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Possibilistic Evaluation of Distributed Generations Impacts on Distribution Networks

Alireza Soroudi, Mehdi Ehsan, Raphael Caire, Member, IEEE and Nouredine Hadjsaid, Senior, IEEE.

Abstract—In deregulated power systems, the distribution network operator (DNO) is not responsible for investment in Distributed Generation (DG) units and they are just concerned about the best architecture ensuring a good service quality to their customers. The investment and operating decisions related to DG units are then taken by entities other than DNO which are exposed to uncertainty. The DNO should be able to evaluate the technical effects of these uncertain decisions. This paper proposes a fuzzy evaluation tool for analyzing the effect of investment and operation of DG units on active losses and the ability of distribution network in load supply at presence of uncertainties. The considered uncertainties are related to load values, installed capacity and operating schedule of DG units. The proposed model is applied on a test system and also a real French urban network in order to demonstrate its functionality in evaluating the distribution expansion options.

Index Terms—Fuzzy modeling, Uncertainty, Load repression, Distributed generation, Active losses.

I. INTRODUCTION

The Distributed Generation (DG), defined as generation plants connected to distribution system, has been given a great deal of attention in the last decade. The technological development, necessities of emission reduction [1], securing the energy supply, flexibility of investment [2], active loss reduction [3], investment deferral [4], [5] and reliability improvement has made DG units an interesting option to meet the load growth. The analysis and quantification of DG effects on distribution networks is of great importance. The proposed models of the literature for quantification of these effects can be widely divided into two frameworks: centrally controlled investment and investment under unbundling rules [6]. In centrally controlled investment, Distribution Network Operator (DNO) is responsible for DG investment in distribution network. In these models, the optimal investment and operating of DG units is calculated and run by DNO [3], [7]. In contrast with centrally controlled investment, the DG investment under unbundling rules is not the duty of DNO and he is just responsible for maintaining the fair access and efficiency of distribution network. In this context, DG investment is done by private sector based on its own interests. In liberalized electricity markets, DG-owners are given some economic signals (like connection charge or loss reduction incentives [2]) to lead DG investment in certain areas and sizes but two important questions arise here, as follows:

1) Are the investment decisions of DG developers exactly the same as what DNO intends?
2) Will DG operators schedule their units according to the DNO’s benefits (if no technical constraint is violated)?

The answer of the first question is quite clear. The decisions of DG developers are determined by technical constraints and also their own interests and are not exactly what DNO desires. Although the production forecasts are traditionally required only at transmission levels, but DNOs should have a clue about the active resources (DG units) which may be added in their territory. The answer of the second question is highly dependent on DG technology and decisions of DG-operators. The DG technologies can be widely categorized into renewable and non-renewable technologies.

In renewable DG technologies, the uncertainties show stochastic behavior because of random nature of their primary source of energy. In [8], an efficient probabilistic load flow method is proposed to take account the distribution system operation uncertainties including daily time varying load, stochastic DG power production, network configuration, and voltage control devices operation. In [9], a probabilistic power flow is proposed to deal with the interdependent uncertainties of wind generation, loads and generation availability. In [10], a stochastic model of wind generation is proposed to allow the coordination of wind and thermal power in an OPF dispatching program. A method based on time series is proposed in [11], [12] to examine the opportunities and challenges offered by renewable power generation in non-firm connection. The time-series steady-state analysis proposed in [13], assesses technical issues such as energy export, losses, and short-circuit levels due to high penetration of renewable energy resources on the distribution network.

In non-renewable DG technologies, normally, DG operators tend to maximize energy production. Consequently, if possible, they will operate at nominal or near nominal capacity but this is just an estimate of what they will decide. The problem is that the DNO cannot be sure about the capacity of DG unit which may be installed in a specific area and its operating schedule; In [14], a probabilistic model based on Monte Carlo method is proposed in which the location and capacity of DG units are known but the operating schedule of these units are uncertain due to the behaviors of DG owners.

The key point is that all of the uncertainties do not necessarily follow basic probabilistic behaviors. In other words,
if the DG technology is non-stochastic but uncertain (such as gas turbine or CHP), the output of these technologies depends on investment/operating decisions of their owner/operators. How should DNO model and evaluate these non-stochastic uncertainties?

This paper tries to answer the last question with proposing a diagnostic model for evaluating the effect of non-stochastic uncertainties of DG units on distribution network performance. The focus is on two indices, namely active power losses and load repression [15] which is an index demonstrating the ability of distribution network in load supplying. The assumptions, constraints and the evaluation indices are described as follows:

A. Assumptions

1) DG Modeling: DG units are modeled as negative PQ loads with constant power factor [6], as follows:

\[
\bar{P}_{i,t,h}^{dg} = \cos\varphi_{i,t,h}^{dg} \times \bar{S}_{i,t,h}^{dg}
\]

\[
\bar{Q}_{i,t,h}^{dg} = \sin\varphi_{i,t,h}^{dg} \times \bar{S}_{i,t,h}^{dg}
\]

where \( \bar{S}_{i,t,h}^{dg} \), \( \bar{P}_{i,t,h}^{dg} \), and \( \bar{Q}_{i,t,h}^{dg} \) denote the apparent, active and reactive fuzzy power generated by DG unit in bus \( i \), demand level \( h \) and year \( t \), respectively.

The assumptions, constraints and the evaluation indices are described as follows:

A. \( \alpha \)-cut method

In engineering problems, the evaluation of a certain quantity is usually in the form of a multi-variable function namely, \( y = f(x_1, \ldots, x_n) \), if \( \bar{x_i} \) are uncertain then \( y \) will be also uncertain, \( \bar{y} = f(\bar{x}_1, \ldots, \bar{x}_n) \). The question is that, knowing the membership functions of uncertain input variables \( \bar{x}_i \), what would be the membership function of \( \bar{y} \)? The \( \alpha \)-cut method [16] answers this question in this way:

For a given fuzzy set \( A \), defined on universe of discourse, \( U \), the crisp set \( A^\alpha \) is defined as all elements of \( U \) which have membership degree to \( A \), greater than or equal to \( \alpha \), as calculated in (1).

\[
A^\alpha = \{ x \in U \mid \mu_A(x) \geq \alpha \}
\]

(1)

The \( \alpha \)-cut of each input variable, \( x^\alpha \), is calculated using (1), then the \( \alpha \)-cut of \( y \), \( y^\alpha \), is calculated as follows:

\[
y^\alpha = (y^\alpha_1, \ldots, y^\alpha_n)
\]

\[
y^\alpha = \min\{x^\alpha_1, \ldots, x^\alpha_n\}
\]

\[
\bar{y}^\alpha = \max\{x^\alpha_1, \ldots, x^\alpha_n\}
\]

This means for each \( \alpha \)-cut, two optimization is done. One maximization to obtain \( y^\alpha \) which is the upper bound of \( y^\alpha \), and one minimization for obtaining the \( y^\alpha \), the lower bound of \( y^\alpha \).

B. Defuzzification

The defuzzification is a mathematical process for converting a fuzzy number into a crisp one [22]. In this paper, the centroid method [19] is used for defuzzification of fuzzy numbers. The defuzzified value of a given fuzzy quantity, \( A \), is calculated as follows:

\[
A^* = \frac{\int \mu_A(x) dx}{\int \mu_A(x) dx}
\]

(3)

III. PROBLEM FORMULATION

The assumptions, constraints and the evaluation indices are described as follows:

A. Assumptions

1) DG Modeling: DG units are modeled as negative PQ loads with constant power factor [6], as follows:

\[
\bar{P}_{i,t,h}^{dg} = \cos\varphi_{i,t,h}^{dg} \times \bar{S}_{i,t,h}^{dg}
\]

\[
\bar{Q}_{i,t,h}^{dg} = \sin\varphi_{i,t,h}^{dg} \times \bar{S}_{i,t,h}^{dg}
\]

where \( \bar{S}_{i,t,h}^{dg} \), \( \bar{P}_{i,t,h}^{dg} \), and \( \bar{Q}_{i,t,h}^{dg} \) denote the apparent, active and reactive fuzzy power generated by DG unit in bus \( i \), demand level \( h \) and year \( t \), respectively.
2) Uncertainty Modeling: The three main sources of uncertainties considered in this paper are electric loads, installed capacity of DG units and their operating schedule. The explanation of each parameter is described as follows:

- **Fuzzy load**: The load variation curve over each year is modeled using multiplication of three parameters. The first one is the base load, $S_{D,base}^f$, in the first year of the evaluation period and each year is divided into $N_{dlf}$ demand levels. A Demand Level Factor, $DLF^f_i$, is assigned to each demand level which is the forecasted value of “load to peak ratio” varying between 0 and 1. The duration of demand level $h$ is denoted by $\tau_h$. The uncertainty of $DLF^f_i$ is modeled using FTN, described as follows:

$$DLF^f_i = (dlf_{min} , dlf_{L} , dlf_{U} , dlf_{max}) \times DLF^f_i$$

Assuming a demand growth rate, $\gamma$, the demand in bus $i$, in demand level $h$ and year $t$ is calculated as follows:

$$\hat{P}_{D,i,t,h}^+ = P_{D,base}^+ \times DLF^f_h \times (1 + \gamma)^t$$

$$\hat{Q}_{D,i,t,h}^+ = Q_{D,base}^+ \times DLF^f_h \times (1 + \gamma)^t$$

where $\hat{P}_{D,i,t,h}^+$ and $\hat{Q}_{D,i,t,h}^+$ are the apparent, active and reactive power demand in bus $i$, demand level $h$ and year $t$; $P_{D,base}^+$ and $Q_{D,base}^+$ are the predicted values of apparent, active and reactive power demand in bus $i$, demand level $h$ and year $t$.

- **Fuzzy installed capacity**: In deregulated environment, the DNO is not responsible for investment in DG units and private sector will invest in the network based on its own interests. The DNO can only analyze the network and identify the interests of DG investors and predict their actions. These facts imply that the capacity of DG units in each bus is not a certain value. In this paper, the installed capacity of DG units are modeled as a FTN, namely $\xi^f_{dg}$, as follows:

$$\xi^f_{dg} = (\xi^{f_{min}}, \xi^{f_L}, \xi^{f_U}, \xi^{f_{max}}) \times Cap_{dg}^f$$

where $Cap_{dg}^f$ denotes the forecasted value of DG capacity to be installed in bus $i$.

- **Fuzzy DG generation**: The generation schedules of DG units are determined by DG owners and are not centrally controlled by DNO. In this paper, the apparent power of DG units are modeled as a FTN, namely $\xi^f_{dg}$, as follows:

$$\xi^f_{dg} = (\xi^{f_{min}}, \xi^{f_L}, \xi^{f_U}, \xi^{f_{max}}) \times Cap_{dg}^f$$

Although the capacity of installed DG in a given bus, $\xi^{f_{min}}$, is uncertain but the DG generation, $\xi^f_{dg}$, can not exceed the installed capacity of DG unit in any $\alpha$-cut. The minimum generated power of DG unit is highly dependent on the decision of its owner and technical characteristics of DG. In $\alpha = 1$, the percentage of $\xi^f_{dg}$ which may DG decrease its generated power is specified by $\epsilon_L$ and in $\alpha = 0$, this is done using $\epsilon_{min}$.

B. Constraints

1) Power flow equations: The power flow equations which must be satisfied for each $\alpha$-cut, in demand level $h$ and year $t$, are as follows:

$$\hat{P}_{net}^{D,f} = \hat{P}_{D,i,t,h}^+ - \hat{P}_{D,i,t,h}^-$$

$$\hat{Q}_{net}^{D,f} = \hat{Q}_{D,i,t,h}^+ - \hat{Q}_{D,i,t,h}^-$$

$$\hat{P}_{net}^{t,h} = \hat{V}_{i,t,h} \sum Y_{ij} \hat{V}_{j,t,h} \times \sin(\delta_{i,t,h} - \delta_{j,t,h} - \theta_{ij})$$

$$\hat{Q}_{net}^{t,h} = \hat{V}_{i,t,h} \sum Y_{ij} \hat{V}_{j,t,h} \times \sin(\delta_{i,t,h} - \delta_{j,t,h} - \theta_{ij})$$

where $\hat{P}_{net}^{D,f}$ and $\hat{Q}_{net}^{D,f}$ are the net active and reactive power injected to the network in bus $i$, in demand level $h$ and year $t$, respectively.

2) Voltage limits: The magnitude of voltage in each bus $i$ in demand level $h$ and year $t$ should be kept between the safe operating limits.

$$V_{min} \leq \hat{V}_{i,t,h} \leq V_{max}$$

where $V_{min}$ and $V_{max}$ are the minimum and maximum safe operating limits of voltage, respectively.

3) Thermal limits of feeders and substation: To maintain the security of the feeders and substations, the flow of current/energy passing through them should be kept below their thermal limit, $I_{th}^f / S_{grid}^{max}$, as follows:

$$I_{th}^f / S_{grid}^{max} \leq S_{th}^{max}$$

where $I_{th}^f$ is the fuzzy current magnitude of feeder $f$ in demand level $h$ and year $t$; $S_{grid}^{max}$ is the fuzzy apparent power passing through substation’s transformer in demand level $h$ and year $t$.

C. Distribution network impact indices

As already explained, in investment under unbundling rules, the responsibility of DNO is maintaining the efficiency and fair access in distribution network. The total active loss of the network is a good measure of efficiency in a distribution network. The economic impact of loss reduction on DNO benefits highly depends on regulatory framework. In some models of the literature, like UK, an incentive-based mechanism exists which encourages the DNOs to reduce their active losses below a given target level [6]. A load repression factor [15] is used in this paper as a measure of fair access and ability of the given network in load supplying which will be explained later. The proposed model calculates the introduced two indices namely, load repression and active losses, as follows:

1) Load Repression: The distribution networks are designed for forecasted values of loads. The DNOs need some evaluation tools to determine the robustness of distribution network against different uncertainties. These uncertainties include investment/operating of DG units and also forecasted values of loads in the network. The load repression index introduced in [15], is used to identify the difference between the possible (predicted) values of load
and what can be supplied in each bus. If these two values are different in a bus, its load is repressed. First of all, the differences between two important concepts are explained and their application will be demonstrated. The predicted value of load in bus \( i \) is obtained by multiplication of three parameters namely, base value of load in bus \( i \), \( S_{i,\text{base}}^D \), forecasted value of demand level factor in demand level \( h \), \( DLF^f_h \), and load growth factor until year \( t \), \( (1 + \gamma)^t \). The forecasted value of load is shown in Fig. 2 and calculated as follows:

\[
S_{i,t,h}^f = S_{i,\text{base}}^D \times DLF^f_h \times (1 + \gamma)^t \tag{12}
\]

![Realizable and forecasted values of apparent power demand in a given bus](image)

Fig. 2. Fuzzy load repression

The distribution network is designed to meet the forecasted values of load, \( S_{i,t,h}^f \), during the planning horizon. As it is already explained, the DNO needs some diagnostic tools to investigate if the ability of network in load supply is robust against different uncertainties. In order to explain the load repression index, two concepts are introduced, as follows:

The first concept is the maximum/minimum possible load due to prediction in each \( \alpha \)-cut, \( S_{i,t,h}^{\alpha,\text{max}}/\alpha, \text{min} \), which are defined as follows:

\[
\begin{align*}
S_{i,t,h}^{\alpha,\text{min}} & = \frac{S^D_{i,\text{base}}}{\alpha} \times DLF^\alpha_h \times (1 + \gamma)^t \leq S_{i,t,h}^{\alpha,\text{min}} \leq S_{i,t,h}^{\alpha,\text{max}} \\
S_{i,t,h}^{\alpha,\text{max}} & = \frac{S^D_{i,\text{base}}}{\alpha} \times DLF^\alpha_h \times (1 + \gamma)^t \\
S_{i,t,h}^{\alpha,\text{min}} & \leq S_{i,t,h}^{\alpha,\text{max}} \leq S_{i,t,h}^{\alpha,\text{max}}
\end{align*}
\tag{13}
\]

It should be noted that the limits introduced in (13) are not calculated values. They are predicted by DNO for describing the behaviors of load in each bus.

The second concept is that, hypothetically, the magnitude of each load can take any value between the limits posed by (13), in each \( \alpha \)-cut, \( S_{i,t,h}^{\alpha,\text{min}} \leq S_{i,t,h}^{\alpha,\text{max}} \leq S_{i,t,h}^{\alpha,\text{max}} \), but because of some technical considerations like voltage limits or thermal capacity of feeders/transformers as mentioned in (10) and (11), the predicted limits may not be reachable. Whenever a load in a bus can not reach its predicted limits, it is called repressed. The maximum/minimum load that can be supplied due to technical constraints, are indicated as \( S_{i,t,h}^{\alpha,\text{min}} \) and \( S_{i,t,h}^{\alpha,\text{max}} \), respectively and depicted in Fig. 2. A method was proposed in [15] for calculating the upper and lower bounds of active/reactive values of loads in each bus. In this paper, it is modified as follows: first, for calculating the upper bound of load in a given \( \alpha \)-cut, in addition to the constraints considered in [15], voltage limits are also considered in calculations. The second issue is that when the uncertainty of a load is concerned, it is mainly toward its magnitude not its power factor. This implies that if the calculation of the active and reactive values of load is done independently then the loads can have any power factor which is not realistic. In this paper, it is assumed that the only uncertain value of the load in each bus is the magnitude of it. The DNO checks the maximum and minimum load in bus \( i \), \( S_{i,t,h}^{\alpha,\text{min}}, S_{i,t,h}^{\alpha,\text{max}} \), which distribution network is able to supply in each demand level, as follows:

\[
\begin{align*}
S_{i,t,h}^{\alpha,\text{min}} & = \min S_{i,t,h}^{\alpha,\text{min}} \\
S_{i,t,h}^{\alpha,\text{max}} & = \max S_{i,t,h}^{\alpha,\text{max}} \\
\text{Subject to:} & \rightarrow (13)
\end{align*}
\]

The load repression index in demand level \( h \) and year \( t \), \( rep_{i,t,h} \), is defined as the sum of the area under the membership function of each load that can not be supplied in a given network (distinguished with grey color in Fig. 2) and calculated as follows:

\[
rep_{i,t,h} = \Delta(S_{i,t,h}^{\alpha,\text{min}}, S_{i,t,h}^{\alpha,\text{max}}) - \Delta(S_{i,t,h}^{\alpha,\text{min}}, S_{i,t,h}^{\alpha,\text{max}}) \tag{15}
\]

where \( \Delta \) is the operator for calculating the surface under the membership function of fuzzy parameter. The total load repression in each year, \( Y_{\text{rep}_i} \), is defined as the sum of the multiplication of load repression in each load level \( h \) by its duration \( \tau_h \) over all load buses of the system, as follows:

\[
Y_{\text{rep}_i} = \sum_{h=1}^{N_d} \sum_{i=1}^{N_s} rep_{i,t,h} \times \tau_h \tag{16}
\]

The total load repression in bus \( i \) over the evaluation period, \( B_{\text{rep}_i} \), is calculated as follows:

\[
B_{\text{rep}_i} = \sum_{t=1}^{T} \sum_{h=1}^{N_d} rep_{i,t,h} \times \tau_h \tag{17}
\]

The total load repression of the distribution network over the evaluation period, \( T_{\text{rep}} \), is calculated as follows:

\[
T_{\text{rep}} = \sum_{t=1}^{T} Y_{\text{rep}_t} \tag{18}
\]

2) Active Losses: The total active losses in each \( \alpha \)-cut is calculated as the sum of all active losses in demand levels of each year, over the evaluation period.

\[
\tilde{P}_{\text{loss}} = \sum_{t=1}^{T} \sum_{h=1}^{N_d} \sum_{i=1}^{N_s} \tilde{P}_{i,t,h} \times \tau_h \tag{19}
\]
For calculating $\tilde{P}_{\text{loss}}$, the $\alpha$-cut concept introduced in section II is used as follows:

$$P_{\text{loss}}^\alpha = (P_{\text{loss}}^\alpha, P_{\text{loss}}^\alpha)$$

$$\mathcal{P}_{\text{loss}}^\alpha = \max_{1 \leq t \leq T} \sum_{h=1}^{N_h} \sum_{i=1}^{N_i} P_{\text{net},i,t,h}^{\alpha} \times \tau_h$$

$$\mathcal{P}_{\text{loss}}^{\alpha} = \min_{1 \leq t \leq T} \sum_{h=1}^{N_h} \sum_{i=1}^{N_i} P_{\text{net},i,t,h}^{\alpha} \times \tau_h$$

subject to:

$$\alpha \rightarrow (13)$$

The mathematical formulation described in this section, is formulated under a GAMS environment [23].

IV. SYSTEM STUDIES

The proposed methodology is applied to two distribution systems to demonstrate its abilities. The first case is a 9-node distribution test system and the second one is a realistic 574-node distribution network.

A. Case-I

The proposed method is applied on a 11-kV, 9-bus distribution network which is shown in Fig.3 [3]. This network is fed through a transformer with $S_{\text{grid}} = 40\text{MVA}$ and has 8 aggregated load points. The rate of load growth, $\gamma$, is considered to be 2%. The technical characteristics of the network can be found in [3]. The evaluation period, $T$, is 5 years and the minimum and maximum value of operating limits of voltage, $V_{\text{min}}, V_{\text{max}}$, are 0.95 and 1.05 pu, respectively. The load duration curve is divided into four demand levels namely, high, normal, medium and minimum, where the forecasted values of them, $DLF_h^i$, are 1, 0.941, 0.866 and 0.686, respectively. The duration of each load level, $\tau_h$, is assumed to be 73, 2847, 2920, 2920 hours, respectively. The demand level factors are described as fuzzy trapezoidal numbers as explained in section III-A2. The specification of membership functions of demand level factors is done by DNO based on his prior experiences. It is not necessary that all of the demand level factors have the same membership functions but here, for simplicity, a non-symmetrical membership function is used for all buses of the network, as follows: In $\alpha = 0$,

$$df_{\text{max}} = (1 + U_{\text{df}}), df_{\text{min}} = (1 - 0.7 \times U_{\text{df}})$$

In $\alpha = 1$,

$$df_{U} = (1 + 0.5 \times U_{\text{df}}), df_{L} = (1 - 0.6 \times U_{\text{df}})$$

where $U_{\text{df}}$ is a factor for demonstrating the severity of uncertainty, varying between zero and one.

The capacity of DG which might be installed in a given bus is not determined by DNO and he should have an estimation about this value. In this paper, it is assumed that the buses which have the possibility of DG investment are identified and the potential DG capacity which may be installed there is predicted. This process is not necessarily precise and is subject to uncertainties associated to the decisions of the DG investors. In the given network, there are three buses which are candidate for DG installation, namely bus 2, 3 and 9. The forecasted values of the DG capacities and their associated uncertainties are given in Table. I.

For example for DG #1, in $\alpha = 1$, the lower bound of the DG capacity is $0.9 \times 400 = 360\text{kVA}$ and the upper bound is $1.05 \times 400 = 420\text{kVA}$. This means that the maximum degree of belief of the planner is that the capacity of DG will have a value between 360 and 420 kVA. In $\alpha = 0$, the lower bound of DG capacity is still zero, this means that the planner can not specify a minimum limit for the capacity of DG that may be installed in the given bus and its upper bound is $1.1 \times 400 = 440\text{kVA}$. This means the DG owner/investor may decide not to invest in DG and the maximum value of capacity which an investor may be interested (or able to) to install in bus $i=2$ is 440 kVA. The same concept holds for the data specified for other DG units. The values of $\epsilon_{\text{min}}$ and $\epsilon_{L}$ in (8) are used to model the operational uncertainties of owner-invested DG units. These values are highly dependent on DG technology and decisions of DG owner for making more profits. For Gas turbine DG units, DG owner tries to produce electricity as much as possible. This means that DNO expects these units to produce power near their capacity limit. In this paper, $\epsilon_{L}$ is considered to be 0.9 for Gas Turbine technology. On the other hand for CHP units, the DG operation is more uncertain because DG owner has two options for making benefits namely, selling power and heat. If DG owner decides to produce heat then he will have to reduce its output power and vice-versa. For CHP units, $\epsilon_{L}$ is considered to be 0.4. For both DG technologies, $\epsilon_{\text{min}}$ is 0. In other words, the maximum belief of DNO ($\alpha = 1$) indicates that the DG owner will produce more than $\epsilon_{L}$ % of its rated capacity but it is not guaranteed and he might produce less or even stop generating power which is less expected but possible ($\alpha = 0$).

The introduced indices are calculated and the effect of load uncertainties on them are investigated.

1) Calculating the technical indices : In order to clarify the application of load repression index, it is calculated when no uncertainty exists in demand level factors, $U_{\text{df}} = 0\%$. It is expected to obtain $T_{\text{rep}} = 0$ because the distribution network is designed for this purpose. The DNO may be interested to
know the answers of the following questions: how much the current network is robust against load uncertainty?; when will be the reinforcement actions required? The load repression indices are recalculated for $U_{df} = 5\%$. As it can be observed in Table II, there is no load repression in the system in the evaluation period. This means the system will face no problem even there is 5% uncertainty in demand. The second index to be calculated is the active losses. This index is calculated using (20) and the crisp value of total active losses is obtained as 17764 MWh. Now the effect of demand uncertainty on the proposed indices is assessed. The uncertainty of demand level factors, $U_{df}$, is varied and its effect on the total yearly load repression, $Y_{rep}$, total load repression in the evaluation period, $T_{rep}$, and finally the total active loss is investigated.

The yearly load repression, $Y_{rep}$, is calculated for different $U_{df}$ and the variation of this parameter is given in Table II. The values of $Y_{rep}$ in Table II show that the network supplies its loads when there is no uncertainty in the predicted values of load ($U_{df} = 0\%$). When the uncertainty increases the load repression occurs in the system. The first load repression occurs in year $t=5$, and ($U_{df} = 25\%$). With the increase of demand uncertainty, the load repression index shows an ascending pattern. The limits of fuzzy loss variation, the crisp values of active losses and total load repression for the given configuration of the network are calculated for different demand level uncertainties are given in Table III.

### Table I

Predicted values of DG capacities and their uncertainties

<table>
<thead>
<tr>
<th>Cases</th>
<th>DG #</th>
<th>DG Technology</th>
<th>Bus</th>
<th>$C_{d,g}$ ($kW$)</th>
<th>$C_{d,g}$ (min)</th>
<th>$C_{d,g}$ (max)</th>
</tr>
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<tr>
<td>I</td>
<td>1</td>
<td>Gas Turbine</td>
<td>2</td>
<td>200 kVA</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Gas Turbine</td>
<td>9</td>
<td>500 kVA</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>CHP</td>
<td>3</td>
<td>1 MVA</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>Gas Turbine</td>
<td>15</td>
<td>500 kVA</td>
<td>0.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Gas Turbine</td>
<td>283</td>
<td>3.5 MVA</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Gas Turbine</td>
<td>344</td>
<td>500 kVA</td>
<td>0.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Gas Turbine</td>
<td>495</td>
<td>3.5 MVA</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>CHP</td>
<td>426</td>
<td>500 kVA</td>
<td>0.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>CHP</td>
<td>163</td>
<td>500 kVA</td>
<td>0.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>

### Table II

The yearly load repression under different uncertainties of demand level factors in Case-I

<table>
<thead>
<tr>
<th>$U_{df}$ (%)</th>
<th>$Y_{rep}$(MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 → 20</td>
<td>t = 1</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>1.14</td>
</tr>
</tbody>
</table>

2) DG penetration level impact investigation: In this section the impact of DG penetration level on crisp active losses and total load repression is analyzed. In this case, it is assumed that the demand uncertainty is $U_{df} = 30\%$. Two different DG scenarios were created and assessed, as follows:

- "Multi DGs" scenario: In this scenario, more than one DG exist in the distribution network. The capacity of each DG unit is assumed to be equal to 1 MW. Different number of DG units are connected to the network and the crisp values of active losses and total load repression are given in Table IV. When DG units are in bus 5 and 7, the load repression is the same as the case when they are installed in bus 4,6 equal to 1412.1 MWh. The values of active losses are different in these two cases. The load repression can be reduced more if the DG units are installed in bus 3,8. With three DG units in 4,8,3 buses the load repression can be eliminated completely.

### Table III

The $T_{rep}$ and active losses under different uncertainties of demand level factors in Case-I

<table>
<thead>
<tr>
<th>$U_{df}$ (%)</th>
<th>$P_{loss}$ (MWh)</th>
<th>$P_{loss}$ (MWh)</th>
<th>$P_{loss}$ (MWh)</th>
<th>$P_{loss}$ (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>10784.2</td>
<td>11639.7</td>
<td>20145.7</td>
<td>27713.2</td>
</tr>
<tr>
<td>10</td>
<td>9846.9</td>
<td>10899.9</td>
<td>21523.7</td>
<td>30518.1</td>
</tr>
<tr>
<td>15</td>
<td>8939.1</td>
<td>10172.0</td>
<td>22951.1</td>
<td>33379.7</td>
</tr>
<tr>
<td>20</td>
<td>8088.6</td>
<td>9485.7</td>
<td>24406.1</td>
<td>36269.0</td>
</tr>
<tr>
<td>25</td>
<td>7292.7</td>
<td>8830.7</td>
<td>25864.9</td>
<td>37762.9</td>
</tr>
<tr>
<td>30</td>
<td>6548.9</td>
<td>8206.4</td>
<td>27336.3</td>
<td>41939.0</td>
</tr>
<tr>
<td>35</td>
<td>5856.0</td>
<td>7612.0</td>
<td>28804.1</td>
<td>44585.7</td>
</tr>
<tr>
<td>40</td>
<td>5213.3</td>
<td>7046.8</td>
<td>29049</td>
<td>49585.7</td>
</tr>
</tbody>
</table>

### Table IV

The $T_{rep}$ and active losses in multi DG scenario in Case-I

<table>
<thead>
<tr>
<th>Bus</th>
<th>Total DG capacity</th>
<th>$T_{rep}$ (MWh)</th>
<th>Crisp Loss (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,9</td>
<td>2</td>
<td>1407.80</td>
<td>28999</td>
</tr>
<tr>
<td>5,7</td>
<td>2</td>
<td>1412.10</td>
<td>29044</td>
</tr>
<tr>
<td>4,6</td>
<td>2</td>
<td>1412.10</td>
<td>29987</td>
</tr>
<tr>
<td>7,9</td>
<td>2</td>
<td>1407.80</td>
<td>29086</td>
</tr>
<tr>
<td>2.6</td>
<td>2</td>
<td>41.38</td>
<td>30265</td>
</tr>
<tr>
<td>2.9</td>
<td>2</td>
<td>37.11</td>
<td>29748</td>
</tr>
<tr>
<td>3.6</td>
<td>2</td>
<td>7.27</td>
<td>29606</td>
</tr>
<tr>
<td>2.3</td>
<td>2</td>
<td>4.27</td>
<td>29803</td>
</tr>
<tr>
<td>3.8</td>
<td>2</td>
<td>3.00</td>
<td>29539</td>
</tr>
<tr>
<td>29.8</td>
<td>3</td>
<td>30.3</td>
<td>29049</td>
</tr>
<tr>
<td>4,8.3</td>
<td>3</td>
<td>0</td>
<td>28875</td>
</tr>
<tr>
<td>3,9.5</td>
<td>3</td>
<td>0</td>
<td>27755</td>
</tr>
<tr>
<td>3,4,9,8</td>
<td>4</td>
<td>0</td>
<td>27556</td>
</tr>
</tbody>
</table>

- “Single DG” scenario: In this scenario, just one DG is installed in different nodes of the distribution network. The capacity of each unit is gradually increased from 0 to 8 MW. The variation of active losses and load repression are depicted in Fig.4. Initially, the active losses gradually decreases with the increase of DG capacity and after a certain value of DG capacity it starts to increase. The impact of DG capacity on $T_{rep}$ is shown in Fig.5. As it can be seen in Fig.5, the existence of DG units in some buses highly affects (reduces) the load repression (like bus 2,3), while the presence of DG units in some buses...
In this paper, the calculated indices are treated in a multi-attribute way but an alternative method for selecting the “most promising” plan would be to translate the load repression and losses into cost values and then choosing the least cost plan. This is valid just when the entity who pays for the investment cost, also pays for active losses and load repressions.

B. Case II: A real 574-bus urban network

The second case is a 20-kV, 574-node distribution system, depicted in Fig.6, which is extracted from a real French urban network. This system has 573 sections with total length of 52.188 km, and 180 load points. This network is fed through one substation. These data have been extracted from reports of Electricité de France (EDF) [24] and more details can be found in [25]. All DG units are assumed to operate with constant power factor equal to 0.9 lag. The forecasted values of the DG capacities and their associated uncertainties are given in Table. I. The other simulation data is the same as case I.

1) Calculating the technical indices: For the given network configuration, the introduced indices are calculated as follows: The yearly load repression, i.e. \( Y_{rep} \), is calculated under different uncertainties of demand level factors and the results are given in Table. VII. The limits of fuzzy loss variation, the crisp values of active losses and total load repression for the given configuration of the network are calculated for different demand level uncertainties are given in Table. VI.

### Table V

<table>
<thead>
<tr>
<th>Expansion plan #</th>
<th>Added circuit</th>
<th>( P_{loss}^{max} ) (MWh)</th>
<th>( T_{rep}^{max} ) (MWh)</th>
<th>Cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>18630.80</td>
<td>51.01</td>
<td>2.40</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>16422.87</td>
<td>6.43</td>
<td>5.40</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>22202.41</td>
<td>47.70</td>
<td>5.85</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>18872.89</td>
<td>54.42</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>18399.20</td>
<td>72.72</td>
<td>2.70</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>17987.59</td>
<td>78.73</td>
<td>4.35</td>
</tr>
</tbody>
</table>

Fig. 4. The DG penetration level impact on active losses in single DG scenario-Case I

It is clear from the analysis that not only the size of DG unit affects the active losses and total load repression but also the location of DG units plays an important role.

3) Planning application: The DNO can find some expansion plans using different techniques. The proposed indices can help him to choose the best expansion plan which is robust against different uncertainties. In this study, 6 expansion plans have been identified using the method proposed in [5], considering two objective functions (minimizing active losses and total costs). These plans are evaluated for different uncertainty demand levels \( U_{dlf} = 0 \rightarrow 40\% \). The maximum values of \( P_{loss}^{max} \) and \( T_{rep}^{max} \) are calculated and given in Table.V. This can be used to quantify the robustness of each plan against uncertainties. For example the plan #4 has the least cost but the maximm Trep under different values of \( U_{dlf} = 0 \rightarrow 40\% \) is 54.42 MWh and \( P_{loss}^{max}=18872.89 \) MWh. If the DNO seeks for the most robust expansion plan he should choose the plan

![Graph showing DG penetration level impact on active losses in single DG scenario-Case I](image1)

![Graph showing DG penetration level impact on load repression in single DG scenario-Case I](image2)
VIII. The most robust expansion plan is #3 because it has the least total repression and also the least crisp value of active loss among all other plans.

Table VII

<table>
<thead>
<tr>
<th>$U_{arr}$ %</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
<th>$t = 4$</th>
<th>$t = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.20</td>
<td>0.21</td>
<td>1.15</td>
<td>1.33</td>
<td>1.56</td>
</tr>
<tr>
<td>15</td>
<td>0.24</td>
<td>0.32</td>
<td>1.58</td>
<td>2.19</td>
<td>3.28</td>
</tr>
<tr>
<td>20</td>
<td>0.27</td>
<td>0.46</td>
<td>1.68</td>
<td>3.15</td>
<td>3.87</td>
</tr>
<tr>
<td>25</td>
<td>0.40</td>
<td>0.52</td>
<td>2.51</td>
<td>4.35</td>
<td>4.94</td>
</tr>
<tr>
<td>30</td>
<td>0.46</td>
<td>0.64</td>
<td>3.56</td>
<td>6.54</td>
<td>5.39</td>
</tr>
<tr>
<td>35</td>
<td>0.54</td>
<td>0.87</td>
<td>3.76</td>
<td>7.78</td>
<td>7.95</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This paper proposes a new possibilistic framework for evaluating the effects of DG units on distribution network performance. The model considers possibilistic modeling of the uncertainties associated to loads and decisions of DG investors including their installed capacity and operating schedule. The proposed technical indices demonstrate the ability of the given distribution network in load supply and also its efficiency at presence of DG units. The new evaluation method is applied on two different distribution systems and its performance is investigated. The proposed model is useful for basic engineering design and as a diagnostic tool for DNOs in evaluating their decisions in network reinforcement or reconfiguration of distribution network at presence of uncertainties associated to DG units and load values. The future work will be focused on modeling the mixed fuzzy and stochastic uncertainties in the proposed framework.

ACKNOWLEDGMENT

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REFERENCES

<table>
<thead>
<tr>
<th>Expansion plan #</th>
<th>Added circuit</th>
<th>max $P_{\text{loss}}^*$ (MWh)</th>
<th>max $T_{\text{rep}}$ (MWh)</th>
<th>Cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n_{32,456}^1$, $n_{508,524}^1$, $n_{262,263}^1$, $n_{471,468}^1$, $n_{14,515}^1$, $n_{57,156}^2$, $n_{334,253}^2$, $n_{41,91}^3$, $n_{337,323}^3$, $n_{229,252}^3$</td>
<td>1694.03</td>
<td>13.54</td>
<td>2.10</td>
</tr>
<tr>
<td>2</td>
<td>$n_{125,117}^4$, $n_{248,25}^4$, $n_{324,325}^4$, $n_{136,143}^4$, $n_{141,124}^4$, $n_{153,113}^4$, $n_{170,183}^4$, $n_{125,117}^4$</td>
<td>1329.56</td>
<td>20.63</td>
<td>3.78</td>
</tr>
<tr>
<td>3</td>
<td>$n_{253,561}^5$, $n_{300,307}^5$, $n_{253,354}^5$, $n_{349,350}^5$, $n_{506,507}^5$, $n_{115,126}^5$, $n_{440,350}^5$, $n_{81,82}^5$</td>
<td>1275.26</td>
<td>4.79</td>
<td>5.21</td>
</tr>
<tr>
<td>4</td>
<td>$n_{229,252}^6$, $n_{127,123}^6$, $n_{242,258}^6$, $n_{173,192}^6$, $n_{351,350}^6$, $n_{374,375}^6$, $n_{353,354}^6$</td>
<td>1669.13</td>
<td>16.27</td>
<td>1.32</td>
</tr>
<tr>
<td>5</td>
<td>$n_{147,343}^7$, $n_{82,88}^7$, $n_{210,96}^7$, $n_{43,360}^7$, $n_{47,340}^7$, $n_{282,285}^7$, $n_{545,430}^7$</td>
<td>1572.03</td>
<td>14.21</td>
<td>1.87</td>
</tr>
</tbody>
</table>