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Application of a Modified NSGA method for Multi Objective Static DG Planning

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Abstract— The integration of distributed generations in distribution network has changed its characteristics. These electric resources are used as an alternative energy resource for main grid. The technical and economical benefits of these units are achieved only when they are optimally sized and placed in the network. In this paper a static mixed integer non-linear model for distributed generation planning is defined and solved using a modified NSGA method (Non-dominated Sorting Genetic Algorithm). Different DG technologies are taken into account and objective functions to be minimized are defined as the total active losses, investment and operational costs and environmental pollutions. The method is applied on a test distribution network and the results are presented and discussed and compared to other methods.

Key words—Static planning, Dispersed generation, Multi Objective Optimization, Active loss reduction, Technical constraint, Emission reduction.

1. INTRODUCTION
WITH the introduction of restructuring concepts to traditional power systems, a great deal of attention has been given to utilization of distributed generations. DG is defined as all small generators, typically ranging from 15 to 10000 KW, scattered throughout a power system, to provide the electric power needed by customers [1]. In most power systems, a large portion of electricity demand is supplied by large-scale generators; this is because of economic advantages of these units over small ones. However, in the last decades, technological innovations and a changing economic and regulatory environment have resulted in a renewed interest for DG units [2]. A study by the Electric Power Research Institute (EPRI) indicates that by 2010, 25% of the new generation will be distributed. Natural Gas Foundation concluded that this figure could be as high as 30% [3]. There are five major factors behind this trend [4]: electricity market liberalization, development in DG technology, constraints on the construction of new transmission lines, reliability enhancement and concerns about environmental aspects [5]. In addition to those indicated before, there are other important benefits in using DG units into distribution systems.

It should be noted that the maximum benefits of DG units can be obtained only if they are optimally placed and sized in the network. Different models and method are proposed in the literature. An optimization method based on ant colony approach is proposed in [7] for reliability enhancement of distribution systems. This method determines the optimal placement of protection devices and distributed generators in radial feeders. A composite reliability index is used as the objective function in the optimization procedure. An ordinal optimization method is proposed in [8] for specifying the locations and capacities of distributed generation (DG) such that a in order to maximize the benefits of distribution system operator due to loss reduction and DG capacity maximization. A new methodology based on nodal pricing is proposed in [9] for optimally allocating DG units in order to reduce losses and voltage profile improvement. In [10], a method for placement of distributed generation (DG) units in distribution networks has been presented based on the analysis of power flow continuation and determination of most sensitive buses to voltage collapse. The objective function is to improve the voltage profile and reduction of power losses. In [11] a method is proposed for DG placement in order to increase the reliability and efficiency in distribution networks. A hybrid GA-OPF method for sitting and sizing a predefined number of DG units was proposed in [12]. This method tries to increase the benefits of distribution system operator by decreasing the active losses and increasing the
incentives received by accepting DG units in their territory. Dg units can also offer an alternative planning approach to distribution network operators to meet the demand growth as an alternative for network investment. A multiyear multi-period optimal power flow is proposed in [13], to determine for the optimal sitting and sizing of DG installation. The objective function is deferring the network investment.

The priority based methods are often used to determine the optimal site and size of DG units. Applying these methods change the multi objective function into a benefit to cost ratio index [14] or an additive utility function [7-13],[15-17] (using a predefined set of weight factors) and then it is tried to maximize (minimize) the constructed single objective problem. The weighted sum approaches cannot guarantee to find all Pareto optimal solutions in the case of non-convex objective spaces [18, 19]. Since it is, in general, difficult to know whether objective space is convex or not, the weighted sum approach should be applied with caution. On the other hand, in some cases, objective functions are incommensurable and can not be added together. Moreover, the obtained solution is very sensitive to the given priorities and if the circumstances in which these are determined changes the problem should be solved again. The shortcomings of these methods has made multi-objective analysis a powerful decision making tool for planners.

This paper proposes a two-stage multi-objective model for integration of DG units in a deregulated environment. At first stage, the proposed heuristic algorithm finds the set of Pareto optimal front and at the second stage a fuzzy satisfying method is used to choose the most beneficial plan which satisfies the technical, environmental and economical criterions. The rest of this paper is organized as follows: The principles of multi objective optimization and Genetic algorithm are discussed in section 2. In section 3, problem formulation is introduced and discussed. The proposed algorithm is presented in section 4. Section 5, introduces the fuzzy satisfying method applied in this paper. Simulation results and conclusions are given in section 6, 7 respectively.

2.PRINCIPLES OF MULTIOBJECTIVE OPTIMIZATION AND GA

In most realistic optimization problems, particularly those applicable in power system, there exists more than one objective function which should be optimized simultaneously.

The main difference between the multi objective optimization and traditional single optimization techniques can be categorized in two areas:
1) Several objective functions are to be optimized at the same time.

2) There exists a set of optimal solutions which are mathematically equally good solutions (it means one of them can not be preferred against others and a trade off shall be made to select one) Instead of a unique optimal solution

Generally, every multi-objective optimization problem consists of a number of objectives and several equality and inequality constraints which can be formulated as (1).

\[
\begin{align*}
\text{Min } F(\vec{x}) &= [f_1(\vec{x}) \ldots f_{N_{\text{objective}}}(\vec{x})] \\
\text{Subject to:} & \\
G(\vec{x}) &= \vec{0} \\
H(\vec{x}) &\leq \vec{0}
\end{align*}
\]

In this context, it is tried to produce a set of trade-offs solutions for decision maker. The set of all optimal solutions which are non-dominated by any other solution, is known as Pareto-optimal set. A set of pareto optimal solutions are given in Fig.1 for a minimizing problem with two objectives in conflict.

**Figure 1**

Each solution in Pareto optimal set has two basic characteristics:

1) For every two solutions belonging to the same pareto front (2) holds:

\[
\forall i \exists j, n: f_n(\vec{x}_i) > f_n(\vec{x}_j) \quad \forall \vec{x}_i, \vec{x}_j \in S
\]

This means for every solution \( \vec{x}_i \) belonging to Pareto front \{S\}, at least one solution exists as \( \vec{x}_j \) which is better than \( \vec{x}_i \) at least in one objective function (named n here). In other words there is no solution in Pareto optimal front which is the best among all members of this set considering all objectives.

2) For every solution belonging to an upper Pareto front and the ones in the lower fronts, (3) holds:

\[
\forall i, n \exists \vec{x}^*: f_n(\vec{x}^*) \leq f_n(\vec{x}_i) \\
\exists m: f_m(\vec{x}^*) < f_m(\vec{x}_i) \\
\vec{x}_i \in S, \vec{x}^* \in S^* < S \\
n, m \in \{1, \ldots, N_{\text{objectives}}\}
\]

This means for every solution \( \vec{x}_i \) belonging to an upper Pareto front, there exists at least one solution in a lower Pareto front named \( \vec{x}^* \) which is not worse than \( \vec{x}_i \) in any objective function.
and is better than it in at least one objective function (named \( m \) here).

The classical approach for finding the Pareto optimal set is preference-based method in which a relative preference vector is used to weight the objectives and change them into a scalar value \[18\]. By converting a multi objective optimization problem into a single objective one, only one optimum solution can be achieved which is very sensitive to the given weights.

It seems that the heuristic algorithms are particularly suitable to solve multi-objective optimization problems, because they deal simultaneously with a set of possible solutions (the so-called population). This allows decision maker to find several optimal solutions (Pareto optimal set) in a single run, instead of having to perform a series of separate runs as in the case of the traditional mathematical methods. To do this, many heuristic algorithms have been proposed like NSGAII \[20\], PSO \[21\]. In all of these algorithms an initial population is created and then it is guided toward the Pareto front. The solutions with smaller distance to Pareto optimal front and more diversity are more preferred. This is repeated through several iteration until the stopping criteria is met. The ultimate goal is to seek the most preferred solution among the Pareto optimal set, in this paper this it done by using a fuzzy satisfying method which will be discussed in section V.

The Genetic Algorithm (GA) is a computational model which is designed to simulate processes in natural systems necessary for evolution. It follows the principles which were first introduced by Holland \[22\]. This algorithm involves three operators named as: Selection, crossover and mutation. GA has been used in many power system applications such as power plant control system design \[23\], Economic dispatch \[24\], and Unit commitment \[25\].

3. PROBLEM FORMULATION

Distributed generation planning consists of various linear and nonlinear sub problems. The concerned problem is proposing a plan which maintains the technical and environmental constraints in an optimum level and minimizing the total associated costs with such a plan.

The Distribution Network Operator (DNO), as the planner, has two alternatives to supply its customers, the first one is purchasing energy from main grid and the second option is using DG in its territory or an optimum combination of them. The aim of distribution system planner is seeking the best configuration among some available DG categories and placing them on suitable buses with considering various factors such as active power losses, investment and operation
costs and environmental emissions. The following assumptions are employed in problem formulation:

3.1. Decision variables
The number of installed DG units from each technology, in each bus, IDG(BUS,DGtech), constitute the set of decision variables of the planning problem.

3.2. Constraints
3.2.1. Dynamics of investment
The best plan is sought which satisfies the forecasted load at the end of the planning horizon while satisfying the technical constraints and minimizing the defined objective functions.

3.2.2. Modeling of DG connection
Connection of a DG unit to a bus is modeled as a negative PQ load with a variable but bounded power factor as shown in Fig.2.

\[
\sqrt{(P_{h}^{Grid})^2 + (Q_{h}^{Grid})^2} \leq GridCaP
\] (4)

Where, GridCap is the capacity of substation transformer which feeds the network, \( P_{h}^{Grid} \) is the active power purchased from grid in demand level \( h \), in MWh and \( Q_{h}^{Grid} \) is the reactive power purchased from grid in demand level \( h \), in MVARh.

3.2.3. Limitation on Transmission system
It shall be noted that the DNO has a physical limitation for the amount of purchased energy form main grid. This means that the total energy purchased from main grid can not exceed the transformer and substation capacity. This constraint is addressed in (4).

3.2.4. Demand level factors and their durations
For getting closer to reality it is assumed that each day has three different demand levels Low, Medium and High with different demand level factors, i.e. \( DLF_i \). The duration of each level will be equal to \( DLF_i \). The variation of demand level factor in different demand levels in 24 hours is depicted in Fig.3. The values of \( DLF_i \) in each demand level demonstrate the ratio of demand in that demand level to medium demand period. This value is 1 for medium demand period and increases in peak load.

3.2.5. Demand growth
The demand of a system is not constant during the planning horizon and it increases with a rate called Yearly demand growth. At the end of the planning horizon the demand has increased with a factor equal to $\prod_{\text{year}=1}^{T} (1 + \text{Growth}_\text{year})$. The demand at each bus in each demand level can be calculated using (5).

$$D_{(i,h)} = \text{Load}_\text{base} \times DLF_h \times \prod_{\text{year}=1}^{T} (1 + \text{Growth}_\text{year})$$ (5)

Where, $\text{Growth}_\text{year}$ is the demand growth factor in each year, $\text{Load}_\text{base}$ is the base load in the beginning of the planning horizon and $D_{(i,h)}$ is the demand in bus $i$ and demand level $h$, at the end of the planning horizon.

### 3.2.6. Variations of electricity price

Obviously the price of purchased energy from grid is competitively determined and is not constant during different demand levels. Forecasting the variation of this parameter would not be an easy job but it is assumed that the variation pattern of this parameter can be modeled by a factor named grid price factor, i.e. $GPF_h$. The variation of grid price demand factor in different demand levels during the 24 hours of a day is modeled using this parameter. As it is already explained for demand level factors, this parameter describes the variation of electricity price in different demand levels. In medium demand period, this quantity is unity and decreases in low demand period and increases in peak demand period as depicted in Fig.4.

### 3.2.7. Limitation on the number of available DGs

It is assumed that the planner can choose between discrete capacity choices up to a maximum capacity from each DG technology. The number of available DG units in each technology is constrained by resource availability or regulation. This is illustrated in (6).

$$\sum_{j=1}^{N_{\text{res}}} IDG_{(i,DGtech)} \leq N_{DGtech}$$ (6)

Where, $IDG_{(i,DGtech)}$ is the number of installed DG in bus $i$, $N_{DGtech}$ is the number of available units of a DG technology.

### 3.2.8. Operating limits of DG units
The total active power generated by each installed distributed generator cannot violate its maximum and minimum allowed operating ranges as shown in (7).

$$P_{(i,DGtech,h)} \leq IDG_{(i,DGtech)} \times P_{DGtech}^{max}$$ (7)

Where, $P_{(i,DGtech,h)}$ is the active power generated by a DGtech in a specific bus and demand level in MWh and $P_{DGtech}^{max}$ is the maximum limit for active power generation of a specific DG technology in MW.

### 3.2.9. Power flow constraints

Besides the economic assumptions and calculations made till now, it is necessary that the technical constraint are also satisfied. The most viable constraints are well known power flow equations that shall be satisfied for each configuration and demand level as indicated in (8).

$$P_i' = V_{i,h} \sum_{j=1}^{n} Y_{ij} V_{j,h} \cos(\delta_{i,h} - \delta_{j,h} - \theta_{ij})$$

$$Q_i' = V_{i,h} \sum_{j=1}^{n} Y_{ij} V_{j,h} \sin(\delta_{i,h} - \delta_{j,h} - \theta_{ij})$$

$$V_{min} \leq V_{i,h} \leq V_{max}$$

Where, $P_i'$ is the net active power injected to bus I, in demand level h. $Q_i'$ is the net reactive power injected to bus I, in demand level h. $V_{i,h}$ is the voltage magnitude of bus I in demand level h. $V_{max}$ is the maximum permissible voltage in the network and $V_{min}$ is the minimum permissible voltage in the network.

### 3.3. Objective functions

#### 3.3.1. Active Losses

Distribution systems are usually designed with just one supply source and this may cause significant active and reactive losses. The active losses mainly depend on the line resistance and currents and are usually referred to as thermal losses. While the line resistances are fixed, the currents are a complex function of the system topology and the location of DG units and system demand level.

$$OF_1 = TAL = \sum_{h=1}^{N_h} AL_h \times DU_h$$ (9)

Where, $AL_h$ is the active loss in demand level $h$ and TAL is the total active loss.

#### 3.3.2. Total Costs

The total cost which DNO has to pay, consists of three components including Total Grid Costs, Installation & Operating costs of DG units. Each component is defined and calculated as follows: As stated before, the DNO has the DG as an alternative in his plans while the main grid (which is
called grid here) is its main source of energy. By purchasing energy from grid, DNO has to pay for it. Total grid payment can be calculated by summation of the cost of both active and reactive energy purchased from grid as stated in (10).

\[
TGC = \sum_{h=1}^{N_h} \left( P_{h}^{Grid} \times PC_{h}^{Grid} + Q_{h}^{Grid} \times QC_{h}^{Grid} \right) \times DU_{h} \times GPF_{h}
\]  

(10)

Where, TGC is the total grid costs that should be paid by DNO, \(PC_{h}^{Grid}\) and \(QC_{h}^{Grid}\) are the costs of active and reactive power purchased from main grid by DNO. \(GPF_{h}\) is the grid price factor in demand level \(h\).

The installation cost of DG units is calculated as the sum of costs associated to all installed DGs over all DG technologies and buses.

\[
DGIC = \sum_{Bus} \sum_{DGtech} IDG_{(i,DGtech)} \times IC_{DGtech}
\]

(11)

Where, DGIC is the installation cost of DG units, and \(IC_{DGtech}\) is the investment cost of a specific DG technology.

The operating costs of DG units can be calculated by summation of cost of active energy generated by each installed DG technology multiplied by its associated operating cost as stated in (12).

\[
DGOC = \sum_{i=1}^{N_{DG}} \sum_{DGtech} \sum_{h=1}^{N_h} DU_{h} \times OC_{DGtech} \times P_{(i,DGtech,h)}
\]

(12)

Where, \(DGOC\) is the total operating cost of DG units, \(OC_{DGtech}\) is the operating cost of each DG technology.

The final objective function is obtained by adding all the introduced cost components. This is done in (13).

\[
OF_2 = DGIC + DGOC + TGC
\]

(13)

3.3.3. Emissions

The total produced emission has two parts:

1. The emission caused by the main grid
2. The emission produced by installed DGs which are being used.

The total emission cost is obtained by summation of these two parts as calculated in (14).
\[ \text{OF}_3 = \text{TE} = \sum_{h=1}^{N_h} DU_h \times \left( \text{EF}_{\text{Grid}} \times P_h^{\text{Grid}} + \sum_{i=1}^{N_{\text{DG}}} \sum_{j=1}^{N_{\text{DG}}} \text{EF}_{\text{DGtech}} \times P_{(i, j, \text{DGtech}, h)} \right) \] (14)

Where, \( \text{TE} \) is the total emission produced in the system, \( \text{EF}_{\text{Grid}} \) is the emission factor of the main grid, \( \text{EF}_{\text{DGtech}} \) is the emission factor of each DG technology.

4. PROPOSED ALGORITHM (FIRST STAGE OF THE ALGORITHM)

4.1. Parameterization of problem for GA

To solve DG planning problem through genetic algorithm, problem parameters must be modeled in terms of genetic parameters:

In the proposed problem the solutions are binary coded. A sample solution vector is shown in Fig.5. Each row is demonstrating the installation and operating of a DG unit. The first column has a binary value 0 or 1. If it is 1 this means that this unit should be installed in the network and if it is 0 this means that this DG unit should not be installed in the network. The next column shows the location of this DG. This will specify in which bus, it should be installed. The remaining columns determine the operating schedule of an installed DG unit. It should be remembered that for increasing the performance of the proposed algorithm, if a DG is not installed (first column is 0) the remaining columns are not analyzed.

4.2. Determination of Pareto front

As stated before to direct the population toward the Pareto optimal front two things shall be noted:

1) Getting closer to Pareto optimal front
2) Maintaining the diversity among the solutions

In single objective optimization it is easily done because there is no need to diversify the solutions and also just one objective function shall be considered for guiding the population. To solve this problem a pseudo fitness value is assigned to each solution as (15)

\[ \text{Pseudo}_i = \text{Paretofront}_i^{-1} + \text{GDiversity}_i \] (15)

Where, \( \text{Pseudo}_i \) is a pseudo fitness for solution \( i \), \( \text{Paretofront}_i \) is the Pareto front number in which the solution \( i \) is located. Finally, \( \text{GDiversity}_i \) is the global diversity of solution \( i \).
The first term in (15) guides the population toward the Pareto optimal front since the solutions which are members of lower fronts get higher fitness while the second term insures the diversity among the solutions. It is calculated as follows:

A local diversity factor is defined according to each objective function then a global diversity factor is introduced. For every objective function, solutions are sorted and distance between the maximum and minimum is calculated using (16)

\[
Maxdistance_k = \max_{i=1}^{N_{\text{Solutions}}} \left( f_k(X_i) - f_k(X_i') \right)
\]

(16)

Where, \(Maxdistance_k\) is the maximum distance between solutions regarding the kth objective function.

As the solutions are sorted, so the first and the last one are the max and min of them. The diversity of every other solution is the average distance of it to its neighbors as shown in (18).

\[
Diversity^i_k = \frac{\left| f_k(X_i) - f_k(X_{i+1}) \right| + \left| f_k(X_i) - f_k(X_{i-1}) \right|}{2 \times Maxdistance_k}
\]

(17)

Where, \(Diversity^i_k\) is the diversity of solution i, regarding the kth objective function.

For the first and the last solution local diversity can be calculated using (18)

\[
Diversity^1_k = \max_{j=2}^{N_{\text{Solutions}}-1} \left( Diversity^j_k \right)
\]

(18)

\[
Diversity^N_{\text{Solutions}} = Diversity^1_k
\]

The global diversity factor for each solution is calculated as the average of its local diversities as shown in (19).

\[
GDiversity^i = \frac{\sum_{k=1}^{N_{\text{Objective}}} Diversity^i_k}{N_{\text{Objective}}}
\]

(19)

### 4.3 Steps of the Algorithm

After analyzing the problem and selection of a proper expression of solution the topology of the algorithm is as follows:

1. Generate an initial random solution.
2. Set iteration = 1.
3. Evaluate the fitness for each member of the population as follows:
• Convert the binary solution into decimal number.
• Evaluate each individual of the population by performing load flow calculations to compute energy loss and voltage magnitudes at each node, total emission and total costs (including installation and operation costs).
4. Determine the pareto front and the global diversity factor for each solution.
5. Compute pseudo fitness for each solution.
6. Sort the population based on pseudo fitness value. Keep the population length limited to $N_{solution}$.
7. If the stopping criterion is met go to step 16, else continue.
8. Consider a predefined top percent of the population as the potential parents (elite set) based on their pseudo fitness.
9. **Probabilistically** select two parents from elite set based on roulette wheel method.
10. Perform crossover and generate two children.
11. Mutate these two children based on mutation probability.
12. If there still more children are needed, go to 9, else continue.
13. Combine the old population and new population to create a single population containing the bests of both.
15. Return to Step 3.
16. End

**The novelties of the proposed method are as follows:**
1. A separate pseudo fitness for the first and the last solutions regarding each objective function. This will enable the algorithm to maintain the diversity of the solution in the solution space.
2. In step 9 where the parents are probabilistically selected using their pseudo fitness. This will help the algorithm for escaping the local minima and increases the number of Pareto solutions found by the algorithm.

**5. Fuzzy Satisfying Method (Second Stage of the Algorithm)**

The next step after obtaining the Pareto front is to select the solution among the candidates. A fuzzy satisfying method has been used to obtain the satisfactory solution for the decision maker.
from the Pareto optimal set. For each solution in the Pareto optimal front, like \( X_i \), a membership function is defined as \( \mu_{f_i}(X_i) \). This value shows the level of which \( X_i \) belongs to the set that minimizes the objective function \( f_i \). \( \mu_{f_i}(X_i) \) Varies between 0 to 1. The membership value “0” indicates incompatibility with the set, while “1” means full compatibility. In other words, the membership function gives a numerical description of how the decision maker is satisfied by which level of achievement of solution with respect to a specific objective function. The decision maker is fully satisfied with \( X_i \) if \( \mu_{f_i}(X_i) = 1 \) and dissatisfied when \( \mu_{f_i}(X_i) = 0 \) [26].

Different types of membership functions have been suggested like linear or exponential ones. Note that by selecting the shape of membership function other than linear type can be used by the decision maker to give more priority to a specific objective function. Here a linear type of membership function has been used for all objective functions as described in (20).

\[
\mu_{f_i}(X) = \begin{cases} 
0 & f_i(X) > f_i^{\text{Max}} \\
\frac{f_i^{\text{Max}} - f_i(X)}{f_i^{\text{Max}} - f_i^{\text{Min}}} & f_i^{\text{Min}} \leq f_i(X) \leq f_i^{\text{Max}} \\
1 & f_i(X) < f_i^{\text{Min}} 
\end{cases}
\]

(20)

Fig. 6 shows the selected membership function in this study.

After defining the membership functions, there are several ways to choose the final solution. Each method considers a different philosophy. The method used in this paper is introduced as follows:

As the planners will live with their plans, a conservative decision can be achieved by trying to find the solution which its Minimum satisfaction is Maximum over all objective functions. Using the MaxMin formulation, the final solution can be found by solving (21).

\[
\text{Max} \left( \text{Min} \left( \mu_{f_i}(X_i) \right) \right)_{i=1}^{N_{\text{Objective}}} = \text{Min} \left( \mu_{f_i}(X_i) \right)_{i=1}^{N_{\text{Solution}}}
\]

(21)

The flowchart of the proposed two-stage algorithm is depicted in Fig.7.

6. SIMULATION RESULTS

The proposed algorithm is applied to an IEEE test feeder [27] with slight modifications which is shown in Fig.8.
In the performed simulations, it is assumed that all buses are candidate for DG installation and more than one DG can be installed in a specific bus.

The proposed framework can be used for different technologies and operation and planning philosophies. Here, without loss of generality, the assumptions used in this work are listed:

1. The planning horizon (T) is 10 years.
2. Transmission system limitation for energy delivery is GridCap=200MVA.
3. The demand growth in each year of planning horizon can be different, it is assumed that it is similar in each year and Growth_{year}=5%. With this assumption, demand growth at the end of the 10th year will be equal to \((1+0.05)^{10}=1.551\).
4. The duration of low, medium and high demand level in each day, are assumed to be 8,12,4 hours respectively. So the duration of these demand levels in a year will be multiplied by 365. These are given in Table 1.
5. Variations of demand and Grid price factors are given in Table 1.
6. Emission factor of main grid (EF_{Grid}) is assumed to be 672 Kg/MWh
7. Cost of active (PC_{Grid}) and reactive energy (QC_{Grid}) purchased from main grid are 60$/MWh, 30$/MVRh respectively.
8. Maximum and minimum voltage for each bus is assumed to be \(V_{Max}=1.05Pu\), \(V_{Min}=0.9Pu\).
9. Cross over probability =70%.
10. Mutation probability =5%.
11. Number of population = 500.
12. Stopping criteria: Max iteration =1000.
13. The characteristics of different DG technologies [28, 29] used in this paper are available in Table 2.

The Pareto optimal solutions of the defined problem are found using the first stage of the proposed algorithm. The variation of all three objective functions 430 solutions is depicted in Fig.9. This figure shows the planner that how choosing different solutions can change the objective functions. He has a set of Pareto solutions (instead of one solution) and can decide what
to choose based on his priorities. If the priorities of planner change, then it is not necessary to resolve the problem and he can easily change its final solution. Fig.9 shows that if the planner desires to reduce the emissions, he should pay more. This means that the emission reduction and cost reduction are in conflict.

Some of the obtained solutions are given in Table 3, these solutions can be categorized and each category have some special features and the planner can choose the best solution based on its preferences over different objective functions. It should be noted that the total emission is given in Tons and Total cost is in *1000 scale.

For example if the planner is more interested in just one of the objective functions the B, C, D categories can be good options. Suppose we are interested in the fist one, the solutions belonging to B have the least OF1. If the planner decides to choose the #1 solution, although it has the greatest satisfaction value over the OF1, but by choosing solution #284 he can obtain almost the same loss with a less cost and of course this solution has a worse environmental emission over the #1.

The C and D category contain the solutions with better OF2, OF3 respectively. The same strategy for obtaining the satisfactory solution among these solutions can be taken.

Some times the planner has to choose the solutions which require satisfying more than one criterion. In these cases, E, F, G categories can help him to choose the best alternative. In E category, OF1, and OF2 have their best values while in F, the OF1, OF3 and finally in G the OF2, OF3 have the appealing values.

For obtaining the final solutions the second stage of the proposed algorithm is used and the final solution is chosen in a manner which the least satisfaction over all objective functions is maximum. Here the solution #495 has this property and given in more detail in Table 4.

In order to verify the computational strength of the proposed solution method, is is compared to other heuristic search methods. The comparison is made in two aspects for different algorithms, namely, number of Pareto optimal solutions found by algorithm and computational time. The proposed genetic algorithm is compared with simulate annealing method, SA [30] , Particle
Swarm Optimization, PSO [31], ordinary NSGA [18] and Tabu Search method, TS [32]. The results are given in Table 5. It can be observed in Table 5, that for a given number of iterations (1000 in this paper), the proposed algorithm is the fastest. Although the computation procedure of this algorithm is offline and it may not be so important for increasing the computation speed but if the distribution network is very large or the number of considered demand levels are more than three then it would be beneficial. The numbers of obtained Pareto optimal solutions are demonstrated in Table 5. The proposed algorithm finds more Pareto optimal solutions in comparison to other heuristic methods. This will provide the planner a broader set of solutions for selecting the final action.

Table 5

7. CONCLUSIONS

In this paper, a multi objective optimization model for integration of DG units into a distribution network in a deregulated power system has been proposed. The main contributions of this study can be summarized as follows:

1) Consideration of different demand levels and their duration instead of assuming the load as a constant parameter during the planning period.
2) As the planning procedure is performed in a liberalized power market, the variations of energy price during different demand levels are considered in the calculations.
3) The ability of providing ancillary services like reactive power for DG units is considered in this work.
4) Different DG technologies with their different technical, economical and environmental characteristics are considered as available alternatives for DG planner.
5) The operation schedule of installed DG units has been incorporated in the optimization process.
6) The proposed algorithm can obtain more non-dominated solution with less computational burden.

The advantages of this model have been demonstrated by applying it on a standard test system. Results presented in this paper indicate that the proposed algorithm can be used to find the optimal sitting and sizing of DG units in a distribution system to reduce the active power losses, operation, investment costs and environmental pollutions simultaneously. Since the proposed method is a posterior one there is no need to resolve the problem if the importance of objective
functions change for the planner. Instead, it gives a broad range of solution to the planning problem, which makes the planner enable to decide what to choose according to the condition he has considered about different objective functions. The strength of the proposed method is demonstrated by comparing it to other heuristic search methods. The comparison shows that the proposed algorithm can find more Pareto optimal solutions in a shorter running time.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GridCap</td>
<td>Capacity of substation transformer which feeds the network</td>
</tr>
<tr>
<td>$p^\text{Grid}_h$</td>
<td>Active power purchased from grid in a specific demand level in MWh.</td>
</tr>
<tr>
<td>$q^\text{Grid}_h$</td>
<td>Reactive power purchased from grid in a specific demand level in MVARh.</td>
</tr>
<tr>
<td>$D_{(i,h)}$</td>
<td>Demand in a specific bus and demand level in MW.</td>
</tr>
<tr>
<td>Load$_{bus}$</td>
<td>The base load of each bus.</td>
</tr>
<tr>
<td>$DLF_h$</td>
<td>The demand level factor in demand level $h$.</td>
</tr>
<tr>
<td>$\text{Growth}_{\text{new}}$</td>
<td>Yearly load growth.</td>
</tr>
<tr>
<td>$DGIC$</td>
<td>DG installation costs in $.</td>
</tr>
<tr>
<td>$IDG_{(i,\text{DGtech})}$</td>
<td>Number of installed DG in a specific bus.</td>
</tr>
<tr>
<td>$IC_{\text{DGtech}}$</td>
<td>Installation cost of a DG technology in $.</td>
</tr>
<tr>
<td>$P_{(i,\text{DGtech},h)}$</td>
<td>Active power generated by a DGtech in a specific bus and demand level $h$, in MWh.</td>
</tr>
<tr>
<td>$Q_{(i,\text{DGtech},h)}$</td>
<td>Reactive power generated by a DGtech in a specific bus and demand level $h$, in MVARh.</td>
</tr>
<tr>
<td>$N_{\text{DGtech}}$</td>
<td>Number of available units of a DG technology.</td>
</tr>
<tr>
<td>$\text{MinPF}_{\text{DGtech}}$</td>
<td>The minimum power factor for a DG technology in producing reactive power.</td>
</tr>
<tr>
<td>$\text{MaxPF}_{\text{DGtech}}$</td>
<td>The maximum power factor for a DG technology in producing reactive power.</td>
</tr>
<tr>
<td>$P_{\text{max}}^{\text{DGtech}}$</td>
<td>Maximum limit for active power generation of a specific DG technology in MW.</td>
</tr>
<tr>
<td>$OC_{\text{DGtech}}$</td>
<td>Operating cost of a DG technology in $/MWh.</td>
</tr>
<tr>
<td>$DGOC$</td>
<td>Total operating cost of all installed DG technology in $.</td>
</tr>
<tr>
<td>$EF_{\text{Grid}}$</td>
<td>DG technology emission factor in kg/MW.</td>
</tr>
<tr>
<td>$EF_{\text{DGtech}}$</td>
<td>Total emission in Ton.</td>
</tr>
<tr>
<td>$TGC$</td>
<td>Total grid cost paid for purchasing active, reactive energy from grid in $.</td>
</tr>
<tr>
<td>$DU_h$</td>
<td>Duration of demand level in hours.</td>
</tr>
<tr>
<td>$GPF_h$</td>
<td>Grid price factor in a specific demand level.</td>
</tr>
<tr>
<td>$PC_{\text{Grid}}$</td>
<td>Cost of active power purchased from grid in $/MWh.</td>
</tr>
<tr>
<td>$QC_{\text{Grid}}$</td>
<td>Cost of reactive power purchased from grid in $/MVARh.</td>
</tr>
<tr>
<td>$TAL$</td>
<td>Total active losses in MWh.</td>
</tr>
<tr>
<td>$AL_h$</td>
<td>Active losses in a specific demand level in MWh.</td>
</tr>
<tr>
<td>$V_{i,h}$</td>
<td>The amplitude of voltage in the $i_{th}$ bus, in demand level $h$, in pu,</td>
</tr>
</tbody>
</table>
\( \delta_{i,h} \) The angle of voltage in the \( i_{th} \) bus in radian, in demand level \( h \)

\( \theta_{ij} \) The angle of \( i_{th} \) row and the \( j_{th} \) column element of admittance matrix in radian.

\( V_{\text{min}} \) Minimum allowed voltage in the \( i_{th} \) bus.

\( V_{\text{max}} \) Maximum allowed voltage in the \( i_{th} \) bus.

\( P_{i,h}^{\prime} \) Total active power injected to \( i_{th} \) bus in MW.

\( Q_{i,h}^{\prime} \) Total reactive power injected to \( i_{th} \) bus in MVAR.

\( P_{\text{pseudo}} \) Pseudo fitness for the \( i_{th} \) solution.

\( \text{Diversity}^k_i \) Diversity of \( i_{th} \) solution in the \( k_{th} \) objective function.

\( G\text{Diversity}^i \) Global diversity of \( i_{th} \) solution among all objective functions.

\( \text{Paretofront}^i \) The pareto optimal front which \( i_{th} \) solution belongs to.

\( \mu_k \) Membership function of the \( k_{th} \) objective function.

\( \text{Maxdistance}^i \) Maximum distance between solutions respecting \( k_{th} \) objective function.

\( N_{\text{Solution}} \) Number of solutions

\( N_{\text{Objective}} \) Number of objective functions

\( N_h \) Number of demand levels

**REFERENCES**


Fig. 1. Classification of a population to \( k \) non-dominated fronts.

Fig. 2. Model of the DG Unit Connected To the \( i \)th Bus.

\[
L_i = P_i + jQ_i
\]

\[
L_2 = \left( P_2 - \sum_{j=1}^{n} P_{DG_j} \right) + j \left( Q_2 - \sum_{j=1}^{n} Q_{DG_j} \right)
\]

\[
L_3 = P_3' + jQ_3'
\]
Fig. 3. Demand Level Factors in 24 hours

Fig. 4. Grid Price Factors in 24 hours
Fig. 5. Proposed vector for modeling the Problem Parameters $\bar{X}_i$.

Fig. 6. Linear type membership function

$$\mu_{f_i}(\bar{X})$$

$$f_i^{\text{min}} \quad f_i^{\text{max}}$$
Stage I

Generate an initial set of $N$ solutions

Iteration=1

Calculate objective functions for each solution

Calculate the pseudo fitness for each solution

Save the best $N$ solutions in memory

Is stopping criteria met?

Yes → Export Pareto optimal solutions

No → m=1

Probabilistically select two antibodies based on their pseudo fitnesses

m=m+1

m<N

Crossover

Mutation

Store new solutions

Stage II

New solutions

Union

Iteration=Iteration+1

Figure 7: Flowchart of two stages of the planning model
Fig. 8. Single Line Diagram of IEEE 34 bus Distribution system

Fig. 9. Pareto Optimal front for three objectives
<table>
<thead>
<tr>
<th>TABLE 1. ASSUMPTIONS</th>
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<tbody>
<tr>
<td>Load Levels</td>
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<tr>
<td>Medium</td>
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<td>High</td>
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<table>
<thead>
<tr>
<th>TABLE 2. CHARACTERISTICS OF CANDIDATE DG UNITS</th>
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<td>DG #</td>
</tr>
<tr>
<td>------</td>
</tr>
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</tr>
<tr>
<td>2</td>
</tr>
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<td>3</td>
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</tr>
<tr>
<td>6</td>
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TABLE 3. CATEGORIZED SOLUTIONS

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<th>Solution #</th>
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<th>OF3</th>
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<th>$\mu_{f2}$</th>
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TABLE 4. FINAL SOLUTION

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<td>P</td>
<td>Q</td>
<td>P</td>
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</table>

**TABLE 5. COMPARING THE PROPOSED METHOD WITH OTHER MULTI-OBJECTIVE SEARCH METHODS**

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<tr>
<th>method</th>
<th>Number of Pareto optimal solutions found</th>
<th>Computational time in Sec</th>
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<tbody>
<tr>
<td>Simulated Annealing [30]</td>
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<td>Particle Swarm Optimization [31]</td>
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<td>ordinary Non-dominated Sorting Genetic Algorithm (NSGA) [18]</td>
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