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Scenario-Based Selection of Pilot Nodes for Secondary Voltage Control

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Abstract—Due to local nature of the voltage and reactive power control, the voltage control is managed in a zonal or regional basis. In this paper a new comprehensive scheme for optimal selection of pilot points is proposed. The uncertainties of operational and topological disturbances of the power system are included to provide the robustness of the pilot node set. To reduce the huge number of probable states (i.e., combined states of load and topological changes) a scenario reduction technique is used. The resulted optimal control problem is solved using a new Immune-based Genetic Algorithm. The performance of the proposed method is verified over IEEE 118-Bus and realistic Iranian 1274-Bus national transmission grids.

Index Terms—Secondary voltage control, uncertainty, pilot node, scenario reduction, Immune-based Genetic Algorithm.

NOMENCLATURE

N_g Number of generators or dynamic Var devices
N_l Number of load buses
N_p Number of pilot nodes
N_s Number of combined load and contingency states
l_i Subscribe stands for i^{th} load state
d_i Subscribe stands for i^{th} contingency state
\pi_{d_i} Probability of occurring of d_i contingency state
\pi_{l_i} Probability of occurring of l_i load state
\pi_c Probability of each combined state
l_m Number of load states
d_m Number of contingency states
m_{ij} Weighting factor for the j^{th} dynamic Var resource
r Factor reflecting the order of the index for removing the masking effect in contingency screening
\Delta Q_G Vector of reactive power generation deviations
\Delta Q_L Vector of reactive power load perturbations
\Delta Q_{GL} Vector of reactive power load perturbations at l_{ij} load state
\Delta V_L Voltage deviation at load buses
\Delta V_P Voltage deviation at pilot nodes
\Delta V_G Voltage deviation at generator stations
S_{GL} Sensitivity matrix describing changes of reactive power at generator nodes w.r.t voltage magnitude changes at load buses
S_{GG} Sensitivity matrix describing changes of reactive power at generator nodes w.r.t their voltage adjustment
S_{LG} Sensitivity matrix describing changes of reactive power at load buses w.r.t voltage magnitude changes at generator nodes
P Binary pilot node matrix denotes the location of pilot nodes among load buses
R S I_i Reactive Support Index for i^{th} contingency
Q_{i}^{no} The unlimited reactive generation of the j^{th} dynamic Var device after contingency i
Q_{i}^{na} The unlimited reactive generation of the j^{th} dynamic Var device in the pre-contingency case
P I_c Performance index for c^{th} combined state
K_{svc} Gain of linear feedback controller to maintain voltage magnitudes at pilot nodes
D_{ij} Electrical distance between node i and j
R Weighting factor to merit voltage control at some load buses

I. INTRODUCTION

A. Motivation And Problem Description

Voltage control could be carried out in a hierarchical way to obtain different goals at different layers. Due to local nature of the voltage and reactive power control, the voltage control is carried out in a regional or zonal basis. In other words, voltage control is performed in different hierarchical layers: primary layer, secondary layer and tertiary layer. The primary voltage control contains local automatic actions such as Automatic Voltage regulators of generators to diminish fast local disturbances (e.g. short circuits). Voltage magnitude is deviated from the desired thresholds by slow load perturbations. Therefore a secondary or zonal voltage control is needed to counteract slow variations of voltage magnitudes inside an electric region. The voltage magnitudes of load points could be controlled by dynamic reactive power resources. Therefore in addition of voltage control of load points it is needed to fair distribution or dispatch of the required reactive power among available resources. In other words, the zonal voltage control is designed to control the voltage magnitudes throughout the electric zones and fair distribution of required reactive power among available resources simultaneously. The time constant of the zonal or secondary voltage control is chosen more than the time constant of the primary control to insure the independency of the hierarchical control layers [1]. The whole grid is separated into distinct areas. Each electric area is represented by a pilot node and some regulating generators. The pilot node is a load point that its voltage magnitude has the maximum similarity to the area voltage profile. According to Fig. 1 by measuring voltage magnitude deviation at the pilot nodes the area’s reactive power requirement is calculated...
by a proportional-integral law. Based on the output of this regulator a zone signal is obtained. Considering the zone signal all regulating generators inside a region will participate in voltage control with the same percentage of reactive power generation. The locations of pilot nodes has a major role in secondary voltage control performance. The optimal pilot node set obtained for a base case configuration cannot be optimal over the all possible operating scenarios. Thus, it is needed to improve the pilot node selection algorithms to make them robust against the uncertainties due to structural or operational changes in actual power system. This need motivates the work reported in this paper.

B. Literature Review

Various approaches have been proposed for pilot node selection problem including heuristic methods [2]–[6] and evolutionary optimization-based methods [7], [8]. In [2], the load buses with higher values of short circuit capacity are selected as pilot nodes. The concept of electrical distance in combination with clustering techniques is the next proposed method [3]. The pilot point selection could be formulated as an optimization model. The objective of this optimization problem is minimizing the voltage deviation throughout all electrical regions under all possible structural or load disturbances. The generator terminal voltage is taken as control variable. Two different approaches have been proposed to solve the optimization model: heuristic methods [4]–[6] and evolutionary technique [7], [8]. Recently some coordinated secondary voltage control schemes have been proposed to improve voltage stability margin or eliminate voltage violations. These scheme assume that the pilot nodes and associated control zones are known [9]–[12]. In [13] two very promising wide-area voltage protection (V-WAP) solutions, able to face stability and security problems in the transmission grid, have been presented with considering operation of secondary and tertiary control schemes according to their hierarchies. The major weakness of the previously proposed methods is the lack of robustness of the set of pilot nodes against structural (i.e. line or generator outages) and operational(i.e. load perturbations) changes in actual power system.

C. Contributions

Any secondary voltage control scheme should satisfy the following requirements:

1) R1: The voltage magnitude of the pilot node should represent the voltage profile of its associated region
2) R2: The regulating generators at each region should be able to provide enough reactive power support to regulate the voltage changes inside the region
3) R3: Each secondary voltage control area should be electrically decoupled from the other control areas
4) R4: The set of pilot nodes should be robust against the uncertainties due to structural or operational changes in actual power system

The gap that this paper intends to fill, is the consideration of forth requirement in addition to the other three requirements. Regarding this issue the uncertainties of loading conditions and the outages of main inter-area transmission lines are taken into account. The independency of the electrical zones is provided using the concept of electrical distance. To reduce the computational burden of the problem and uncertainty modeling a scenario reduction technique is developed. The pilot selection problem is a mixed integer nonlinear optimization problem. Therefore, in this paper, a new hybrid Immune-Genetic Algorithm is proposed to solve the optimal pilot node selection problem.

D. Paper Organization

The rest of this paper is organized as follows. In section 2, the detailed formulation of the secondary voltage control is presented. The proposed evolutionary algorithm is described in Section 3. In Section 4 the proposed scheme will be simulated for IEEE 118-Bus test system and a large scale realistic transmission network (Iranian 1274-Bus national grid). The conclusions are given in section 5.

II. PILOT NODE SELECTION FORMULATION

The overall structure of the multilayer voltage control is shown in Fig. 1. By measuring the voltage deviation at pilot point a zone signal, i.e. $N$, is obtained using a proportional integral controller. The zone signal forces all regulating generators to have the same participations in voltage control (i.e. same percentages of reactive power generation). Using the decoupled power flow model of the steady state system equations the linearized model of the zonal voltage control could be formulated via (1)-(3).

\[
\begin{bmatrix}
\Delta Q_G \\
\Delta Q_L
\end{bmatrix} =
\begin{bmatrix}
S_{GG} & S_{GL} \\
S_{LG} & S_{LL}
\end{bmatrix}
\begin{bmatrix}
\Delta V_G \\
\Delta V_L
\end{bmatrix}
\]

(1)

\[
\Delta V_L = J_1 \Delta Q_L + J_2 \Delta V_G
\]

(2)

where

\[
J_1 = S_{LL}^{-1}J_2 = -S_{LL}^{-1}S_{LG}
\]

(3)
Due to hierarchical nature of the SVC, the primary control is reached its steady state before the initiation of the secondary layer and so on. The lack of sufficient measurement and communication infrastructures at all load buses necessitates the existence of a minimum number of pilot nodes that their measurements are sufficient to control the voltage profile over all electrical zones. A linear controller in which the initial post-contingency pilot node voltage deviations written as a function of the set-point changes of regulating units is formulated as follows.

\[ \Delta V_G = K_{svc} \Delta V^0_L \]  

(4)

where \( \Delta V^0_L \) denotes voltage deviation at load buses without secondary control. The voltage magnitudes are measured only at pilot nodes:

\[ \Delta V_P = P \Delta V^0_L \]  
\[ \Delta V_G = K_{svc} \Delta V_P = K_{svc} P J_1 \Delta Q_L \]  

(5)

(6)

The pilot node matrix is defined as follows.

\[ P = [p_{ij}]_{N_p \times N_l} \]  
\[ p_{ij} = \begin{cases} 
1 & \text{if bus } i \text{ is the } j^{th} \text{ pilot node} \\
0 & \text{otherwise} 
\end{cases} \]  

(7)

(8)

The voltage deviation of load points could be defined as a function of controller gain, \( K_{svc} \), and pilot node matrix, \( P \), as follows.

\[ \Delta V_L = J_1 \Delta Q_L + J_2 K_{svc} P J_1 \Delta Q_L \]  
\[ = (I + J_2 K_{svc} P) J_1 \Delta Q_L \]  

(9)

The objective function or performance index of the secondary control scheme, \( PI \), could be defined as the total weighted sum of squares of voltage deviations throughout the network. For a given load disturbance given by \( \Delta Q_L \), it could be formulated via (10)-(13).

\[ PI = (\Delta V_L)^T R (\Delta V_L) \]  

(10)

\[ PI = \text{trace}[R (I + J_2 K_{svc} P) G (I + J_2 K_{svc} P)^T] \]  

(11)

where

\[ G = (J_1) \Lambda (J_1)^T \]  
\[ \Lambda = (\Delta Q_L) (\Delta Q_L)^T \]  

(12)

(13)

A. Uncertainty Modeling

The selected pilot nodes should provide a desired level of robustness against load and contingency uncertainties. In this section, a general procedure is proposed to model the uncertainties of load perturbations and structural changes (i.e. outages).

1) Load Uncertainty Modeling: In fact, the load disturbance has a random nature and so the performance index is a random variable. Therefore, to consider this randomness, the deterministic performance index should be replaced by a probabilistic index. Here, three load levels have been considered: Peak-Load, Off-Peak Load, and Light Load. The load perturbation around each load level has a normal distribution as shown in Fig. 2. The load will be divided into different levels using a clustering technique, utilizing the central centroid sorting process. All these states are defined as a percentage of the base-case loading. Mathematically each load state i.e. \( l_i \), is described with its \( \Delta Q_L \) and probability of occurrence, i.e. \( \pi_l \).

2) Topological Uncertainty Modeling: The outage(i.e. topological changes) modeling is carried out in two consequent steps. In the first step the contingency screening is done and the contingency modeling will then be followed in the second step.

- Contingency Screening: The most credible contingencies should be screened and weighted based on their probabilities. Reactive Support Index (RSI) proposed in [14], has been used for contingency ranking based on the capability of the power system in voltage and reactive power support. The RSI index, for a given contingency, is defined as the additional amount of reactive generation required to get from the base-case saddle-node point to the contingency nose-point. The Reactive Support Index is defined as the extra amount of reactive generation from all the existing dynamic VAr resources (e.g. generation, SVC, etc.) in which the reactive limits at the dynamic VAr devices are removed [14].
where $RSI_i$ is the relative $RSI$ index for contingency $i$. $Q_{ij}^{no}$ and $Q_{ij}^{vo}$ are calculated with open reactive limits of dynamic Var devices [14].

- Contingency States: After carrying out contingency analysis, in the first stage, a list of the most critical contingencies are selected. In the second stage, based on the forced outage rate or other historical information, the probability of each screened contingency is calculated. Here, without losing generality, only the severe contingencies are selected based on the value of $RSI$ index calculated in (14). In this paper, only the outages of tie-line transmission lines are taken into account. The normal state is considered as a state in which all equipments are in-service. Mathematically each contingency state i.e. $d_i$, is described with its $RSI_i$ and probability of occurrence, i.e. $\pi_{di}$.

**B. Combined Load and Contingency States**

It is assumed that the load and contingency states are independent so the states are combined to construct the whole set of states as follows:

$$\pi_c = \pi_l \times \pi_d$$

(15)

The total number of states, i.e., $N_s$, will be $l_n \times d_n$. For a large scale power system the huge number of scenarios(i.e. states) will increase the computational burden of the optimization task, enormously. In this paper a scenario reduction technique is implemented to reduce the total number of scenarios without losing much information of the original set of scenarios. The scenario reduction technique has been used in risk-averse decision making [15] and electricity market [16]. The formulation of the scenario reduction technique could be found in Appendix.

For uncertainty modeling the performance index could be formulated via (16) to (19).

$$PI = \sum_{c=1}^{N_s} \pi_c P|_c$$

(16)

$$P|_c = \text{trace}[R_c(I + J_{2c}K_{svc}^{c}P)G_c(I + J_{2c}K_{svc}^{c}P)^T]$$

(17)

where

$$G_c = (J_{1c})\Lambda_c(J_{1c})^T$$

(18)

$$\Lambda_c = (\Delta Q_{Le})(\Delta Q_{Le})^T$$

(19)

**C. Optimal Gain**

For a given pilot matrix, $P$, the controller gain, $K_{svc}$, is optimized with any integral control law, provided that the gain matrix, $K_{svc}^{c}$ verifies (4). The optimal gain of controller could be determined by two different strategies. In the first strategy the total voltage deviation over all load buses is minimized without minimizing voltage changes of regulating units, while in the second one the voltage deviation of pilot nodes is forced to be zero by minimizing voltage changes of regulating units. The optimal gain of linear controller for both strategies for each combined state is determined via (20) and (21), respectively.

$$\frac{\partial P|_c}{\partial K^{c}_{svc}} = 0 \Rightarrow K_{svc}^{*} = [J_{2c}^TR_{1c}^{-1}]^{-1}J_{1c}^T R_{1c} P^{T}[P^{T} R_{1c}]^{-1}$$

(20)

$$\Delta V_{P} = 0 \Rightarrow K^{*}_{svc} = [P J_{2c}]^{-1}([P^{T} J_{2c} P])^{-1}$$

(21)

**D. Optimization problem**

1) **Constraints:** Many constraints could be included in the pilot selection problem. Here, two main constraints are included to provide the independency of pilot nodes and to respect the limits of reactive power generations and terminal voltage changes of generators as well as voltage deviation of pilot nodes after implementing control actions. To provide the independency between electrical zones or pilot nodes, each two pilot pair should have a minimum electrical distance as follows.

$$D_{ij}^{c} = -\text{Log}(\alpha_{ij}^{c} \times \alpha_{ji}^{c}) \quad i, j = 1, ..., n_l$$

(22)

$$\alpha_{ij}^{c} = \frac{\partial V_{ij}^{c}}{\partial Q_{ij}} \quad i, j = 1, ..., n_l$$

(23)

2) **Optimization Problem:** By considering all probable scenarios of loading conditions and topological changes of the network, the optimal pilot set is the one that has the minimum cost for all loading conditions over the base-case and contingency configurations. Therefore the $P|_c$ is rewritten to consider all load and network states as follows.

$$\min_{P} P|_c = \sum_{c=1}^{N_s} \pi_c P|_c$$

(24)

subject to

$$P|_c = \text{trace}[R_c(I + J_{2c}K_{svc}^{c}P)G_c(I + J_{2c}K_{svc}^{c}P)^T]$$

(25)

$$K_{svc}^{c} = \begin{cases} [J_{1c}^T R_{1c}^{-1}]^{-1} J_{1c}^T R_{1c} P^{T}[P^{T} R_{1c}]^{-1} & 1st \ law \\ P J_{2c}^{-1} P^{T} J_{2c}^{-1} N_{S} P & 2nd \ law \end{cases}$$

(26)

$$D_{ij}^{c} \geq D_{ij}^{min}$$

(27)

$$\Delta V_{G_{min}} \leq \Delta V_{G} \leq \Delta V_{G_{max}}$$

(28)

$$\Delta Q_{G_{min}} \leq \Delta Q_{G} \leq \Delta Q_{G_{max}}$$

(29)

$$\Delta V_{L_{min}} \leq \Delta V_{L} \leq \Delta V_{L_{max}}$$

(30)

where $\Delta Q_{G}$, $\Delta V_{G}$, and $\Delta V_{L}$ are determined via (1), (5), and (9). Indeed due to the random natures of load and topological disturbances the secondary voltage control is a stochastic mixed integer non-linear optimization problem. In this paper, an Immune-GA-Based Technique is proposed to solve the optimization problem.
III. PROPOSED IMMUNE-GA METHOD

Immune Algorithm is a heuristic method which imitates the human’s reaction against external invasions. This algorithm has been successfully applied to pattern recognition [17] and multi-objective DG planning problem [18], [19]. In Immune algorithm, the objective functions and their associated constraints are assumed to be antigens and the solutions act as antibodies. Affinity factors are defined as the ability of antibodies (solutions) in recognizing (optimizing) the antigens (objective functions and constraints). Immune algorithm it is an iterative methodology which starts with an initial set of solutions and improves its performance. The Immune algorithm has two important operators namely, cloning and mutation [19]. The cloning operator reproduces the antibodies with a change proportional to their ability in recognizing the antigens (affinity) [19], [20]. The mutation operator applies some perturbation on antibodies in hope to find better ones. The mutation probability is related to the inverse value of the affinities. In order to enhance the strength of the algorithm, crossover operator [21] of GA is proposed in the present work to overcome the lack of memory in immune algorithm. To do this, in the cloning phase, the algorithm selects two solutions (instead of one) and performs the crossover operation. It then generates two new solutions and passes them to mutation operator. Mutation operator uses the value of affinity factor of the selected parents (i.e. antigens) as a measure for mutating them. The proposed solution algorithm is described as the following steps:

Step 1. Generate initial $N_{pop}$ solutions
Step 2. Set $Iteration = 1$
Step 3. Calculate the objective function (affinity factor) for each antibody using (24)
Step 4. If $Iteration < \text{maximum number then end} \text{else continue}$
Step 5. Keep the best antibodies in memory
Step 6. Set the cloning counter, i.e. $m$, equal to 1
Step 7. Select two antibodies ($p$ and $q$) as the parents among the antibodies stored in memory, using roulette wheel based on their affinities
Step 8. Calculate the number of cloning replica, i.e. $k_m$, and mutation probabilities based on the average values of parent affinities. The value of $k_m$ is determined as follows:

$$k_m = \text{round}(\Gamma \times \frac{OF_p + OF_q}{2\text{max}(OF_n)} \times N_{pop}) \quad (31)$$

Where, $\Gamma$ is a controlling factor and round is the function which gives the nearest integer number
Step 9. Clone the selected parents selected in Step.7, for $k_m$ times, by applying the crossover and mutation operators and produce new antibodies
Step 10. Store the new generated antibodies
Step 11. If the cloning counter is below the memory size, then increase cloning counter and go to Step.7; else, construct the new antibody set using the union of newly generated antibodies and the antibodies of memory, increase the iteration and go to Step.3

IV. SIMULATION RESULTS

The proposed model is simulated over IEEE 118 Bus test case and the realistic 1274-Bus transmission grid of Iran. The obtained optimal patterns of pilot nodes are compared to previously proposed methods. It is assumed that all load buses are candidates to be selected as pilot node. The results of simulations are presented for the second control law. Solving the (24) gives $21(\text{contingency states}) \times 10(\text{load states}) = 210$ states for IEEE 118 and $31(\text{contingency states}) \times 10(\text{load states}) = 310$ for 1274-Bus Iranian grid. It is clear that the calculation process for all these states imposes a heavy computational burden. In order to overcome this problem, a scenario reduction technique is implemented to reduce the number of states (see Appendix for more details) [15].

A. Load States

Ten independent load levels are chosen based on the clustering technique and utilizing the central centroid sorting process [22]. The proposed method in [22] verifies that choosing ten equivalent load levels (states), with different probabilities $\pi_i$, provides a reasonable trade-off between accuracy and fast numerical evaluation. The states have been described as a fraction of base case loading as given in Table I. This load states are applied to both test cases without losing generality.

B. Contingency States

Contingency screening is carried out based on the value of RSI index as described before. The results of RSI calculations for both test cases are shown in Table II and Table III.
The 20 reduced states by using the scenario reduction technique are given in Table IV and Table V.

C. Case 1: IEEE-118 Bus test case

The proposed model is applied to IEEE-118 bus test case. The loading data of this test case are modified based on [23]. Generation units with low reactive power capacities are converted to load buses.

1) IEEE-118 Bus Without Uncertainty Modeling: For this case the number of population is assumed as $N_P = 50$. Other optimization parameters such as clonal factors, crossover and mutation rates are assumed adaptively. The results are given in Table VI. The best objective function using the proposed method is compared with other heuristic and intelligent techniques. The optimal cost for the optimal pilot set is $43.81$.

2) IEEE-118 Bus With Uncertainty Modeling: In this case the load and contingency states are considered as given in Table I and Table II. The 20 reduced combined states are given in Table VII. Choosing more than 20 reduced scenarios does not provide better voltage profile over the grid.

D. Case 2: Iranian National Transmission Grid

Iranian national transmission grid consists of 1274 nodes, 551 generation units and 724 load points. The standard transmission voltages are 400 kV and 230 kV. The 400 kV backbone is shown in Fig. 6. The load, contingency, and voltage deviations are given in Fig. 5. It can be seen that the pilot set obtained by the proposed method provide better voltage profile over the grid.

The optimal cost for the optimal pilot set is $918.3$.
reduced combined states are given in Table I, Table III, and Table V. The obtained results are given in Table VIII. The total 32 pilot points are given for each region with and without uncertainty modeling. Referring to the single line diagram of the Iranian National Grid it can be seen that the obtained results have been distributed throughout the network.

V. CONCLUSION

The previously proposed model of secondary voltage control was modified to take into account topological and operational disturbances. The optimization model as a full integer programming problem was solved using a new Immune-GA based algorithm which was robust and could find better solutions with low computational burden by considering load and structural uncertainty. To reduce the computational burden the total number of states was reduced by a scenario reduction technique. The proposed scheme was applied to IEEE 118-Bus test case and Iranian 1274-Bus transmission grid and the obtained results verified the robustness of the proposed method.

<table>
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<tr>
<th>New State No</th>
<th>Original State No</th>
<th>Load State (% of BaseCase Load)</th>
<th>Contingency State (RMS Value)</th>
<th>Probability</th>
</tr>
</thead>
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<tr>
<td>148</td>
<td>1</td>
<td>0.0000</td>
<td>0.0452</td>
<td>0.0452</td>
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<tr>
<td>148</td>
<td>2</td>
<td>0.0000</td>
<td>0.0452</td>
<td>0.0452</td>
</tr>
<tr>
<td>148</td>
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<td>0.0000</td>
<td>0.0452</td>
<td>0.0452</td>
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<td>10</td>
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Fig. 4. Voltage deviation of load buses for different sets of pilot nodes without uncertainty modeling, 118-Bus system

<table>
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<tr>
<th>Solution Method</th>
<th>Optimal Pattern</th>
<th>Performance Index</th>
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<td>Greedy Search [6]</td>
<td>14,77,92,38,56,103,23,47,71,60</td>
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<td>0.9163 x 10^-2</td>
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Fig. 5. Voltage deviation of load buses for different sets of pilot nodes with uncertainty modeling, 118-Bus system

APPENDIX: SCENARIO REDUCTION TECHNIQUE

The purpose of scenario reduction is selection of a set, i.e. \( \Omega_s \), with the cardinality of \( N_{\Omega_s} \), from the original set, i.e. \( \Omega_f \) [16]. This procedure should be done in a way that makes a trade off between the loss of the information and decreasing the computational burden [24]. The scenario reduction technique used in this paper is described as the following steps [15]:

1. Construct the matrix containing the distance between each pair of scenarios \( c(w, \bar{w}) \)
step. 2 Select the first scenario \( w_1 \) as follows:

\[
\begin{aligned}
\arg &\min _{w_j,w_j',J} \sum _{w \in \Omega _J} \pi _w \epsilon _j (w, w') \end{aligned}
\]

\[
(32)
\]

\( \Omega _S = \{ w_1 \} , \Omega _J = \Omega _J - \Omega _S \)

step. 3 Select the next scenario to be added to \( \Omega _S \) as follows:

\[
\begin{aligned}
\arg &\min _{w_j,w_j',J} \sum _{w \in \Omega _J} \pi _w \min _{w' \in \Omega _S} \epsilon _j (w, w') \end{aligned}
\]

\[
(33)
\]

\( \Omega _S = \Omega _S \cup \{ w_n \} , \Omega _J = \Omega _J - \Omega _S \)

step. 4 If the number of selected set is sufficient then end and go to step 2; else continue.

step. 5 The probabilities of each non-selected scenario will be added to its closest scenario in the selected set.

step. 6 End.

REFERENCES


