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Rayleigh-Lamb Wave Detection of Two-dimensional Defects in Metal Plates

Nondestructive testing of engineering structures is essential to ensure safety. Ultrasonic C-scanning uses waves propagating normal to a plane surface. This is accurate, but slow for planar structures. Longitudinal waves would allow quicker testing of such structures. This paper considers the reflection of low to moderate frequency Rayleigh-Lamb waves by a through width lozenge-shaped defect in an isotropic plate. A numerical method is used. It is shown that such longitudinal ultrasonic waves can be used with success to detect and locate structural defects if care is exercised in frequency selection.

1. Introduction

Conventional ultrasonic non-destructive evaluation (NDE) of plate-like structural elements typically relies on the through thickness propagation of waves. This is generally quite time consuming for condition monitoring of large engineering structures. An alternative approach to testing, particularly for thin structures such as plates and pipes, is to propagate ultrasonic waves along the major plane of the component. This can result in significant time saving since the transducer is only passed along the edge of the structure, rather than passing over its entire area. In plates, such longitudinally propagating waves are referred to as Rayleigh-Lamb waves.

2. Mathematical Analysis

The basic mathematics of Rayleigh-Lamb waves have been extensively studied [1]. For the present analysis, the equations were expressed in dimensionless form, after [2]. In this way, the fundamental characteristic of any mode of vibration becomes its dimensionless frequency, given by:

\[ \Omega = \frac{\omega h}{c_l} \]  

where \( \omega \) is the angular frequency \((\text{rad} \cdot \text{s}^{-1})\), \( h \) is the plate half thickness \((\text{m})\), and \( c_l \) is the speed of transverse waves \((\text{m} \cdot \text{s}^{-1})\) and is a function of material properties. At low frequencies \((\Omega < 1.5)\), there are only two possible modes of vibration: one symmetric \((S_0)\), and one anti-symmetric \((A_0)\). At low frequencies, the \( S_0 \) wave is virtually non-dispersive, while the \( A_0 \) wave is highly dispersive. This leads us to concentrate on the \( S_0 \) waves, since non-dispersive behaviour gives more coherent wave trains.

3. Modelling

To study the reflection of Rayleigh Lamb waves by the defect, a finite element model was created [3]. The modelled aluminium plate has a length of \( 80 \times \text{plate half thickness} \). The interest here is in symmetric modes, so only half the thickness of the plate was modelled, with a symmetry boundary condition imposed on the bottom surface of the model to represent the plate contour line. The defects too are symmetrical about the midplane of the plate, and are lozenge/diamond shaped \((\Box)\). For any defect height \( 2d \) (dimension perpendicular to midplane), the width of the defect at the centreline is varied between 0 (vertical crack) and \( 2d \) (square defect, included angle of 90°).

The plate was meshed with quadrilateral plane strain isoparametric elements, with vertex nodes and 3x3 integration points. A mesh size in the direction of propagation of \( 0.25 \times \text{plate half thickness} \) was chosen, giving 320 elements along the plate. 20 elements were used to mesh the half thickness of the plate. 10 elements to mesh the crack face, and 10 to mesh the intact part of the plate above the crack. This gave adequate mesh density on the face of the defect. The ratio of defect height to plate thickness \((d/h)\) was varied by changing the vertical position of the eleventh node. Defect width was varied for each height by changing the horizontal position of the first node. Nodes 2-10 were then linearly placed between nodes 1 and 11. 20 'infinite' elements were used along the non-cracked end of the plate to provide a quiet boundary to the finite element model.

Since the face of the crack represents a free surface, it is traction free. However, the incident wave will excite stresses on the surface of the crack. The reflected wave must therefore excite the plate such that it cancels out...
these stresses on the free surface with equal but opposite stresses. In this model, a stress equation representing the $S_0$ wave was programmed into the model through a user subroutine which applied the stress to the free surface of the crack. The intact portion of the plate, above the crack, was fixed in the $x$-direction by a plane of symmetry.

The stresses were applied for frequencies up to $\Omega = 1.52$. Defects were of height 0.2 and 0.4 of the plate thickness. Defect width was varied from zero (vertical crack) to defect height (45° leverage). The displacement amplitudes of waves excited in the plate were then compared to those of the corresponding incident wave. The ratio of these two amplitudes was used to define a reflection coefficient.

4. Results and Discussion

Selected results obtained from the model are shown graphically below. On the left are results from a defect height of 0.2 × plate thickness, on the right 0.4 × plate thickness. Seven representative frequencies are plotted on each graph. On the left of each graph, the results closest to those of a vertical crack. Results from an analytic method [2], for $\Omega = 1$ and a vertical crack gave reflection coefficients of 4% for $d = 0.2$, and 16% for $d = 0.4$. This shows good agreement with the plotted results. The taller defect produces a larger reflection coefficient at all frequencies, as would be expected. Also, higher frequencies produce larger reflection coefficients for defects of small width. However, it must be noted that higher frequencies lose some of their efficacy when the defect becomes more oblique to the incident wave form (wider), particularly for the taller (0.4) defect, almost to the point of disappearing.

5. Conclusions

The usefulness of low/mid frequency symmetric Rayleigh-Lamb waves for the detection of embedded symmetric 2-D flaws in aluminium plate has been characterised by means of a dimensionless index. It has been shown that physically measurable reflection coefficients ($> 10\%$) can be obtained for defects of height 0.4 of plate thickness using the non-dispersive low frequency longitudinal $S_0$ wave. The importance of frequency selection has been indicated. For higher frequencies ($\Omega > 1$), it is possible for oblique defects to reflect below a detection threshold of 10%.

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6. References


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