantly negatively priced in stock returns in down-markets. Although perhaps initially counter-intuitive, we describe the theoretical support for such a finding in the literature. Our results also reveal a strong role for liquidity, size and momentum factors in explaining the cross-section of U.K. stock returns.

JEL Classification: G11; G12.
Key Words: asset pricing; idiosyncratic risk; turnover; conditional beta.

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1. Introduction

Idiosyncratic, or non-systematic, risk arises due to asset price variation that is specific to a security and is not driven by a systematic risk factor common across securities. It is typically estimated using a pricing model with explicit risk factors and obtained as the residual unexplained variation. In this paper we revisit the question of whether idiosyncratic risk is priced in a large cross-section of U.K. stocks. A distinguishing feature of our paper is that we incorporate a conditional relationship between systematic risk (beta) and return in our tests for which we find strong evidence. This in turn reveals conditionality in the pricing of idiosyncratic risk. We control for other stock risk characteristics such as liquidity (which we decompose into systematic and idiosyncratic liquidity), size and momentum risks which may explain some idiosyncratic risk.

The role of idiosyncratic risk in asset pricing is important as investors are exposed to it for a number of reasons, either passively or actively. These include portfolio constraints, transaction costs that need to be considered against portfolio rebalancing needs or belief in possessing superior forecasting skills\(^1\). Assessing if and how idiosyncratic volatility is priced in the cross-section of stock-returns is relevant in order to determine if compensation is earned from exposure to it. Controlling for systematic risk factors and other stock characteristics, if the expected risk premium for bearing residual risk is positive, it may support holding idiosyncratic difficult-to-diversify stocks and other undiversified portfolio strategies. Conversely, negative pricing of idiosyncratic risk clearly points to increased transaction costs to achieve a more granular level of portfolio diversification to offset it. Idiosyncratic risk is important and large in magnitude, and accounts for a large proportion of total portfolio risk.\(^2\) A better characterization of it will

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\(^1\) Portfolio constraints include the level of wealth, limits on the maximum number of stocks held or restrictions from holding specific stocks or sectors. Funds with a concentrated style willingly hold a limited number of stocks. Even large institutional portfolios benchmarked to a market index rarely hold all stocks in the index but typically purchase a subset of stocks and use techniques to minimize non-systematic exposures.

\(^2\) Campbell et al. (2001) for a US sample find firm-level volatility to be on average the largest portion (over 70\%) of total volatility, followed by market volatility (16\%) and industry-level volatility (12\%). Our results are broadly consistent, with the firm-level component accounting on average for over 50\% of total variance, with the rest evenly split between the market and industry components.
improve the assessment of portfolio risk exposures and the achievement of risk and return objectives.

Notwithstanding this, traditional pricing frameworks such as the CAPM imply that there should be no compensation for exposure to idiosyncratic risk as it can be diversified away in the market portfolio. However, this result has been challenged both theoretically and empirically. Alternative frameworks relax the assumption that investors are able or willing to hold fully diversified portfolios and posit a required compensation for idiosyncratic risk. Merton (1987) shows that allowing for incomplete information among agents, expected returns are higher for firms with larger firm-specific variance. Malkiel and Xu (2002) also theorise positive pricing of idiosyncratic risk using a version of the CAPM where investors are unable to fully diversify portfolios due to a variety of structural, informational or behavioural constraints and hence demand a premium for holding stocks with high idiosyncratic volatility. In empirical testing several studies find a significantly positive relation between idiosyncratic volatility and average returns; Lintner (1965) finds that idiosyncratic volatility has a positive coefficient in cross-sectional regressions as does Lehmann (1990) while Malkiel and Xu (2002) similarly find that portfolios with higher idiosyncratic volatility have higher average returns.

However, the direct opposite perspective on the pricing of idiosyncratic risk, that of a negative relation between idiosyncratic volatility and expected returns, has also been theorised and supported by empirical evidence. One theory links the pricing of firm idiosyncratic risk to the pricing of market volatility risk. Chen (2002) builds on Campbell (1993 and 1996) and Merton’s (1973) ICAPM to show that the sources of assets’ risk premia (risk factors) are the contemporaneous conditional covariances of its return with (i) the market, (ii) changes in the forecasts of future market returns and (iii) changes in the forecasts of future market volatilities. In particular, this third risk factor, which the model predicts has a negative loading, indicates that investors also demand compensation in the form of higher expected return for the risk that an asset will perform poorly when the
future becomes less certain, i.e., higher (conditional) market volatility. Ang et al. (2006) argue that stocks with high idiosyncratic volatilities may have high exposure to market volatility risk, which lowers their average returns, indicating a negative pricing of idiosyncratic risk in the cross-section. If market volatility risk is a (orthogonal) risk factor, standard models of systematic risk will mis-price portfolios sorted by idiosyncratic volatility due to the absence of factor loadings measuring exposure to market volatility risk. In empirical testing on US data, Ang et al. (2006) find that exposure to aggregate volatility risk accounts for very little of the returns of stocks with high idiosyncratic volatility (controlling for other risk factors) which, they say, remains a puzzling anomaly. We add to this literature by investigating the pricing of idiosyncratic volatility in a large sample of U.K. stocks in conditional market settings and controlling for other risk factors and stock characteristics in the cross-section.

Much like the mixed predictions concerning the pricing of idiosyncratic risk arising from theoretical frameworks, empirical findings are also quite mixed. For instance, Malkiel and Xu (2002), Chua et al. (2010), Basu and Martellini (2007) and Fu (2009) detect a positive relationship between idiosyncratic volatility and returns while Ang et al. (2006), Li et al. (2008), Arena et al. (2008) and Ang et al. (2009) find evidence in support of a negative relationship. Furthermore, a conditional idiosyncratic component of stock return volatility is found to be positively related to returns by Chua et al. (2010) and Fu (2009), while conflicting results are found in Li et al. (2008). Despite the use of a variety of theoretical models of agents’ behaviour, pricing models and testing techniques, the debate is still open.

3 Conversely, assets with high sensitivities (covariance) to market volatility risk provide hedges against future market uncertainty and will be willingly held by investors, hence reducing the required return.

4 Jacobs and Wang (2004) develop a consumption-based asset pricing model in which expected returns are a function of cross-sectional (across individuals) average consumption growth and consumption dispersion (the cross-sectional variance of consumption growth). The model predicts (and the evidence supports) a higher expected return the more negatively correlated the stock’s return is with consumption dispersion. An intuitive interpretation is that consumption dispersion causes agents to perceive their own individual risk to be higher. Hence a stock which is sensitive to consumption dispersion offers a hedge, will be willingly held and consequently has a lower required return. Stocks with high idiosyncratic volatilities may have high exposure to consumption dispersion, which lowers their average returns, indicating a negative pricing of idiosyncratic risk in the cross-section.
as to whether idiosyncratic risk is a relevant cross-sectional driver of return, and if it is, whether the relationship with returns is positive or negative. There is evidence that several other cross-sectional risk factors interact with residual risk effects, such as momentum, size and liquidity suggesting that a large part of it might be systematic rather than idiosyncratic (Malkiel and Xu (1997, 2002), Campbell et al. (2001), Bekaert et al. (2012) and Ang et al. (2009)).

While the majority of empirical work deals with U.S. data, in recent years the analysis has been extended to international markets although datasets tend to be more limited covering markets with fewer stocks, sparse trading and have some difficulties relating to market microstructure and data quality (see for instance Ince and Porter (2006) on overall data deficiencies, Bartram et al. (2009) on issues with classification and especially volume and Bekaert et al. (2007) on illiquidity in emerging markets). In order to draw reliable cross-sectional inferences we need an ample and extensive pool of stocks, traded in a single market: we focus on such a U.K. dataset while obtaining results of general interest in terms of sample selection, methodological approach and empirical results.

We contribute to the literature on idiosyncratic risk in a number of ways. A key distinction in our paper is that we take account of the conditional relation between beta and returns (Pettengill et al. 1995), for which we find strong evidence. We use a large sample of 1,333 stocks after rigorous data cleaning to remove serious data deficiencies. Finally, we highlight a strong role for liquidity (turnover) in explaining the cross-section of stock returns.

The paper is set out as follows: section 2 describes the selection and treatment of data while section 3 describes our testing methodology. Results are discussed in section 4 while Section 5 concludes.
2. Data Treatment and Selection

Our starting universe includes all stocks listed on the London Stock Exchange between January 1990 and December 2009 – a period long enough to capture economic cycles, latterly the ‘financial crisis’ and alternative risk regimes. We collect monthly prices, total returns, volume, outstanding shares and static classification information from Datastream. We also collect 1-month GBP Libor rates. Serious issues with international equity data have been highlighted in the literature (Ince and Porter, 2006). These include incorrect information, both qualitative (classification information) and quantitative (prices, returns, volume, shares etc), a lack of distinction between the various types of securities traded on equity exchanges, issues of coverage and survivorship bias, incorrect information on stock splits, closing prices and dividend payments, problems with total returns calculation and with the time markers for beginning and ending points of price data and with handling of returns after suspension periods. Ince and Porter (2006) also flag problems caused by rounding of stock prices and with small values of the return index. Most (not all) of the problems identified are concentrated in the smaller size deciles and this issue would significantly impact inferences drawn by studies focusing on cross-sectional stock characteristics. We thus apply great care to mitigate these problems by defining strict data quality filters to improve the reliability of price and volume data and to ensure results are economically meaningful for investors. Summarizing, we carry out three steps:

First, we review all classification information with a mix of manual and automatic techniques. Second, we cross-check all static information against a second data source, Bloomberg. Static classification data is key to establishing whether a stock passes certain minimum criteria to be included in the final sample. We find this to be unsatisfactory in

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5 “Manual” means, in many cases, a name-by-name, ISIN-by-ISIN check of the data, or the retrieval and incorporation of data from company websites. In this first step we exclude (i) investment trusts and other types of non-common-stock instruments, eliminating securities not flagged as equity in Datastream, (ii) securities not denominated in GBP, (iii) unit trusts, investment trusts, preferred shares, American depositary receipts, warrants, split issues, (iv) securities without adjusted price history, (v) Securities flagged as secondary listings for the company, (vi) stocks identified as non-UK under the Industry Classification Benchmark (ICB) system, (vii) securities without a minimum return history of 24 months and (viii) non-common stock constituents, mis-classified as common-stock, by searching for key words in their names - for instance, collective investment funds are have been identified and excluded.
Datastream. Third, we cross-check all time-series information (prices, returns, shares, volume) against Bloomberg, correcting a large number of issues and recovering data for a significant number of constituents that were missing. We remain with a large sample of 1,333 stocks from an initial sample of 7,968. (Full details of our data cleaning procedures are available from the authors on request).


We use a broadly similar two step procedure to that of Fama and MacBeth (1973) to test for the pricing of cross-sectional risk factors. The first step is to run a time series regression of the form

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \omega_{i,t} \quad i = 1, 2, \ldots n$$

(1)

where $R_{i,t}$ is the excess return (over the risk free rate) on asset $i$ at time $t$, $R_{m,t}$ is the excess return on the market portfolio, $\beta_i$ represents the systematic risk of asset $i$, $\omega_{i,t}$ represents idiosyncratic variation in asset $i$ and $n$ is the number of stocks in the cross-section. We estimate Eq (1) each month using a backward looking window of 24 months, rolling the window forward one month at a time. (The exception is in the initial months of a stock's series where we start with a minimum of 12 monthly returns and gradually expand the window until reaching 24 months). We collect the series of $\hat{\beta}$ and generate

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6 The extremely high error rate in Datastream and the much higher reliability of stock-level data in Bloomberg raises the question of why we do not simply use Bloomberg as our data source. There are various reasons including that only Datastream allows queries for bulk data with a common characteristic (i.e. all stocks listed on the London exchange) and licensing issues.

7 We provide only a brief outline of this well-known procedure here.

8 The data frequency, backward looking window length and forward rolling frequency vary in previous literature. For instance, Malkiel and Xu (2002) and Spiegel and Wang (2005) employ monthly data with a backward looking window of 60 months, Li et al. (2008) use windows of 3, 6 and 12 months, Hamao et al. (2003) use monthly data over a 12 month window. A number of studies such as Ang et al. (2009) and Bekaert et al. (2007) use daily data over one month. Brockman et al. (2009) use both daily data and monthly data. We use monthly data for consistency with our following cross-sectional analysis and a window length of 24 months as sufficiently long to ensure reliable risk estimators in each window but short enough to capture changing risk over time.
estimates of idiosyncratic risk, $\sigma$, using the series of the residuals $\hat{\epsilon}_{i,t}$ based on three alternative approaches:

i. as the standard deviation of the series of $\hat{\epsilon}_{i,t}$ over the 24 months rolling window

ii. fitting a GARCH(1,1) to the series of $\hat{\epsilon}_{i,t}$ over the 24 months window and

iii. generating each month a forecast of the conditional volatility of $\hat{\epsilon}_{i,t}$ from a GARCH(1,1) fitted over the 24 month window. The model is

$$\hat{\sigma}_{i,t}^2 = \pi_0 + \pi_1 \hat{\sigma}_{i,t-1}^2 + \pi_2 \hat{\epsilon}_{i,t-1}^2$$

(2)

In the second stage, a cross-sectional regression is estimated each month of the form

$$R_{i,t} = \gamma_{0,t} + \gamma_{1,t} \hat{\beta}_{i,t-1} + \gamma_{2,t} \hat{\sigma}_{i,t-1} + u_{i,t}$$

(3)

where $u_{i,t}$ is a random error term. Subscript $t-1$ denotes that $\hat{\beta}$ and $\hat{\sigma}$ are estimated in the 24 month window up to time $t-1$. It is advisable to obtain systematic and idiosyncratic risk estimates from (1) from month $t-1$ through month $t-24$ and then relate these to security returns in month $t$ in Eq (3) in order to mitigate the Miller-Scholes problem.

This procedure provides estimates of $\gamma_{0,t}$, $\gamma_{1,t}$, and $\gamma_{2,t}$ each month.

Under CAPM, $H_{10}: \gamma_{0,t} = 0$, $H_{20}: \gamma_{1,t} = R_{M,t}$ and $H_{30}: \gamma_{2,t} = 0$. Under normally distributed i.i.d. returns, $t_\gamma = \frac{\bar{\gamma}}{\sigma_{\gamma}}$ is distributed as a student’s t-distribution with $T-1$ degrees of

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9 We also estimate a version of (1) with additional Fama-French (1992) type size and value factors. However, we find that our later results around the pricing of idiosyncratic risk based on the single factor market model and the Fama-French model are very similar. (This result has also been found in US datasets (Malkiel and Xu, 2002, Guo and Savickas, 2008)). We report results using the market model estimates. All additional results are available on request.

10 Miller and Scholes (1972) find that individual security returns are marked by significant positive skewness so that firms with high average returns will typically have large measured total or residual variances as well. This suggests caution when using total or residual variance as an explanatory variable, as substantiated in practice by Fama and MacBeth (1973) who found total risk added to the explanatory power of systematic factor loadings in accounting for stock mean returns only when the same observations were used to estimate mean returns, factor loadings and total variances. Similar results were obtained by Roll and Ross (1980) in their tests of the Arbitrage Pricing Theory.
freedom where $T$ is the number of observations, $\bar{\gamma}$ and $\sigma_\gamma$ are the means and standard deviations respectively of the time series averages of the cross-sectional coefficients estimated monthly. The CAPM asserts that systematic risk is positively priced and this may be tested empirically by $H_0 : \gamma = 0$ versus $H_A : \gamma > 0$.

However, there is a problem when researchers test the model empirically using ex-post realized returns rather than the ex-ante expected returns upon which the CAPM is based. When realized returns are used Pettengill et al. (1995) argue that a conditional relationship between beta and return should exist in the cross-section of stocks. This arises because the model implicitly assumes that there is some non-zero probability that the realized market return, $R_{m,t}$, will be less than the risk free rate, i.e., $R_{m,t} < R_f$ as well as some non-zero probability that the realized return of a low beta security will be greater than that of a high beta security\textsuperscript{11}.

\textsuperscript{11} The CAPM asserts that the expected values of the distributions of asset $i$'s returns and the market returns are related as follows: $E(R_{i,t}) = R_f + \beta_{i,t} [E(R_{m,t}) - R_f]$ where $E(R_{i,t})$ is the expected return on security $i$ at time $t$, $R_f$ is the known return on a risk free asset over time $t$, $\beta_{i,t}$ is the security beta at time $t$ and $E(R_{m,t})$ is the expected market return at time $t$. The model implicitly assumes that $E(R_{m,t}) > R_f$ as otherwise all investors would hold the risk free asset. Therefore, the model implies that in the cross-section of security returns $E(R_{i,t})$ is a positive function of $\beta_{i,t}$. There is a problem, however, when researchers test the model using realized returns instead of expected returns. Pettengill (1995) argues that the model also implicitly assumes that there is some non-zero probability that $R_{m,t} < R_f$, where $R_{m,t}$ is the realized market return as otherwise no investor would hold the risk free asset. The CAPM itself does not describe a relationship between $R_{i,t}$ and $\beta_{i,t}$ when $R_{m,t} < R_f$ as it does the positive relationship between $E(R_{i,t})$ and $\beta_{i,t}$. Pettengill (1995) also argues that a further implication of the CAPM is that while a high beta security has a higher expected return than a low beta security to compensate for higher systematic risk, there must be some non-zero probability that the realized return of the low beta security will be greater than that of the high beta security as otherwise no investor would hold the low beta security. Pettengill et al (1995) suggest a reasonable inference is that this realization occurs when $R_{m,t} < R_f$. The implication of this is that there should be a positive (negative) relationship between beta and realized return when the excess market return is positive (negative). While the CAPM does not imply this relationship, the relationship is consistent with the market model, Markowitz (1959), Sharpe (1963), Fama (1968).
Pettengill et al. (1995) propose a conditional relationship between beta and return of the form\(^\text{12}\)

\[
R_{i,t} = \lambda_{0,t} + \lambda_{1,t} D \hat{\beta}_i + \lambda_{2,t} (1 - D) \hat{\beta}_i + e_{i,t}
\]

(4)

where \(R_{i,t}\) is the realised excess return on stock \(i\) in month \(t\), \(D\) is a dummy variable equal to one (zero) when the excess market return is positive (negative). Equation (4) is estimated each month. The model implies that either \(\lambda_{1,t}\) or \(\lambda_{2,t}\) will be estimated in a given month depending on whether the excess market return is positive or negative. The hypotheses to be tested are \(H_{\lambda,0} : \lambda = 0\) and \(H_{\lambda,1} : \lambda > 0\) where \(\bar{\lambda}_1\) and \(\bar{\lambda}_2\) are the time series averages of the cross-sectional coefficients estimated monthly. These hypotheses can be tested by the t-tests of Fama and MacBeth (1973).

Our final testing model incorporating a conditional beta/return relationship, idiosyncratic risk and the rolling backward looking estimation window is of the following form,

\[
R_{i,t} = \lambda_{0,t} + \lambda_{1,t} D \hat{\beta}_i + \lambda_{2,t} (1 - D) \hat{\beta}_i + \lambda_{3,t} \hat{\sigma}_{i,t-1} + \eta_{i,t}
\]

(5)

where \(\eta_{i,t}\) is a random error term. The time series averages of the lambda coefficients are then calculated and statistical significance tested.

3.1 Additional Cross-sectional Stock Characteristics: Size, Liquidity and Momentum.

A number of other cross-sectional influences have been shown to interact with residual risk and we attempt to control for these by augmenting Eq (5) at time \(t\) with appropriate proxies. In addition to size and value, liquidity and momentum have been documented in the literature to be particularly relevant. Malkiel and Xu (1997) report evidence of a strong

\textsuperscript{12} Fletcher (2000) conducts similar tests of a conditional relationship between the returns of 18 international equity indices and a global market index.
relationship between idiosyncratic volatility and size, suggesting that the two variables may be partly capturing the same underlying risk factors. Similar findings are reported in Malkiel and Xu (2002), Chua et al. (2010) and Fu (2009). Further evidence that some of the results obtained in the literature on the pricing of residual risk are sensitive to the contribution of small and illiquid stocks is found in Spiegel and Wang (2005) where liquidity is shown to interact strongly with idiosyncratic risk.

We use market capitalization at time $t$ as a measure of size. Our preferred proxy for liquidity is turnover. Turnover is defined as the volume of shares traded per period divided by the total number of shares outstanding and has often been employed in cross-sectional studies to gauge the impact of trading on returns. Spiegel and Wang (2005) show that measures of liquidity based on volume are more reliable than cost-based measures, which are heavily impacted by microstructure issues and vary greatly across market participants. Turnover varies over time at both the market-wide level and at stock level. Hence, here we decompose it into a systematic component and an idiosyncratic component. We decompose turnover by estimating a time series regression for each stock of the form

$$\text{TURN}_{i,t} = \phi_0 + \phi_i \text{TURN}_{MKT,t} + \theta_{i,t}$$  \hspace{1cm} (6)

over a 24 month backward looking window and rolling the window forward one month at a time as before. $\text{TURN}_{MKT,t}$ is the market-cap weighted average of individual stocks’ turnover at time $t$. While $\phi_i$ measures the sensitivity of each stock’s turnover to market-wide turnover, $\theta_{i,t}$ is a measure of turnover that is unique to each firm. We augment Eq (5) at time $t$ with $\hat{\phi}_i$ estimated over $t-1$ to $t-24$ and with $\hat{\theta}_{i,t-1}$. We find for the most part that time-variation in stock turnover comes from the systematic component.

A strong relationship between momentum returns and idiosyncratic volatility has been documented in Ang et al. (2006), Basu and Martellini (2007), Li et al. (2008) and Arena et al. (2008). We measure momentum as the cumulative return over the past 3
months. This is the measurement period that yields the most significant winners/losers spread in Li et al. (2008). As a robustness check, we also use a 6 month measure. Our final testing models augment Eq (5) and add the measures of turnover, size and momentum as stock characteristics in the cross-sectional regression.

4. Empirical Results

We begin by estimating cross sectional regressions each month \( t \) over the entire sample period. The cross sectional regressions examine the pricing of systematic risk, \( \beta \), idiosyncratic risk, \( \sigma \), as well as other stock characteristics including liquidity (turnover), size and momentum. As described in Section 3, \( \beta \) and \( \sigma \) are estimated over the previous 24 months. Table 1 presents results for several forms of our baseline model. We report the time series averages of the coefficients from the monthly cross-sectional regressions with their p values in parentheses. In these initial tests in Table 1 we ignore the possible conditionality of the beta/return relationship.

From Table 1 beta risk is negatively priced across all models. It is statistically significant at the 5% significance level in the case of three models and is significant at the 10% significance level in all except two of the models. This finding is contrary to the positive pricing of systematic risk predicted by CAPM. We return to this further below. As described in Section 3 we test alternative measures of idiosyncratic risk. From Table 1, “IVOL-m” estimates idiosyncratic risk as the standard deviation of the series of \( \hat{\varepsilon}_{i,t} \) in Eq. (1) over the 24 months rolling window while “E(IVOL)” estimates idiosyncratic risk by generating each month a forecast of the conditional volatility of \( \hat{\varepsilon}_{i,t} \) from a GARCH(1,1) fitted over the 24 month window. The findings around the pricing of idiosyncratic risk are mixed where the pricing coefficient switches from positive to negative. However, in all cases the results indicate that idiosyncratic risk is not significantly priced. We find that these results are consistent across the alternative measures of idiosyncratic risk. Again we return to this for further discussion below.
As described in Section 3, we use turnover as a proxy for liquidity and decompose total turnover into systematic turnover and idiosyncratic turnover. From Table 1, we find that total turnover ("TURN") is positively and significantly priced at the 5% significance level across all models. This is an interesting counter-intuitive, although robust finding, indicating that in the cross section of stocks higher turnover is associated with higher returns\textsuperscript{13}. However, while total turnover is found to be significant, its systematic component ("$\beta \text{TURN}$") and idiosyncratic component ("I-TURN") are individually statistically insignificant. Size as a risk factor is intuitively signed and the market value measure ("MKTVAL") is highly significant. One \textit{a priori} concern is of a possible interaction between total turnover and market value but Table 1 provides evidence of distinct effects. Finally, we find that momentum is an important determinant of the cross section of stock returns where its coefficients are consistently intuitively signed and statistically significant.

A key feature of our paper is our investigation of a possible conditional relationship between beta risk and return as proposed by Pettengill et al (1995). The author’s argument indicates that there should be a positive (negative) relationship between beta and realized return when the excess market return is positive (negative). We examine this argument by estimating Eq (5) each month and then calculating the time series averages of the conditional coefficients on beta risk, $\lambda_1$ and $\lambda_2$, from the cross sectional regressions. We report results in Table 2 and Table 3 for down-markets and up-markets respectively. The time series averages of the coefficients on idiosyncratic risk, turnover, size and momentum from the cross sectional regressions each month will be unaltered from the values reported in Table 1. However, in order to explore other possible conditional pricing effects, in Tables 2 and 3 we also report the time series averages of the coefficients on idiosyncratic risk, turnover, size and momentum from the monthly cross sectional regressions in down-markets and up-markets separately.

\textsuperscript{13} Lu and Hwang (2007) also find that liquid stocks significantly outperform illiquid stocks in the UK and report that their observed effect explains the value premium. They measure liquidity using a variation of the Amihud (2002) measure - the absolute daily return divided by daily volume.
We find strong support for a conditional beta risk/return relationship. From Table 2 we see that in the cross section of stock returns, high beta stocks underperform low beta stocks in down-markets while the opposite is true in up-markets from Table 3. This is a robust finding where it is statistically significant at the 5% significance level across almost all models tested. Our findings indicate that the distinction between expected returns and actual returns has crucial implications for testing the relationship systematic risk and returns.

Interestingly, our empirical results also point to conditionality in the pricing of idiosyncratic risk, i.e., in down-markets versus up-markets. In down-markets from Table 2 idiosyncratic risk is negatively priced – a significant finding at the 5% significance level and robust across all forms of the model and across all idiosyncratic risk measures. Although perhaps initially counter-intuitive, this finding is consistent with the theory put forward by Chen (2002) and Ang et al (2006) as outlined in Section 2 which predicts that idiosyncratic volatility risk is negatively priced due to its link with market volatility risk. Specifically, the Chen (2002) model predicts a negative loading on the covariance between a stock’s return and changes in the forecasts of future market volatilities indicating that investors demand compensation in the form of higher expected return for the risk that an asset will perform poorly when the future becomes less certain. Ang et al (2006) argue that stocks with high idiosyncratic volatilities may particularly exhibit this characteristic. Our results strongly support the possibility that this negative pricing effect is further accentuated in down-markets. The implication of our results is that investors need to pursue high levels of diversification to insulate their portfolios from the negative pricing effect of idiosyncratic risk and this is particularly the case in down-markets.

In up-markets (Table 3), the coefficient on idiosyncratic risk is positive though generally not significant at the 5% significance level. The contrasting findings around the conditional pricing of idiosyncratic risk in down-markets (Table 2) versus up-markets (Table 3) explains why the unconditional results in Table 1 are of mixed sign on the idiosyncratic risk pricing coefficient and are insignificant. Hence, our results highlight the
importance of incorporating the conditionality of the beta/return relationship in empirical work when using realised returns.

Turning to the other cross-sectional factors, we find that the positive relation between turnover and returns reported in Table 1 is stronger in up-markets while we find no evidence of this relation in down-markets. A very similar phenomenon is found in relation to size risk as measures by market value. Momentum plays an important role in explaining the cross-section of stock returns though this effect is seen more strongly in down-markets.

5. Conclusion
In this paper we test whether firm idiosyncratic risk is priced in a large cross-section of U.K. stocks. An important contribution is that we do this allowing for a possible conditional relationship between systematic risk (beta) and returns, i.e., conditional on whether the excess market return is positive or negative. In our tests we control for other stock characteristics such as liquidity, size, and momentum risks which may explain some idiosyncratic risk. We find strong evidence of a conditional beta/return relationship, incorporating which in turn reveals some conditionality in the pricing of idiosyncratic risk. Specifically, we find that idiosyncratic risk is significantly negatively priced in down-markets but positively priced in up-markets, though not significantly so. Our findings around this negative pricing support extant theories that predict that idiosyncratic volatility risk is negatively priced due to its link with market volatility risk. Our results also reveal a strong role for liquidity, size and momentum factors in explaining the cross-section of stock returns.
References


Table 1. Regressions of returns on cross-sectional stock characteristics

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Table 1 shows the results of our two-step asset pricing tests. In the first step we run a time series regression of stock returns on a market factor for each stock over the previous 24 months to estimate systematic risk. Idiosyncratic risk is estimated from the residuals of this regression. In the second step we regress stock returns on beta and idiosyncratic risk as well as on factors for liquidity, size and momentum risk in a cross-sectional regression. We roll this two-step procedure forward one month at a time. We report the time series average of the coefficients in the cross-sectional regressions with p-values in parentheses. The variables in the cross-sectional regressions are as follows: ‘Const’ is the intercept, ‘Beta’ is systematic risk, ‘IVOL-m’ is idiosyncratic risk estimated as the standard deviation of the residuals from the time series regressions over the 24 months rolling window while ‘E(IVOL)’ is idiosyncratic risk estimated by fitting a GARCH(1, 1) to the variance of the residuals and generating each month a forecast of the conditional volatility, ‘TURN’, ‘β TURN’ and ‘I-TURN’ represent total, systematic and idiosyncratic turnover risk respectively, ‘MKT VAL’ is market value representing the size factor while ‘Mom 3m’ and ‘Mom 6m’ are momentum factors based on stock returns over the previous 3 months and 6 months respectively. The time series average of the R² values from the rolling cross-sectional regressions are also shown. Highlighting indicates significance at the 5% significance level.
Table 2 shows the results of the same two-step asset pricing tests reported in Table 1. We report the time series average of the coefficients in cross-sectional regressions estimated when the excess market return is negative. p-values are in parentheses. The time series average of the R^2 values from the rolling cross-sectional regressions are also shown. Highlighting indicates significance at the 5% significance level.
Table 3 shows the results of the same two-step asset pricing tests reported in Table 1. We report the time series average of the coefficients in cross-sectional regressions estimated when the excess market return is positive. p-values are in parentheses. The time series average of the $R^2$ values from the rolling cross-sectional regressions are also shown. Highlighting indicates significance at the 5% significance level.