A Probabilistic Modeling of Photo Voltaic Modules and Wind Power Generation Impact on Distribution Networks

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Abstract—The rapid growth in use of renewable intermittent energy resources (like wind turbines and solar panels) in distribution networks has increased the need for having an accurate and efficient method of handling the uncertainties associated to these technologies. In this paper, the unsymmetrical two point estimate method (US2PEM) is used to handle the uncertainties of renewable energy resources. The uncertainty of intermittent generation of wind turbine, photo voltaic cells and also electric loads, as input variables, are taken into account. The variation of active losses and imported power from the main grid are defined as output variables. The U2PEM is compared to Symmetrical two point estimate method (S2PEM), Gram Charlier and Latin Hypercube Sampling (LHS) where Monte Carlo Simulation (MCS) is used as a basis for comparison. The validity of the proposed method is examined by applying it on a standard radial 9-node distribution network and a realistic 574-node distribution network.

Index Terms—Monte Carlo simulation, PV cells, wind turbine, active losses, point estimate method.

I. INTRODUCTION

A. Motivation and problem description

The penetration level of Distributed generation (DG) units is increasing on power distribution networks across the world. In deregulated power systems, the Distribution Network Operators (DNOs) are responsible for maintaining the reliability and efficiency of distribution networks. They usually do this by performing investment in network components and applying some active loss reduction policies. The role of Distributed Generation (DG) units in decreasing the operating and investment costs is crucial. The DNOs need some tools to investigate the impact of DG units specially those intermittent ones like wind turbines and photo voltaic modules. The motivation of this study is to provide such a tool for DNOs in order to model the uncertainties of intermittent power generations of wind turbines, photo voltaic cells and also electric load values. It should not only reduce the computational burden but also maintain the accuracy of computation procedure.

B. Literature review

The benefits of DG units like active loss reduction [1], [2], reducing the energy costs in the short term [3], emission reduction [4], distribution system service restoration [5], reliability improvement [6], [7] and incrementing the load balance factor of radial distribution networks [8] have been highly discussed in the literature. In recent years, the production of clean energy (renewable ones) by small power producers is encouraged [9]. The power produced by these technologies may be used in the market, in addition to being consumed locally [10]. The problem is that the generated power of renewable energy resources like wind turbines and photo voltaic cells are exposed to uncertainties. The probabilistic methods are widely used in power system operations and planning to deal with a variety of uncertainties [11]. The probabilistic power flow (PPF) is a tool which handles the uncertainties associated with input data of traditional power flow problem. A great deal of attention has been paid to the PPF problem in the literature. The PPF was first introduced in 1976 [11]. In [12], a convolution based technique was applied to consider the interdependent demands. In [13], a linearized set of load-flow equations were introduced to reduce the complexity of the problem. In [14], a combined Monte Carlo simulation technique and linearized power flow equations was employed. A Cumulant based method was proposed in [15] to deal with PPF problem. An enhancement to the traditional Cumulant method was implemented in [16], named Limit corrected Cumulant method (LCCM) which specifically addressed errors in the existing Cumulant method. This method produces multiple probability density functions (PDFs) and finds the final PDF combining the obtained PDFs. A hybrid Cumulant and Gram-Charlier expansion theory was introduced in [17] to reduce the computational time while maintaining a high degree of accuracy. In [18], an efficient Point Estimate Method (PEM) was proposed to handle the uncertainties of bus injections and line parameters. Four different versions of PEM were tried and tested in [19]. In [20], a Monte-Carlo simulation based method was applied to the nonlinear three-phase load flow equations of distribution networks including wind farms. A Latin Hypercube sampling (LHS) combined with Cholesky decomposition method (LHS-CD) was proposed in [21] or state space pruning [22] to reduce the computational burden of MCS. In [23], a model based on PEM was used to take into account the correlated wind power resources and load values.

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C. Contributions

The contributions of this paper are two fold:
1) To reduce the computational burden while keeping the precision.
2) To analyze the impact of PV modules and wind turbines in the network, simultaneously.

D. Paper organization

The problem formulation is described in Section II. Section III presents the method used for modeling and dealing with uncertainties. The described model is applied on two distribution networks and the simulation results are presented and discussed in Section IV. Finally, conclusions are outlined in Section V.

II. PROBLEM FORMULATION

The assumptions used in problem formulation, constraints and the uncertainty modeling are explained as follows:

A. Constraints

1) Power flow equations: The power flow equations to be satisfied are:

\[ P_{i}^{\text{net}} = -P_{i}^{D} + P_{i}^{w/pv} \]
\[ Q_{i}^{\text{net}} = -Q_{i}^{D} + Q_{i}^{w/pv} \]

\[ P_{i}^{\text{net}} = V_{i} \sum_{j=1}^{N_{b}} Y_{ij} V_{j} \cos(\delta_{i} - \delta_{j} - \theta_{ij}) \]
\[ Q_{i}^{\text{net}} = V_{i} \sum_{j=1}^{N_{b}} Y_{ij} V_{j} \sin(\delta_{i} - \delta_{j} - \theta_{ij}) \]

where \( P_{i}^{\text{net}}, Q_{i}^{\text{net}} \) are the net injected active and reactive power to bus \( i \), respectively. The \( P_{i}^{w/pv}, Q_{i}^{w/pv} \) are the active and reactive power of wind turbine/PV cells in bus \( i \).

2) Voltage limits: The voltage magnitude of each bus, i.e. \( V_{i} \) should be kept between the operating limits, as follows:

\[ V_{\min} \leq V_{i} \leq V_{\max} \]

where \( V_{\min} \) and \( V_{\max} \) are the minimum and maximum operating limits.

3) Feeders and substation capacity limit: To maintain the security of the feeders and substation, the flow of current/energy passing through them should be kept below the feeders/substation capacity limit as follows:

\[ I_{\ell} \leq I_{\ell}^{T} \]

where \( I_{\ell} \) is the current passing through feeder \( \ell \) and \( I_{\ell}^{T} \) is the maximum allowable current in feeder \( \ell \). For substation capacity constraint, the same philosophy holds, as follows:

\[ S_{\text{grid}}^{\text{grid}} \leq S_{tr} \]

\( S_{\text{grid}}^{\text{grid}} \) is the apparent power passing through substation transformer. The \( S_{tr} \) is the capacity of substation transformer.

B. Uncertainty modeling

In this study, the uncertain parameters are electric load in each bus and also the generated power of wind turbines and PV cell modules. The uncertainty modeling of each parameter is described as follows:

1) Electric load modeling: The electric load of each bus is modeled as a normal PDF:

\[ PDF(S_{i}^{D}) = \frac{1}{\sqrt{(2\pi\sigma_{i}^{D})}} \exp\left(-\frac{(S_{i}^{D} - \mu_{i}^{D})^{2}}{2\sigma_{i}^{D}}\right) \]

where \( S_{i}^{D} \) is the apparent power demand in bus \( i \) and \( \mu_{i}^{D}, \sigma_{i}^{D} \) are the mean and variance of demand in bus \( i \), respectively.

2) Wind Turbine generation pattern modeling: The generated power of a wind turbine highly depends on the wind speed in the site. There are various methods to model wind behavior like time-series model [26], data mining algorithms [27] or clustering approach [28]. In this paper, the variation of wind speed, i.e. \( v \), is modeled as a Weibull PDF and its characteristic function which relates the wind speed and the output of a wind turbine.

\[ PDF(v) = \left(\frac{k}{c}\right)(\frac{v}{c})^{k-1} \exp\left(-\left(\frac{v}{c}\right)^{k}\right) \]

where \( k \) and \( c \) are the shape and scale factor of the Weibull PDF of wind speed, respectively [28].

The generated power of the wind turbine is determined using its speed -power curve as follows:

\[ P_{i}^{w} = \begin{cases} 0 \quad \text{if } v \leq v_{in} \text{ or } v \geq v_{out} \\ \frac{v_{out}^{3}}{v_{rated}^{3} - v_{in}^{3}} P_{i,r}^{w} \quad \text{if } v_{in} < v \leq v_{rated} \\ \frac{v^{3}}{v_{rated}^{3} - v_{in}^{3}} P_{i,r}^{w} \quad \text{else} \end{cases} \]

Where, \( P_{i,r}^{w} \) is the rated power of wind turbine installed in bus \( i \), \( P_{i,r}^{w} \) is the generated power of wind turbine in bus \( i \), \( v_{in} \) is the cut-in speed, \( v_{out} \) is the cut-out speed, and \( v_{rated} \) is the rated speed of the wind turbine. The speed-power curve of a typical wind turbine is depicted in Fig. 1.

Fig. 1. The idealized power curve of a wind turbine
3) **Photo voltaic cell pattern modeling** : The generated power of a photo voltaic module depends on three parameters namely, solar irradiance, ambient temperature of the site and finally the characteristics of the module itself. The solar irradiance is modeled using a Beta PDF described as follows:

\[
PDF(s) = \begin{cases} 
\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \times s^{\alpha - 1} \times (1 - s)^{\beta - 1} & \text{if } 0 \leq s \leq 1, 0 \leq \alpha, \beta \\
0 & \text{else}
\end{cases}
\]

where \( s \) is solar irradiance kW/m\(^2\); \( \alpha, \beta \) are parameters of the Beta probability distribution function;

\[
P_{\text{PV}}(s) = N \times FF \times V(s) \times I(s)
\]

\[
FF = \frac{V_{\text{MPP}} \times I_{\text{MPP}}}{V_{\text{oc}} \times I_{\text{sc}}}
\]

\[
V(s) = V_{\text{oc}} - K_e \times T_e
\]

\[
I(s) = s_a \times [I_{\text{sc}} + K_i(T_e - 25)]
\]

\[
T_e = T_A + s_a \times \frac{N_{\text{OT}} - 20}{0.8}
\]

where \( T_e \) is the cell temperature in \(^\circ\)C; \( T_A \) is the ambient temperature in \(^\circ\)C; \( K_e, K_i \) are voltage and current temperature coefficient \( V/C^\circ, A/C^\circ \), respectively; \( N_{\text{OT}} \) is the nominal operating temperature of PV cell in \(^\circ\)C; \( FF \) is the fill factor; \( I_{\text{sc}} \) is the short circuit current in A; \( V_{\text{oc}} \) is the open circuit voltage in V; \( I_{\text{MPP}} \) and \( V_{\text{MPP}} \) are the current/voltage at maximum power point in A, V; finally, \( s_a \) is the average solar irradiance \(^2\).

**C. Output variables**

In this paper, two variables are of interest namely, Purchased active power from main grid, i.e. \( P_{\text{grid}} \) and active power losses, i.e. \( P_{\text{loss}} \). The total active loss is calculated as follows:

\[
P_{\text{loss}} = \sum_{i=1}^{N_b} p_{\text{act}}^i
\]

where \( N_b \) is the number of all buses in the network.

**III. UNCERTAINTY HANDLING METHOD**

**A. Monte Carlo method**

The Monte Carlo method is a technique that uses random numbers and their probability density function to solve problems. This method is often used when the model is complex, nonlinear, or involves many uncertain parameters. A simulation can typically involve over 10000 evaluations of the model, a task which is computationally expensive. Monte Carlo method can be summarized as below:

Step.1 Create a parametric model \( Y = h(x_1, x_2, ..., x_n) \)

Step.2 Generate a set of random inputs using their PDF \( X^i = (x_{1i}, x_{2i}, ..., x_{ni}) \)

Step.3 Evaluate the model and calculate the \( Y^i \)

Step.4 Repeat steps 2 and 3 for \( i = 1 \) to \( N \)

Step.5 Analyze the results using histograms, summary statistics, confidence intervals and so on

The Monte Carlo method is usually used for validation of the proposed methods in the literature for solving the PPF.

**B. Two point estimate method**

The symmetrical two point estimate method (S2PEM), which has the symmetrical location of two sampling points, is described in the following steps [19]:

Step.1 Determine the number of uncertain variables, \( n \)

Step.2 Set \( E(Y) = 0, E(Y^2) = 0 \)

Step.3 Set \( k = 1 \)

Step.4 Determine the location of concentration points \( \epsilon_{k,i} \) and their probabilities, i.e. \( P_{k,i} \), as follows:

\[
\epsilon_{k,i} = (-1)^{i+1} \sqrt{\frac{n}{\lambda_{k,i}}} \quad (11)
\]

\[
P_{k,i} = \frac{1}{2n} \quad (12)
\]

Step.5 Determine the concentration points \( x_{k,i} \), as follows:

\[
x_{k,i} = \mu_{x_k} + \epsilon_{k,i} \times \sigma_{x_k} \quad (13)
\]

Where, \( \mu_{x_k} \) and \( \sigma_{x_k} \) are the mean and standard deviation of \( x_k \), respectively.

Step.6 Run the deterministic power flow for both \( x_{k,i} \), as follows:

\[
Y = [x_1, x_2, ..., x_{k,i}, ..., x_n] \quad (14)
\]

\[
i = 1, 2 \quad (15)
\]

Step.7 Calculate \( E(Y) \) and \( E(Y^2) \) using:

\[
E(Y) \equiv \sum_{k=1}^{n} \sum_{i=1}^{2} P_{k,i} h(x_1, x_2, ..., x_{k,i}, ..., x_n) \quad (15)
\]

\[
E(Y^2) \equiv \sum_{k=1}^{n} \sum_{i=1}^{2} P_{k,i} h^2(x_1, x_2, ..., x_{k,i}, ..., x_n) \quad (15)
\]

Step.8 Calculate the mean and standard deviation as follows:

\[
\mu_Y = E(Y) \quad (16)
\]

\[
\sigma_Y = \sqrt{E(Y^2) - E(Y)^2} \quad (17)
\]

In unsymmetrical two point estimate method (US2PEM), the location of each sampling point is determined as follows:

\[
\epsilon_{k,i} = \frac{\lambda_{k,3}}{2} + (-1)^{i+1} \sqrt{n + \frac{\lambda_{k,3}^2}{2}} \quad (17)
\]

\[
P_{k,i} = (-1)^i \frac{\epsilon_{k,3-i}}{n \zeta_k} \quad (18)
\]

\[
\zeta_k = 2 \sqrt{n + \frac{\lambda_{k,3}^2}{2}} \quad (18)
\]

\[
\lambda_{k,3} = \frac{M_3(x_k)}{\sigma_{x_k}^3} \quad (18)
\]

where \( M_3(x_k), \lambda_{k,3} \) are the third moment and skewness of variable \( x_k \), respectively.
IV. APPLICATION STUDY

Two case studies have been analyzed in this section: the first one is a radial 9-bus distribution network and the second one is a realistic 574-node distribution network. The results obtained by the S2PEM and US2PEM are compared with different methods namely, Monte Carlo, Latin Hypercube sampling (LHS) \[21\] and Gram Charlier method \[17\]. The technical characteristics of wind turbines and PV modules are described in Table I and II, respectively \[2\]. The Weibull parameters of the wind speed in each wind farm for case-I are assumed to be $c = 8.78$, $k = 1.75$. The Beta parameters of solar radiation are assumed to be $\alpha = 6.38$, $\beta = 3.43$. In this study, for modeling the uncertainty of wind turbine, 10000 wind samples are generated using (6) then they are passed through speed-power curve of the wind turbine as depicted in Fig.1. The output of the wind turbine is shown in Fig.2 and used for representing the generated wind power.

![Wind Turbine Output](image1)

**Table I**

<table>
<thead>
<tr>
<th>Bus</th>
<th>No of PV modules</th>
<th>No of WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>500</td>
<td>1</td>
</tr>
</tbody>
</table>

In Table III the number of DG resources and their installation buses are specified. The results obtained for this case, include the mean and standard deviation values of active losses and imported power grid, absolute values of errors and also the running time as presented in Table IV.

The PDF of active power losses and imported power from the main grid are depicted in Fig.5 and Fig.4, respectively.

![PDF of Imported Power](image2)

**B. Case II: A realistic 574-bus urban French network**

The second case is a 20-kV, 574-node distribution system, depicted in Fig.6 which is extracted from a real French urban network. This system has 573 sections with total length of 52.188 km, and 180 load points. This network is fed through one substation. These data have been extracted from reports of Electricité de France (EDF) \[29\] where more details can be found in \[30\].

The speed-power characteristics of the wind turbines in case II are the same as case I and in Table V the number of DG resources and their installation bus are provided.

![Network Diagram](image3)
### TABLE IV
Comparison of results in Case-I (the values are in MW)

<table>
<thead>
<tr>
<th>Method</th>
<th>( \mu_{\text{loss}} )</th>
<th>( \sigma_{\text{loss}} )</th>
<th>( \mu_{\text{P grid}} )</th>
<th>( \sigma_{\text{P grid}} )</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS</td>
<td>0.8764</td>
<td>0.0000</td>
<td>33.5080</td>
<td>0.3705</td>
<td>946.25</td>
</tr>
<tr>
<td>S2PEM</td>
<td>0.8415</td>
<td>0.0000</td>
<td>34.4988</td>
<td>0.3841</td>
<td>1.4000</td>
</tr>
<tr>
<td>LHS</td>
<td>0.8450</td>
<td>3.5809</td>
<td>2.8122</td>
<td>2.6613</td>
<td>3.108</td>
</tr>
<tr>
<td>Gram Charlier</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th order</td>
<td>0.8482</td>
<td>5.2228</td>
<td>0.0210</td>
<td>2.3310</td>
<td>0.3705</td>
</tr>
<tr>
<td>5th order</td>
<td>0.8510</td>
<td>2.9005</td>
<td>0.0200</td>
<td>2.2779</td>
<td>2.6818</td>
</tr>
<tr>
<td>6th order</td>
<td>0.8993</td>
<td>2.6105</td>
<td>0.0209</td>
<td>2.0501</td>
<td>8.3152</td>
</tr>
<tr>
<td>7th order</td>
<td>0.8558</td>
<td>2.3949</td>
<td>0.0209</td>
<td>1.8451</td>
<td>9.8642</td>
</tr>
<tr>
<td>US2PEM</td>
<td>0.8579</td>
<td>2.1145</td>
<td>0.1028</td>
<td>1.6606</td>
<td>0.1720</td>
</tr>
</tbody>
</table>

### Table V
The installed DGs in Case-II

<table>
<thead>
<tr>
<th>Bus</th>
<th>No of PV modules</th>
<th>No of WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>283</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>344</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>495</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>426</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>163</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The results obtained for this case includes the mean and standard deviation values of active losses and imported power grid, absolute values of errors and also the running times have been all presented in Table VI. The PDF of imported power from main grid is depicted in Fig. 6.

### TABLE VI
Comparison of results in Case-II (the values are in MW)

<table>
<thead>
<tr>
<th>Method</th>
<th>( \mu_{\text{loss}} )</th>
<th>( \sigma_{\text{loss}} )</th>
<th>( \mu_{\text{P grid}} )</th>
<th>( \sigma_{\text{P grid}} )</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS</td>
<td>0.2565</td>
<td>0.0000</td>
<td>8.0918</td>
<td>0.2483</td>
<td>1264.3480</td>
</tr>
<tr>
<td>S2PEM</td>
<td>0.2691</td>
<td>3.9082</td>
<td>7.8688</td>
<td>2.7559</td>
<td>16.4800</td>
</tr>
<tr>
<td>LHS</td>
<td>0.2470</td>
<td>3.7226</td>
<td>8.3047</td>
<td>2.6315</td>
<td>650.2130</td>
</tr>
<tr>
<td>Gram Charlier</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th order</td>
<td>0.2646</td>
<td>3.1578</td>
<td>7.8988</td>
<td>2.3852</td>
<td>79.6498</td>
</tr>
<tr>
<td>5th order</td>
<td>0.2618</td>
<td>2.0469</td>
<td>8.2640</td>
<td>2.1279</td>
<td>81.3450</td>
</tr>
<tr>
<td>6th order</td>
<td>0.2607</td>
<td>1.6454</td>
<td>8.2479</td>
<td>1.9292</td>
<td>83.8510</td>
</tr>
<tr>
<td>7th order</td>
<td>0.2527</td>
<td>1.4714</td>
<td>7.9554</td>
<td>1.6852</td>
<td>90.1354</td>
</tr>
<tr>
<td>US2PEM</td>
<td>0.2591</td>
<td>1.0298</td>
<td>8.2106</td>
<td>1.4679</td>
<td>21.3450</td>
</tr>
</tbody>
</table>

### V. Conclusion
In this paper, the uncertainty handling of wind turbines, PV modules and electric load values is studied. Three different PDFs are used to describe the behaviors of aforementioned uncertainties. A Weibull, Beta and normal PDFs are chosen for modeling the uncertainties of wind speed, solar radiation and electric load, respectively. An unsymmetrical two point estimate method (U2PEM) is applied to handle the uncertainties of the mentioned variables. The Monte Carlo simulation is used for obtaining an accurate result as a basis for comparison. The simulation results showed that the values obtained by the
U2PEM are more accurate than other methods and also a great reduction in computational burden was observed.

REFERENCES


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