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A possibilistic-probabilistic tool for evaluating the impact of stochastic renewable and controllable power generation on energy losses in distribution networks- A case study

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Abstract

This paper proposes a hybrid possibilistic-probabilistic evaluation tool for analyzing the effect of uncertain power production of distributed generations (DG) on active losses of distribution networks. The considered DG technologies are Gas and wind turbines. This tool is useful for Distribution Network Operators (DNOs) when they are faced with uncertainties which some of them can be modeled probabilistically and some of them are described possibilistically. The generation pattern of DG units change the flow of lines and this will cause in change of active losses which DNO is responsible for compensating it. This pattern is highly dependent on DG technology and also decisions of DG operators which is an entity other than DNO. For wind turbines, this pattern is described using a weibull probability distribution function (PDF) for wind speed along with the power curve of the wind turbine but for other controllable DG technologies like gas turbines, it’s not an easy job to provide a PDF to describe the generation schedule. On the other hand, the values of loads can not be always described using a PDF so the possibilistic (fuzzy) description can be helpful in such cases. In order to demonstrate the effectiveness of the proposed tool, it is applied to a realistic distribution system and the results are analyzed and discussed.

Key words: Monte Carlo Simulation, Distributed Generation, Fuzzy, Uncertainty, Activ Losses

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1. Introduction

Distributed Generations (DGs) are defined as electric resources interconnected to the distribution networks [1]. The integration of DG units in distribution network was highly studied in the last decade. There are many technical [2], socio-economic [3] and environmental [4] reasons behind this trend. In this paper, the focus is on the effect of DG units on active loss of distribution networks. The effect of conventional DG technologies on active power losses are investigated in the literature. In [5], an efficient method is proposed for determination of optimal sizes and location of DG units by using the Kalman filter algorithm. In [6], a goodness factor is proposed which implies the contributions of a DG unit to active and reactive power losses in the distribution system are modeled using. In [7], a MINLP-based optimization model is proposed to determine the appropriate location of DG units in a distribution network in order to decrease active losses and fuel costs. In [8], the optimal location and size of DG units are obtained using a highly efficient ordinal optimization method. In these methods, the considered DG technology is controllable and non-renewable. The technological development and the importance of using clean technologies has made the renewable energies more fascinating for Distribution Network Operators (DNOs) specifically because they are inexhaustible and nonpolluting. These technologies include hydro, wind [9], solar [10], biomass [11] and tidal. Among these renewable energies, wind technology has evolved very rapidly over the past decade and reduction of capital costs, improvement of reliability, and efficiency has made the wind power able to compete with conventional power generation [12]. The renewable DG technologies like wind have special characteristics due to their main source of energy. Obviously, the primary energy source of a wind turbine is wind. The wind speed is not a constant quantity during the operation of wind turbine and is highly dependent on climate condition of the area which wind turbine is installed there. It is the reason that they exhibit uncertainty and variability in their output [13]. In the literature, some methods are proposed to model the impact of these uncertainties on distribution network performance. In [14], different scenarios are constructed based on the PDF of uncertain values and then a method is proposed to determine the optimal combination of different
renewable technologies for minimizing active losses. In [15], a powerful tool was proposed based on Monte Carlo Simulation for modeling the uncertainties of the uncertainties in
the locations, exported energy and penetration level, the states(onoroff) of the DG units. It is assumed that the mentioned uncertainties follow a probability distribution function and the mentioned PDF is available for DNO. In some cases, there is no PDF available for describing the behavior of an uncertain parameter. In these cases, the Fuzzy arithmetic [16] can be used. In [17, 18], a fuzzy power flow model is proposed for modeling the uncertainty of loads in a network. They use the extension principle of fuzzy to find the membership function of output when the input values are fuzzy and their membership functions are available. The contribution of this paper is as follows:

- An evaluation tool is proposed for DNOs which helps them to deal with the effect of stochastic (probabilistic) and fuzzy (possibilistic) uncertainties of renewable and conventional DG technologies on active energy losses, simultaneously.

To do this, the introduced principle of fuzzy load flow in [18] are used to model the uncertainties of loads and generation of non-stochastic DG technologies. For modeling the stochastic behavior of wind turbines, the Monte Carlo Simulation is used.

The paper is structured as follows: section 2, describes the possibilistic method used for uncertainty modeling, section 3, presents the concepts of Monte Carlo simulation for probabilistic uncertainty modeling; the problem formulation is described in section 4. The proposed solution algorithm is presented in section 5. In section 6, the proposed model is applied on an actual distribution network and the simulation results are given and discussed. Finally, conclusions are drawn in section 7.

2. Possibilistic Uncertainty Modeling

In possibilistic evaluation frameworks, for each uncertain value, i.e. \( \tilde{A} \), a membership function, i.e. \( \mu_A(x) \), is defined which describes that how much each element, i.e. \( x \), of universe of discourse, i.e. \( U \), belongs to \( \tilde{A} \). Different types of membership functions can be used for describing the uncertain values. Here, fuzzy trapezoidal numbers (FTN) with a notation \( \tilde{A} = (a_{min}, a_L, a_U, a_{max}) \) are used as shown in Fig.1.
2.1. $\alpha$-cut Method

In engineering problems, the evaluation of a certain quantity is usually in the form of a multivariable function namely, $y = f(x_1, \ldots, x_n)$, if $\tilde{x}_i$ are uncertain then $y$ will be also uncertain, $\tilde{y} = f(\tilde{x}_1, \ldots, \tilde{x}_n)$. The question is that, knowing the membership functions of uncertain input variables $\tilde{x}_i$, what would be the membership function of $\tilde{y}$. The $\alpha$-cut method [16] answers this question in this way:

For a given fuzzy set $\tilde{A}$, defined on universe of discourse, i.e. $U$, the crisp set $A^{\alpha}$ is defined as all elements of $U$ which have membership degree to $\tilde{A}$, greater than or equal to $\alpha$, as calculated in (1).

$$A^{\alpha} = \{ x \in U \mid \mu_A(x) \geq \alpha \}$$

(1)

$$A^{\alpha} = (A^{\alpha}, \overline{A})$$

The $\alpha$-cut of each input variable, i.e. $x^{\alpha}_i$, is calculated using (1), then the $\alpha$-cut of $y$, i.e. $y^{\alpha}$ is calculated as follows:

$$y^{\alpha} = (\underline{y}^{\alpha}, \overline{y}^{\alpha})$$

(2)

$$\underline{y}^{\alpha} = \min f(X^{\alpha})$$

$$\overline{y}^{\alpha} = \max f(X^{\alpha})$$

$$X^{\alpha} = [x^{\alpha}_1, \ldots, x^{\alpha}_n]$$

$$X^{\alpha} \in (X^{\alpha}, \overline{X})$$

(3)

This means for each $\alpha$-cut, two optimization is done. One maximization for obtaining the upper bound of $y^{\alpha}$, i.e. $\overline{y}^{\alpha}$, and one minimization for obtaining the lower bound of $y^{\alpha}$, i.e. $\underline{y}^{\alpha}$.

2.2. Defuzzification

The defuzzification is a mathematical process for converting a fuzzy number into a crisp one [16]. In this paper, the centroid method [19] is used for defuzzification of fuzzy numbers. The defuzzified value of a given fuzzy quantity, i.e. $\tilde{A}$, is calculated as follows:

$$A^* = \frac{\int \mu_A(x).x \, dx}{\int x \, dx}$$

(4)
3. Probabilistic Uncertainty Modeling

Some of the uncertain input parameters follow a of Probability Distribution Function (PDF), such as the value of wind which follows a weibull PDF [20]. Monte Carlo Simulation (MCS) is a powerful tool for analyzing the uncertainties which follow any PDF.

3.1. Monte Carlo Simulation

The main concept of MCS method is described as follows: suppose a multi variable function, namely $y$, $y = f(Z)$, where $Z = [z_1, ..., z_m]$, in which $z_1$ to $z_m$ are random variables with their own PDF. The problem is, knowing the PDFs of all input variables, i.e. $z_1$ to $z_m$, what would be the PDF of $y$? The MCS acts as follows [21]: First of all, it will generate a value, i.e. $z^e_i$, for each input variable $z_i$, using its own PDF and form the $Z^e = [z^e_1, ..., z^e_m]$ and then calculates the value of $y^e$ using $y^e = f(Z^e)$. This process will be repeated for a number of iterations. The trend of the output, i.e. $y$, will determine its PDF.

4. Problem Formulation

The assumptions for modeling the two types of uncertainties, constraints and the objective functions are described as follows:

4.1. Uncertainty Modeling

As already explained, the uncertain parameters are divided into two groups: possibilistic and probabilistic. In possibilistic uncertainty group the value load in each bus, DG generation which are not stochastic (controllable with decisions of their owners). The second group contains the stochastic generation of wind turbines which is probabilistically modeled. The description of the parameters of each group is as follows:

Possibilistic Parameters:

- load: It is assumed that the DNO can just describe it with a membership function.

\[ S^{D}_{h,i} = S^{D}_{i,j} \times DLF_{h} \times (D_{\min}, D_{L}, D_{U}, D_{\max}) \] (5)
Where, $S_{i,f}^D$ is the apparent forecasted value of peak load in bus $i$ and $DLF_h$ is the demand level factor in demand level $h$ which takes values between 0 and 1. Finally, $S_{h,i}^D$ is the fuzzy value of demand in bus $i$ and demand level $h$.

- **DG generation pattern**: The amount of energy which a controllable DG unit injects into the network is uncertain and usually it depends on the decisions of DG owner so the DNO can not have a PDF of it if there is not much historic data about it. The output power of a controllable DG unit is modeled using a membership function as follows:

$$P_{h,i}^{dg} \in C_{i}^{dg} \times (\zeta_{\text{min}}, \zeta_L, \zeta_U, \zeta_{\text{max}}) \quad (6)$$

Where, $C_{i}^{dg}$ is the capacity of DG unit installed in bus $i$ and $P_{h,i}^{dg}$ is the active power of a DG unit in bus $i$ in demand level $h$.

**Probabilistic Parameters:**

- **Wind Turbine generation pattern**: The generation schedule of a wind turbine highly depends on the wind speed in the site. The variation of wind speed, i.e. $v$, can be modeled using a weibull [22] PDF and its characteristic function which relates the wind speed and the output of a wind turbine [23].

$$PDF(v) = \left( \frac{k}{c} \right) \left( \frac{v}{c} \right)^{k-1} \exp\left( -\left( \frac{v}{c} \right)^k \right) \quad (7)$$

Where, $k$ is the shape factor and $c$ is the scale factor of the weibul PDF of wind speed in the zone under study.

The generated power of the wind turbine is determined using its characteristics as follows:

$$P_{h,i}^{\text{wind}} = \begin{cases} 
0, & \text{if } v \leq v_{in}^c \text{ or } v \geq v_{out}^c \\
\frac{v-v_{in}^c}{v_{out}^c-v_{in}^c} P_{i,r}^{\text{wind}}, & \text{if } v_{out}^c \leq v \leq v_{rated} \\
P_{i,r}^{\text{wind}}, & \text{else}
\end{cases} \quad (8)$$

Where, $P_{i,r}^{\text{wind}}$ is the rated power of wind turbine installed in bus $i$, $P_{h,i}^{\text{wind}}$ is the generated power of wind turbine in bus $i$ and demand level $h$, $v_{out}^c$ is the cut out speed, $v_{in}^c$ is the cut in speed and $v_{rated}$ is the rated speed of the wind turbine.
4.2. Constraints

The constraints that should be satisfied are described in this section, as follows:

4.2.1. Power Flow Constraints

The power flow equations must be satisfied in each demand level $dl$ and year $t$, as follows:

$$P_{h,i}^{net} = -P_{h,i}^D + \sum_{dg=1}^{N_{dg}} P_{h,i}^{dg} + \sum_{wind=1}^{N_{wind}} P_{h,i}^{wind}$$

$$Q_{h,i}^{net} = -Q_{h,i}^D + \sum_{dg=1}^{N_{dg}} Q_{h,i}^{dg}$$

$$P_{h,i}^{net} = V_{h,i} \sum_{j=1}^{N_b} Y_{ij} V_{h,j} \cos(\delta_{h,i} - \delta_{h,j} - \theta_{ij})$$

$$Q_{h,i}^{net} = V_{h,i} \sum_{j=1}^{N_b} Y_{ij} V_{h,j} \sin(\delta_{h,i} - \delta_{h,j} - \theta_{ij})$$

Where, $P_{h,i}^{net}$ and $Q_{h,i}^{net}$ are the net active and reactive power injected to the network in bus $i$.

4.2.2. Operating Limits of DG units

The power factor of DG unit is kept constant in all demand levels as follows:

$$\cos \phi_{dg} = \frac{P_{h,i}^{dg}}{\sqrt{(P_{h,i}^{dg})^2 + (Q_{h,i}^{dg})^2}} = \text{const.}$$

4.2.3. Voltage Profile

The voltage of each bus in each demand level $h$ should be kept within the safe operating limits.

$$V_{min} \leq V_{h,i} \leq V_{max}$$

$V_{min}$ and $V_{max}$ are the minimum and maximum permissible limits of voltage, respectively.
4.2.4. Thermal Limits of feeders and substation

To maintain the security of the feeders and substation, the flow of current passing through them should be kept below their thermal limit in each demand level. The thermal limits of existing feeders and substation are $I_{\ell}$ and $S_{tr}$, respectively. This constraint is described as follows:

\begin{align*}
I_{h}^{\ell} & \leq I_{\text{max}}^{\ell} \\
S_{h}^{\text{grid}} & \leq S_{\text{max}}^{tr}
\end{align*}

Where, $I_{h}^{\ell}$ is the current magnitude of feeder $\ell$ in demand level $h$, $I_{\text{max}}^{\ell}$ is the thermal limit of feeder $\ell$, $S_{h}^{\text{grid}}$ is the apparent power imported from main grid in demand level $h$ and finally $S_{\text{max}}^{tr}$ is the thermal capacity of substation.

4.3. Active Losses

The total active loss of the network is equal to the sum of all active power injected to each bus, as follows:

\begin{align*}
\text{Loss} = \sum_{h=1}^{N_{h}} \left( \sum_{i=1}^{N_{b}} P_{h,i}^{\text{net}} + P_{h}^{\text{grid}} \right) \times \tau_{h}
\end{align*}

Loss $\in (\text{loss}_{\text{min}}, \text{loss}_{\text{L}}, \text{loss}_{\text{U}}, \text{loss}_{\text{max}})$

\begin{align*}
\alpha &= 0 \\
\text{loss}_{\text{min}} &= \min \text{Loss} \\
\text{loss}_{\text{max}} &= \max \text{Loss} \\
\alpha &= 1 \\
\text{loss}_{\text{L}} &= \min \text{Loss} \\
\text{loss}_{\text{U}} &= \max \text{Loss}
\end{align*}

Subject to:

(5) to (12)

Where, $\tau_{h}$ is the duration of demand level $h$ and $N_{h}$ is the number of demand levels.
5. Proposed Model for Mixed Possibilistic-Probabilistic Uncertainty Modeling

The DNO is always faced with situations where the nature of the uncertainties are neither pure probabilistic nor possibilistic. In such cases, a tool is needed to handle the uncertainties. In this paper an evaluation tool is proposed to deal with such situations as follows: The DNO has an index function which is multivariable, i.e. \( f(X,Z) \), where the possibilistic uncertain parameters are represented by vector \( X \) and probabilistic uncertain values are given by vector \( Z \). To deal with such variables they are decomposed into two groups and are dealt with separately as the following steps:

- **Step.1** : For each \( z_i \in Z \), generate a value using its PDF, i.e. \( z_i^e \)
- **Step.2** : Calculate \((\bar{y}^\alpha)^e\) and \((\underline{y}^\alpha)^e\) as follows:

\[
\begin{align*}
(\bar{y}^\alpha)^e &= \min f(Z^e, X^\alpha) \\
(\underline{y}^\alpha)^e &= \max f(Z^e, X^\alpha)
\end{align*}
\]

\( St : \)

\( X^\alpha \in (\underline{X}^\alpha, \bar{X}^\alpha) \)

These steps are repeated to obtain the PDF of the parameters of the output’s membership function. In fact, when both types of the uncertainties exist in the input variables then the parameters of the membership function are stochastic.

6. Simulation Results

The proposed methodology is applied to a 574-bus realistic distribution network which is shown in Fig.2. This network consists of a 20 kV substation with capacity limit, i.e. \( S_{tr} = 20MVA \) and 573 feeders with 180 load points. The average load and power factor at each load point are 55.5 kW and 0.9285, respectively. There are six DG units present in the network where four of them are dispatchable (by non-DNO entities) and two of them are wind turbines. The DG capacities and their location in the network are given in Table.2. The shape and scale factors of the weibull PDF of wind speed and the other simulation parameters are given in Table.1. It is assumed that there are 24 demand levels.
in each year with equal duration of $\tau_h = 365 \text{hr}$. The variation of demand level factors are depicted in Fig.3.

The proposed algorithm is applied to the introduced network and the total active losses are obtained. The total active losses of the network can be presented in two ways: the first method is representing it by a trapezoidal fuzzy number in which each parameter of the membership function is a probabilistic quantity and has a PDF or histogram. In Fig.4, the probability distribution function of four parameters are shown. These parameters, i.e. $(\text{Loss}_{\text{min}}, \text{Loss}_L, \text{Loss}_U, \text{Loss}_{\text{max}})$, describe the total loss in the network. The histogram of variation of each parameter is shown in Fig.5, describes the distribution of samples in the total Monte Carlo experiments which has been 20000 experiments in this study.

The second method for representing the total active loss is calculating the crisp value of active loss using (4) in each Monte Carlo experiment and then obtaining the distribution of this quantity as shown in Fig.6.

The cumulative distribution function of crisp value of total losses is also shown in Fig.7. For example if DNO wants to know the probability of having more than a specific loss in their network. For example, that might be a question that what is the probability of having more than 3100 MWh loss in the network? Referring to Fig.7, the probability that total loss exceeds 3100 MWh, is equal to 0.4388. Further statistical information of total active losses, is given in Table.3, including the minimum, maximum, mean value and the standard deviation of them. The developed hybrid tool attempts to overcome limitations in evaluating the network losses when different sources of uncertainty exist. Specially when renewable and conventional DG units are present in the network. This hybrid approach makes the DNO enable to evaluate active losses when there are stochastic generations and also controllable DG units in the network. The simulation results can help the DNO to have estimation about the amount of money he should pay for compensation of active losses. Although the proposed analysis is offline and the running time of the algorithm is not of major concern but the computation burden of the proposed algorithm can be reduced using load flow techniques developed for radial distribution networks and also the variance reduction methods proposed for reducing the number of necessary Monte Carlo experiments. This will highly increase the speed of the proposed algorithm.
7. Conclusion

A method combining probabilistic and possibilistic uncertainty modeling is proposed for evaluation of active losses in the distribution network. In doing so, it uses the Fuzzy (alfa-cut) and Monte Carlo Simulation to model the uncertainties. Its use would be to enable DNOs to evaluate the effect of different DG technologies on the technical performance of the distribution network. The model considers probabilistic presentation of wind speed using a weibull PDF and possibilistic description of loads and generation pattern of dispatchable DG units for modeling the uncertainties. The proposed tool is implemented on a realistic distribution network to demonstrate its ability.

List of Symbols

\( S_{i,f} \) Apparent forecasted value of peak load in bus \( i \)

\( S_{h,i} \) Apparent fuzzy value of demand in bus \( i \) and demand level \( h \)

\( P_{h,i}^{dg} \) Active power of a DG unit in bus \( i \) in demand level \( h \)

\( P_{grid} \) Active power imported from grid

\( S_{grid}^{h} \) Apparent power magnitude passing through substation in demand level \( h \)

\( Y_{ij} \) Admittance magnitude between buses \( i \) and \( j \)

\( \theta_{ij} \) Admittance angle between buses \( i \) and \( j \)

\( I_{\text{max}}^{\ell} \) Capacity limit of existing feeder \( \ell \)

\( C_{i}^{dg} \) Capacity of DG unit installed in bus \( i \)

\( S_{\text{max}}^{tr} \) Capacity limit of existing transformer

\( I_{h}^{\ell} \) Current magnitude passing through feeder \( \ell \) in demand level \( h \)

\( v_{\text{out}}^{c} \) Cut out speed of the wind turbine

\( v_{\text{in}}^{c} \) Cut in speed of the wind turbine
$$DLF_h \quad \text{Demand level factor in demand level h}$$

$$P_{wind}^{h,i} \quad \text{Generated power of wind turbine in bus i and demand level h}$$

$$V_{min} \quad \text{Minimum operating limit of voltage}$$

$$V_{max} \quad \text{Maximum operating limit of voltage}$$

$$P_{net}^{h,i} \quad \text{Net active power injected to bus i in demand level h}$$

$$Q_{net}^{h,i} \quad \text{Net reactive power injected to bus i in demand level h}$$

$$N_b \quad \text{Number of buses in the network}$$

$$N_{\ell} \quad \text{Number of feeders in the network}$$

$$N_h \quad \text{Number of demand levels}$$

$$Q_{DG}^{h,i} \quad \text{Reactive power of DG unit in bus i in demand level h}$$

$$v_{rated} \quad \text{Rated speed of the wind turbine}$$

$$P_{wind}^{i,r} \quad \text{Rated power of wind turbine installed in bus i,}$$

$$k \quad \text{Shape factor of weibul PDF of wind speed}$$

$$c \quad \text{Scale factor of the weibul PDF of wind speed}$$

$$V_{h,j} \quad \text{Voltage magnitude of bus j in demand level h}$$

$$\delta_{h,j} \quad \text{Voltage angle of bus j in demand level h}$$

References


List of Figure Captions:

• Fig.1. Fuzzy Trapezoidal Number

• Fig.2. Geographical view of the actual distribution system under study

• Fig.3. The variations of $DLF_h$ in each demand level

• Fig.4. Probability distribution functions of total loss membership function

• Fig.5. Histogram of total loss membership function

• Fig.6. Histogram of crisp value of total losses

• Fig.7. Cumulative distribution function of crisp value of total losses
Table 1: Data used in the study

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Table 2: DG capacities and locations

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Table 3: Statistics of values of total active loss in MWh

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<td>CrispLoss</td>
<td>2837.1</td>
<td>4060.3</td>
<td>3101</td>
<td>34.91</td>
</tr>
</tbody>
</table>

Figure 1: Fuzzy Trapezoidal Number

Figure 2: Geographical view of the actual distribution system under study
Figure 3: The variations of $DLF_h$ in each demand level

Figure 4: Probability distribution functions of total loss membership function

Figure 5: Histogram of total loss membership function
Figure 6: Histogram of crisp value of total losses

Figure 7: Cumulative distribution function of crisp value of total losses