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<th>Efficient immune-GA method for DNOs in sizing and placement of distributed generation units</th>
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An Efficient Immune-GA Method for DNOs in Sizing and Placement of Distributed Generation Units

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Abstract

This paper proposes a hybrid heuristic optimization method based on genetic algorithm and immune systems to maximize the benefits of distribution network operators (DNOs) accrued due to sizing and placement of distributed generation (DG) units in distribution networks. The effects of DG units in reducing the reinforcement costs and active power losses of distribution network have been investigated. In the presented method, the integration of DG units in distribution network is done considering both technical and economical aspects. The strength of the proposed method is evaluated by applying it on a small and a realistic large scale distribution network and the results are compared with analytical classic and other heuristic methods and discussed.

Index Terms

Distributed generation, Immune algorithm, genetic algorithm, active loss reduction, DG placement.

* Correspondence to: Alireza Soroudi, Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran, e-mail: (soroudi@ee.sharif.edu, alireza.soroudi@gmail.com). Azadi Ave., P. O. Box 11365-9363 Tel : (Office) +98(21) 66165954 Fax : +98(21) 66023261
I. INTRODUCTION

The application of heuristic search methods in power system problems is highly studied in the last three decades. These methods mimic the behaviors of the nature to find solution for problems which are not easy to solve by classic methods. One of these problems is the interconnection of Distributed Generation (DG) units to distribution networks. This problem is innately mixed integer and non-linear which cannot be easily solved using classic methods. The DG units change the flow over the feeders of the distribution network by injecting active and reactive power to their interconnection node. They may cause different benefits for Distribution Network Operators (DNOs) as reported in the literature, such as: network investment deferral [1], [2], active loss reduction [3]–[6], environmental emission reduction [7] and reliability improvement [8]. These benefits are obtained just if they are connected in the appropriate sizes and locations. Different technical constraints can limit the connection of DG units to the distribution network. These constraints are namely, voltage profile [5], [9], [10], conductor capacity limits [5], [9]–[11] and penetration level of DG units [12] and also stability of the network [13].

In addition to the technical constraints mentioned above, there are some regulatory constraints for DNOs which may change their potential benefits of using DG unit. The regulatory frameworks are different in different countries but they can be widely classified into two groups: DG owned and unbundled DNO [14]. In DG-owned DNO category, the DNO is permitted to invest in DG units and obtain the technical and economical benefits of them. Although he gains the opportunity to make decision about the size and location of DG units but he should pay for this investment. In contrary to the first category, the second category prohibits the DNO of DG ownership. The DG units are installed, owned and operated by other entities instead of DNO. It is obvious that DNO can obtain benefits/suffer from existence of DG units in his territory but he can not decide about the size and location of these units. He can only propose to the regulators to reflect the cost or benefits of DG units in the tariffs of DG owners and energy consumers. In this category, the only objective function of DG owners is the economical benefits they can gain. This makes a chance for DNO to identify the technical characteristics of the network and provide
economic signals for investors to know how much and where they should invest. Most of the reported models and methods of the literature deal with the DG sizing and placement problem in a DG-owned DNO environment, like [3], [9], [12], [15], [16]. In [3], a fuzzy goal programming is proposed to solve a multi-objective model which minimizes the active losses and improves voltage profile. The algorithm used for solving the proposed method is GA. The benefit of DNO due to network investment deferral is not considered in this work. In [9], an interesting model is proposed which considers the effect of load modeling on the decision about size and capacity of DG units. The objective function is defined as the minimization of active and reactive losses in addition to maximizing the distance of operating point of the system from the technical limits such as voltage limit and also capacity limits of feeders. Although it might be interesting to get away from the edge of technical limits of bus voltage and capacity of feeders, however, the planners have a clearly defined upper and lower limit on voltage and also upper limit for feeders and they are happy to plan by making full use of the voltage and feeder bandwidth of the given system. In [12], an integrated model for determination of total capacity of installed DG units and their appropriate connection point is proposed in which the sum of network reinforcement costs, operating and investment costs of DG units and finally the total loss costs are minimized. This model is solved using an analytical method. In [15], a cost to benefit ratio is proposed for DG planning problem. Four cost values are tried to be minimized namely, operating and investment cost of DG units, cost of energy losses, cost of purchasing electricity from the main grid and finally the cost of energy not supplied. It is assumed that DNO can make investment in DG units and also he is responsible for purchasing energy from the main grid and provides it for the customers in its territory. In [16], a fuzzy model is proposed for DG placement which considers the effect of uncertainties of load and electricity price. The optimal placement is done for minimizing the cost of investment and operation of DG units and also the cost of purchasing electricity from the main grid for compensating the active losses. The technical and economic risks are also minimized using a NSGA [17] method. In these methods, the total capacity of DG units is determined where the capacity of each DG unit is chosen from a predefined set. This assumption is made to make
the problem more realistic since DG units are commercially available in certain sizes but this will make the solution getting far from the optimum value. Few literature deal with unbundled DNO [1], [6], [14], [18], [19]. The reference [6], proposes a method based on Kalman Filter algorithm to find the optimal size of DG units to reduce active losses the location of these units are found by clustering the buses in networks into the number of DG units and selecting the largest bus in each cluster as the best candidate bus for DG installation. In [18], a hybrid GA-OPF is proposed to find the place and also the capacity of a predefined number of DG units to increase the incentives received by DNO due to network reinforcement deferral and loss reduction. In this model, the mixed integer non-linear problem is divided into two parts. The Genetic Algorithm (GA) finds the appropriate place for the given set of DG units and the OPF finds the best capacities to be installed there. In [19], an ordinal optimization approach for reducing the search space of the proposed problem in [18] which shows improvement in the results. In this paper, a heuristic method named Immune-GA (IGA) is proposed to find the optimal size and placement of DG units in an unbundled DNO environment. Finding the appropriate size and location of DG units does not mean that the DNO is going perform this investment or oblige the private investor to invest as the obtained results. The final results are useful for DNO to identify the potential benefits that DG units may offer to him in a given distribution network and how much he will get if he can encourage the investors to install DG units as the optimal plans. This encouragement can be done by reducing the connection tariffs of DG units or other mechanisms. This paper is set out as follows: section II presents problem formulation, section III sets out the proposed solution method for solving the problem. The application of the proposed model and the simulation results are presented in section IV and finally, section V summarizes the findings of this work.

II. PROBLEM FORMULATION

The DG sizing and placement is done for a predefined number of DG units, i.e. $N_{dg}$. The decision variables are the binary decision variable, i.e. $\xi_{dg}^i$, which shows the installation of a DG unit in bus $i$ and also the capacity of installed DG, i.e. $S_{dg}^i$, in bus $i$. The assumptions made in problem formulation,
constraints and the objective function are explained next.

A. Assumptions

- The DG units are considered as negative load, which are directly connected to the load points.

A nomenclature of symbols and abbreviations is defined at the end of the paper.

B. Constraints

1) Power Flow Constraints: The power flow equations that should be satisfied for each sizing and placement scheme are as follows:

\[ P_{i}^{net} = -P_{i}^{D} + \sum_{dg} P_{i}^{dg} \]

\[ Q_{i}^{net} = -Q_{i}^{D} + \sum_{dg} Q_{i}^{dg} \]

\[ P_{i}^{net} = V_{i} \sum_{j=1}^{N_b} Y_{ij} V_{j} \cos(\delta_{i} - \delta_{j} - \theta_{ij}) \]

\[ Q_{i}^{net} = V_{i} \sum_{j=1}^{N_b} Y_{ij} V_{j} \sin(\delta_{i} - \delta_{j} - \theta_{ij}) \]

2) Operating limits of DG units: The DG units should be operated considering the limits of their primary resources, i.e.:

\[ P_{i}^{dg} \leq P_{lim}^{dg} \]

The power factor of DG unit is kept constant [4], as follows:

\[ \cos \phi_{i}^{dg} = \frac{P_{i}^{dg}}{\sqrt{(P_{i}^{dg})^2 + (Q_{i}^{dg})^2}} = \text{const.} \]

3) Voltage profile: The voltage magnitude of each bus should be kept between the operation limits, as follows:

\[ V_{min} \leq V_{i} \leq V_{max} \]
4) Capacity limit of feeders: To maintain the security of the feeders, the flow of current passing through them should be kept below their capacity limit, as follows:

\[ I_\ell \leq T_\ell \]  

Where, \( I_\ell \) is the current passing through feeder \( \ell \) and \( T_\ell \) is the capacity limit of feeder \( \ell \).

5) Number of installed DG units: It is tried to find the optimal size and location of a predefined number of DG units in a given network. The total number of all installed DG units should be equal to a given number, i.e. \( N_{dg} \), as follows:

\[ \sum_{i=1}^{N_b} \xi_{dg}^i = N_{dg} \]  

C. Objective Function

The proposed model maximizes the total benefits of DNO which is the sum of two incentives, namely, total incentive of network reinforcement deferral and total loss reduction incentive, as follows:

\[ \max \{ OF \} \]

subject to:

\[ (1) \rightarrow (6) \]

The values of incentives due to network reinforcement deferral and total loss reduction are formulated next.

1) Total incentive for active loss reduction: Different schemes exist for considering the effect of loss reduction on the benefits of DNO. One of the appropriate models reported in the literature is calculating the difference between total loss of the system before and after DG placement [1], [5], [14], [16], [18]. In some models [16], the DNO should pay/receive equal to the electricity price multiplied by amount of loss reduction/increase and in some models [1], [5], [14], [18] a fix incentive, i.e. \( \psi \), is paid to DNO for each MWh reduction of active losses. This paper uses the second model as follows:
\[
\mu_l = \psi \times (Loss^{nodg} - \sum_{i=1}^{N_b} P_{net}^{i,t})
\]

(7)

Where, \(Loss^{nodg}\) is the active loss when no DG unit is installed in the network.

2) Total incentive for network reinforcement deferral: The network investment deferral effect of DG units is one of the important technical and economical values of DG units for DNO. This effect is even known as “non-wire solution” [20], to meet the load growth. One method for exact calculation of this deferral is integrated planning models [12] in which network reinforcement and DG planning are performed simultaneously. The other methods use simplifying assumptions by assuming that each MVA of installed DG reduces the need for reinforcing substation and feeders [1], [18]. In this model, the incentive due to investment deferral in network is proportional to the total installed DG in the network, as follows:

\[
\mu_n = \gamma \times \sum_{i=1}^{N_b} P_{dg}^i
\]

(8)

Where, \(\gamma\) is the coefficient of incentive for each MW of installed DG units.

The objective function is calculated as follows:

\[
OF = \mu_l + \mu_n
\]

(9)

The DG placement problem defined here is a mixed integer non-linear problem. Heuristic search methods have been successful in solving such problems. A new hybrid Immune-GA algorithm is proposed for solving the defined problem, in next section.

III. THE PROPOSED HYBRID IMMUNE-GA

Immune Algorithm (IA) is a heuristic method which is inspired of the behaviors of human immune system in detecting external invasions. This algorithm is known as a powerful computation tool in pattern recognition [21]. In this context, the objective function and constrains are known as “antigens” and the solutions construct the “antibodies”. This algorithm has three key operators, namely, affinity calculation,
clonal selection and mutation [22], [23]. A brief explanation of each operator is given here. The affinity is a factor which demonstrates the ability of each antibody to detect the antigens (optimizing the objective function). In other words, affinity factor is the fitness of each antibody [24], the higher affinity, the better performance. The clonal selection is a process for reproducing the new antibodies from the old antibodies based on their affinity factor. The more affinity factor of an antibody is, the more it will be cloned [25]. This operator is used to give a chance to each antibody for survival and reproducing new replicas. The mutation operator is applied on each antibody in each cloning process. The mutation operator of AI is the same as mutation operator of GA with a difference. In GA, the mutation probability is constant [26] but in IA, it is proportional to the inverse value of affinity factor for each antibody. The more affinity factor of an antibody is, the less it will be mutated. The process of IA is described as the follows: an initial population of antibodies is generated and then the affinity factor for each antibody is calculated. The new set of antibodies is evaluated and ranked based on their affinity factors. The new population is constructed by the highest ranked antibodies. This process is repeated until the terminating condition is reached. The AI treats each antibody based on its affinity factor by cloning and mutating it. This may prevent each antibody from communicating with other antibodies and using their desired characteristics in identifying the antigens (optimization the objective function and its constraints). To overcome this shortcoming of IA, the crossover operator of GA [26] is applied and a hybrid IA and GA named IGA is proposed. The flowchart of proposed IGA is depicted in Fig.1. The steps of the proposed Immune genetic algorithm are as follows:

Step 1. Generate an initial set of antibodies with a size of N
Step 2. Set Iteration=1
Step 3. Calculate the objective function for each antibody using (9) and assign it as its affinity factor
Step 4. If the maximum number of iteration is reached, then end; else continue
Step 5. Keep the best N antibodies in memory (for controlling the population size)
Step 6. Set the cloning counter, i.e. m, equal to 1
Step 7. Select two antibodies (p and q) as the parents among the antibodies stored in memory, using roulette wheel [27] based on their affinities.

Step 8. Calculate the number of cloning replica, i.e. $k_m$, and mutation probabilities based on the average values of parent affinities. The value of $k_m$ is determined as follows:

$$k_m = \text{round}(\beta \times \frac{AF_p + AF_q}{2 \text{max}(AF_n)} \times N)$$  \hspace{1cm} (10)

Where, $\beta$ is a controlling factor and round is the function which gives the nearest integer number.

Step 9. Clone the selected parents selected in Step 7, for $k_m$ times, by applying the crossover and mutation operators and produce new antibodies. (It should be noted that since the cloning procedure uses the mutation operator, this will prevent the algorithm from premature convergence).

Step 10. Store the new generated antibodies.

Step 11. If the cloning counter is below the memory size, then increase cloning counter and go to Step 7; else, construct the new antibody set using the union of newly generated antibodies and the antibodies of memory, increase the iteration and go to Step 3.

IV. SIMULATION RESULTS

The proposed IGA methodology is programmed in MATLAB running on an Intel®Core™2 Duo Processor T5300 (1.73 GHz) PC with 1 GB RAM. It is applied to two distribution systems to demonstrate its abilities. The first case is a 69-node system and the second one is a realistic 574-node distribution network.

A. Case I: 69-bus network

In this case, the distribution network under study is a 11-kV, 69-bus system as depicted in Fig.2. The technical data of this network can be found in [18]. All DG units are assumed to operate with constant power factor equal to 0.9 lag. The loss reduction incentive, i.e. $\psi$, and network deferral incentive, i.e. $\gamma$, are highly dependent on the system under study but here for comparing the proposed method with the
other published results, these are assumed to be $48 \frac{\text{kWh}}{\text{MWh}}$ and $2.5 \frac{\text{kW}}{\text{kWh/year}}$ \cite{1}, \cite{18}, \cite{19}, respectively. The thermal capacity of lines, i.e. $I_L$, are assumed to be 3 MVA. The other simulation parameters are provided in Table. I. The active loss of the network is 0.228 MW when no DG units exists in the network, i.e. $\text{loss}_{\text{nodg}}$. The simulations are done for different number of DG units (three, five, seven and nine) and the results obtained by proposed IGA method.

1) Comparing with other methods: The maximization of objective function introduced in section II is categorized as mixed-integer nonlinear programming (MINLP). The placement schemes proposed by IGA method is compared with classical optimization model and intelligent evolutionary ones. For classical method, the model is solved in Generalized Algebraic Modeling Systems (GAMS) \cite{28}, which is a high-level programming platform, using DIscrete COntinuous OPTimization (DICOPT) solver. The evolutionary methods include GA-OPF \cite{18}, Ordinal Optimization (OO) \cite{19}, Particle Swarm Optimization (PSO) \cite{29}, pure Genetic Algorithm (GA) \cite{30}, Immune Algorithm \cite{23}. The optimal solutions of each method are tabulated in Table.II to V, for different number of DG units. The best solution found by each method is depicted in Fig.3. The technical performance of these methods are compared in Table.VI. This table presents a comparison among the results of the proposed algorithm IGA and other methods for 100 random trials. In Table.VI, the smallest and the largest values of the maximized objective function (total incentives) are referred to as the “Best and Worst” solution, respectively. Comparison of the best and worst solutions of the proposed optimization algorithm (IGA) with the corresponding those of the other methods confirms the effectiveness of the proposed method. Additionally, Table.VI provides the standard deviation and average value of the objective function, based on the proposed method and the other ones.

2) Advantages and Drawbacks of the proposed method: In this section the advantages and drawbacks of the proposed algorithm are discussed, as follows:

- Advantages: The average value of objective function in the proposed IGA method is greater than other analyzed methods while it has lower standard deviation. This means that the IGA is more robust comparing to other heuristic methods like GA-OPF, OO, PSO, GA, Immune methods. The IGA uses
the best features of GA and Immune algorithm simultaneously then trapping in a local optimum is avoided. Finally, the running time of the proposed IGA method, given in the last column of Table VI, is less than GA-OPF, GA, PSO, Immune. The cloning procedure gives the local search capability to the algorithm and avoids premature convergence. The best solution of IGA is also better than the the solution found by GAMS/DICOPT (classical method). This is because of the inherent mixed integer nonlinear nature of the problem which makes it hard for classical methods to find the global optimum for a given solution. The classical methods are very sensitive to the initial starting points assigned to the variables specially in MINLP problems.

- **Drawbacks:** The main drawback of the proposed method is that there is no proof for finding the global optimum solution in a given mixed integer non-linear problem. This problem also exists in classical methods because they are very sensitive to the starting point of the decision variables (initial values). Another drawback lies in the computational burden and running time that would inevitably increase for a larger distribution system (like other heuristic algorithms). It was already demonstrated in Table VI that the running time of IGA is more than OO and GAMS method. It should be noted that although this computation is off-line and will not be a serious problem for planner but can be reduced by using fast distribution load flow techniques [31] proposed in the literature. Finally, the proposed framework is just applicable for dispatchable DG technologies and do not have stochastic nature like Wind Turbines or Photovoltaic cells. The planning procedure for stochastic DG technologies is inherently different and can be found in the literature as described in [32].

**B. Case II: Actual 574-bus network**

The second case is a 20-kV, 574-node distribution system in south-west of France which is shown in Fig.4. This system has 573 sections with total length of 52.188km, and 180 load points. This network is fed through one substation. All DG units are assumed to operate with constant power factor equal to 0.9 lag. The loss reduction incentive, i.e. $\psi$, and network deferral incentive, i.e. $\gamma$, are assumed to be the same as case I. The active loss of the network, at presence of no DG units in the network, i.e. $\text{loss}_{\text{nodg}}$
is 0.31027 MW. The simulations are done for different number of DG units (three to twenty DG units) and the results obtained by proposed IGA are depicted in Fig.5. The percent of appearance of each bus in different optimal placement schemes are shown in Fig.6. For example bus number 573 appears in 61% of the optimal placement patterns and bus number 251 is present in 55.5% of them and so on. It is always interesting for DNO to know which bus is more appropriate for DG placement even if DNO is not responsible for DG placement. In cases where the non-DNO entities perform the DG investment, the DNO can not force them to invest in a specific bus but he can guide them (using some incentives) to act as he desires. This shows how the proposed algorithm can help the DNO to evaluate the potential benefits of the DG units in distribution networks. The average running time (in 20 trials) is about 5hrs and 23 minutes.

V. Conclusion

This paper proposes a novel hybrid Immune-GA method for optimal placement of DG units. The defined objective function is maximizing the total incentives received by DNO due to active loss reduction and network investment deferral. The proposed optimization method is applied to a test system and realistic distribution network and its flexibility and effectiveness is demonstrated. The simulation results explicitly show the computational efficiency of the proposed method in finding the optimal DG placement and sizing schemes in distribution networks. The proposed method is not only useful when the DNO performs the DG placement and sizing but also when other non-DNO entities perform DG investment. In such cases, the DNO can provide them some economic incentive signals based on the proposed method to guide their decisions. In the present work, only a simplified method of DG impact on network investment deferral was considered, however for more comprehensive study, considering precise impact of DG investment on network upgrade deferral and also the effect of protection system on limiting the ability of distribution network in absorbing new DG units are necessary to be investigated, while considering the computation burden reduction. These interesting topics which will be included in our future works.
LIST OF SYMBOLS ANDABBREVIATIONS

Indices

\( i, j \) Bus

\( \ell \) Feeder

Constants

\( \gamma \) Network investment deferral incentive

\( \psi \) Active loss reduction incentive

Variables

\( P^D_i \) Active power demand in bus \( i \)

\( P^d g_i \) Active power injected by a \( d g \) in bus \( i \)

\( Y_{ij} \) Admittance magnitude between bus \( i \) and \( j \)

\( \theta_{ij} \) Admittance angle between bus \( i \) and \( j \)

\( S^d g_i \) Apparent power of \( d g \) installed in bus \( i \)

\( A F_n \) Affinity factor of \( n^{th} \) solution

\( P^D_{i,\text{base}} \) Base active power demand in bus \( i \)

\( Q^D_{i,\text{base}} \) Base reactive power demand in bus \( i \)

\( S^D_{i,\text{base}} \) Base apparent power demand in bus \( i \)

\( I_\ell \) Current magnitude of \( \ell^{th} \) feeder

\( V_{\min} \) Lower operation limit of voltage

\( \overline{P}^d g \) Maximum operating limit of a \( d g \)

\( P^\text{net}_i \) Net active power injected to bus \( i \)

\( Q^\text{net}_i \) Net reactive power injected to bus \( i \)

\( N_b \) Number of buses in the network
\( N \) Number of population

\( \cos \phi^{dg} \) Power factor of a \( dg \)

\( Q_i^{dg} \) Reactive power injected by a \( dg \) in bus \( i \)

\( Q_i^D \) Reactive power demand in bus \( i \)

\( T_\ell \) Capacity limit of existing feeder \( \ell \)

\( V_{\text{max}} \) Upper operation limit of voltage

\( V_i \) Voltage magnitude in bus \( i \)

\( \delta_i \) Voltage angle in bus \( i \)

REFERENCES


**TABLE I**

**DATA USED IN THE STUDY**

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**TABLE II**

**DG LOCATION AND CAPACITIES FOR 3 DG UNITS IN CASE I**

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\begin{align*}
\mu (\ell h^{-1}) & = 10.048 \quad 9.852 \quad 9.983 \quad 9.611 \quad 9.227 \quad 9.904 \quad 9.987 \\
\mu_s (\ell h^{-1}) & = 1.382 \quad 1.718 \quad 1.182 \quad 1.186 \quad 1.04 \quad 1.358 \quad 1.601 \\
\text{Total} & = 11.43 \quad 11.57 \quad 11.165 \quad 10.797 \quad 10.267 \quad 11.262 \quad \underline{11.588}
\end{align*}
### TABLE VI

**Computational performance comparison between the proposed IGA and other methods for 100 trials in Case I**

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Fig. 2. Single-line diagram of the 69-bus distribution network in case I
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Fig. 5. The variation of different incentives with DG numbers in case II

Fig. 6. Percent of appearance of each bus in optimal allocation schemes in case II