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Response of a Simply Supported Beam with a Strain Rate Dependent Elasticity Modulus when Subjected to a Moving Load

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ABSTRACT: The response of a simply supported beam model is simulated under single moving load at different velocities. The beam is discretized into small elements and strain and displacement measurements are obtained at each time step. Contrary to previous work based on a constant modulus of elasticity, here the strain measurements use a time-variant (dynamic) modulus of elasticity. A time-variant modulus influences the bridge response, being more significant at highest velocities.

KEYWORDS: Modulus of Elasticity; Strain rate; Dynamic response; Beam; Concrete.

1 INTRODUCTION

Generally, a constant modulus of elasticity is used to calculate the load effects caused by moving loads. Existing researches [1, 2] have shown the moving loads causing the changed modulus that subsequently affects stress-strain relationship and concrete strength properties. Dynamic loads commonly impact on all the properties of the concrete structure but most noticeably its tensile and compressive strengths. Above a certain strain rate, the modulus starts to increase to reach the ultimate value for constant modulus [3]. Most of the literatures confirm the modulus increases with a higher strain rate due to dynamic load. Previously, dynamic load tests for concrete are undertaken by using an impact hammer on a concrete specimen with a specified strain rate [4, 5]. Strain rates applied are generally high and in the range of $10^{-3}$ to $10^{-4}$ s⁻¹, which correspond to values slightly higher than that of an earthquake. However, strain rates due to a moving load are much lower than ones derived from the impact hammer.

Furthermore, Soroushian and Obaseki [6] and Fu, et al. [7] proved that impact loading has small influence on the modulus of reinforced concrete and negligible effect on modulus of steel [8]. The CEB Information Bulletin No. 187 has been recently used [9, 10] to conduct dynamic analysis on concrete and reinforced concrete specimens. Results showed significant increase in the tensile strength, and ultimate tensile stress and strain. Strain rates used were in the range of 1 s⁻¹ and 50 s⁻¹, which are considered for soft and hard impact. For concrete, the modulus of elasticity increased moderately at high strain rates, in comparison compressive strength changes were more noticeable.

A relationship between strain, strain rate and modulus of elasticity is provided by CEB-FIP Model Code [11] given by:

$$E_d / E_c = (\dot{\varepsilon}/\dot{\varepsilon}_0)^{0.026}$$  \hspace{1cm} (1)

where $E_d$ (time-variant modulus) and $E_c$ (constant modulus) are respectively the modulus of elasticity due to impact and static load, $\dot{\varepsilon}$ and $\dot{\varepsilon}_0$ are the strain due to impact and static loads, respectively, and 0.026 is a constant that relates the strain rate to the time-variant modulus. The strain rate limit, $\dot{\varepsilon}_0$, is a constant and can be taken as $3 \times 10^6$ s⁻¹ in tension region.

In compression, the value of $\dot{\varepsilon}_0$ is different and given by $30 \times 10^6$ s⁻¹. Bischoff and Perry [3] provided different limits of constant strain rate that range from $10^6$ to $6 \times 10^7$ s⁻¹ for static load and $2 \times 10^7$ s⁻¹ to $6 \times 10^8$ s⁻¹ for quasi-static load.

To date, practically and theoretically little or no consideration is given to the problem of moving load across a beam and how it impacts the modulus of elasticity. As such, this investigation is carried out. For that, the load is moving and applied at nodes on the beam, and the relationship between the strain rate at selected cross sections and time is considered. The remainder of the paper is organised as following. The mathematical model employed in the simulations is described in next section. Then the level of the strain is investigated to calculate the time-variant modulus and strain rate. Finally, the impact of the use of a strain-rate dependent modulus on the bridge response is evaluated by comparing to the traditional practice based on the use of a constant modulus.

2 MATHEMATICAL MODEL EMPLOYED IN SIMULATIONS

The model involved a simple beam (having the length $L$) nodal load ($P$) moving at a velocity ($v$ m/s) is adopted to investigate an effect of changing strain (Figure 1).

![Figure 1. Moving load model](image-url)

The beam is modelled with 100 finite elements with 2 degrees of freedom at each node. Stiffness and mass matrices are defined for each element and then assembled into the global stiffness and mass matrices. Complex damping is usually ignored because it has negligible influence on the overall results of strain and displacement [12]. Moreover, the time of impact on the structure due to the load is relatively short. As
such a linear damping model is adopted in this case. The global damping matrix is expressed as a linear combination of stiffness and mass matrices in the form of the following equation [12]:

$$[C_g] = a_0[M_g] + a_1[K_g]$$

(2)

where $C_g$ is the damping matrix, $M_g$ is the mass matrix, and $K_g$ is the stiffness matrix. $a_0$ and $a_1$ are respectively the Rayleigh coefficients of first and second modes of the bridge. These coefficients are adjusted as to provide a damping ratio of 3% for typical small to medium sized bridges [13].

In this investigation, the beam is 10 m long with a cross-sectional area by 10.4 m², which is made of concrete having a constant modulus of $35 \times 10^4$ N/m² and a density of 2400 kg/m³. The single point load $P$ of 100 kN is applied at nodes. Therefore, a first natural frequency of the beam is 11.25 Hz.

Time step defined in the calculations is 0.001 s. The total time that the load is applied on the nodes was determined based on loading velocity, and the equation of motion of the finite element model (FEM) is solved to find the displacements at each node. At each time step, the strain is obtained from the displacements using shape functions and the time-variant modulus is subsequently calculated for each element based on the CEB-FIP equation [Equation (1)]. The modulus of elasticity is calculated for each fiber within a cross-section (as each fiber has a different strain rate), and then an equivalent modulus of elasticity determined for each cross-section. These equivalent modulus of elasticity are then used to populate the global stiffness matrix before equations of motion are solved in the time step that followed. A more detailed description of the model can be found in Aied and González [14].

3 LEVELS OF STRAIN IN A MOVING LOAD PROBLEM

The increase in strain rate depends on the mechanical properties of the structure, as well as the magnitude and velocity of the load. In Figure 2, the rate of change in strain is plotted in the main vertical axis together with the strain-rate dependent modulus in the secondary vertical axis for each time-step. The load is 100 kN travelling at 25 m/s (Figure 2a) and 5 m/s (Figure 2b). These velocities present typical values of vehicles crossing a bridge at fast and slow speeds. The properties of the beam are as stated above. In Figure 2a, strain rate exceeds the specified static limit for most of the run except at 0.1 s where the strain rate is 0.0, and for the first 0.2 s the strain rate is positive and vice versa for the last 0.2 s. Maximum difference between constant and time variant modulus is 8%, which occurs at a strain rate of $7 \times 10^{-5}$ s⁻¹. In Figure 2b, the strain rate exceeds the static limit but the increase is much less than that in Figure 2a. A maximum difference of 4% corresponding to a strain rate of $1.35 \times 10^{-5}$ s⁻¹ is obtained. Also, the strain rate reaches almost a constant value after 1 s (Figure 2b).

Throughout work of Aied and González [14] on investigating effect of the velocity on the strain rate, the results showed the maximum time-variant modulus of elasticity increase rapidly when the velocity changes from 5 m/s to 25 m/s (Figure 4). However, gradual increase was found for a velocity in range of 25 m/s to 60 m/s. For example, the maximum time-variant modulus of elasticity were 4% ($3.65 \times 10^{10}$ N/m² vs. $3.5 \times 10^{10}$ N/m²), 8% and 10.2% corresponding to 5 m/s, 25 m/s and 60 m/s when compared to the constant modulus of elasticity. Thus, the strain rate at 25 m/s was of interest.

The relationship between strain rate and ratio of time-variant to constant modulus of elasticity is illustrated by Figure 4 using the formulation of CEB-FIP (equation 1). This presents how the strain rate is related to the time-variant modulus. The ratio of modulus rapidly increases between strain rate of $3 \times 10^{-6}$ (close to 0) and $0.5 \times 10^{-4}$ with 8% increase, and after that the ratio gradually increases. Figure 4
shows the intersection points of strain rates at 25 m/s (5.5×10⁻⁵ s⁻¹ and -5×10⁻⁵ s⁻¹) and 5 m/s (1.5×10⁻⁵ s⁻¹ and -0.5×10⁻⁵ s⁻¹), showing how faster velocity produces higher strain rates modulus ratio.

![Figure 4](image)

Figure 4. Ratio of time-variant modulus to constant modulus versus strain rate; maximum strain rate at 25 m/s (dotted line) and 5 m/s (dashed line)

Strains at beam sections other than mid-span shown in Figures 3 and 4, are subject to strain rates associated to different values of modulus of elasticity. Generally the strain rate gets smaller the closer to the support. A time-variant modulus of elasticity that depends on strain rate is used to calculate the strain and displacement at each time step. Beam properties and sections under investigation are varied in order to investigate the impact of a varying modulus on different scenarios.

3.1 Strain and displacement at mid-span section

Strain rate and time-variant modulus are calculated at every single discretized beam cross-section for every simulation. Figure 5 compares strain at the mid-span section when using a constant modulus (based on ‘static’ or low strain rates) or a time-variant modulus at a velocity of 25 m/s and load of 100kN. As the time-variant modulus of elasticity is greater than the constant one, strain based on the time-variant modulus of elasticity is somewhat smaller than ones based on the constant modulus of elasticity (Figure 5). For this particular case, when the load is near or on the mid-span point the strain based on the time-variant modulus is the same as the ones of constant modulus because at mid-span the strain rate reaches 0.0 and the time-variant modulus is the same as the constant modulus of elasticity (Figure 2a). However, strain derived from time-variant modulus is 3% higher than one based on the constant modulus near the mid-span. Additionally, the largest divergence between the strain from the time-variant modulus and one from the constant modulus occurs at the peaks of excitation response (8%).

Similarly, displacements calculated from the time-variant modulus are also smaller than ones derived from the constant modulus (Figure 6). Maximum variation between displacements using a time-variant modulus and displacement using constant modulus are at 0.16 s (9%). When the load is on the reference point (mid-span) both displacements are close although displacement using a time-variant modulus is slightly higher.

![Figure 5](image)

Figure 5. Strain based on a time-variant modulus and strain based on a constant modulus versus time at mid-span of the beam

![Figure 6](image)

Figure 6. Displacement at mid-span when using a time-variant modulus or a constant modulus

3.2 Strain at different locations

As the load crosses the beam, strain and strain rate varies throughout the structure. Figure 7 presents the strain at the lower fibre of quarter span cross-section and Figure 8 presents the strain at different cross-sections on the beam. All sections provide a similar output and differences between using a constant or a time-variant modulus are small. On the quarter span calculations, the strain using time-variant modulus is sometimes higher than strain using constant modulus and at other points the strain using time-variant modulus is higher (at times 0.12 s and 0.22 s). A maximum difference of 3% is obtained close to maximum strain using a constant modulus. When the load is exactly on quarter span strain using constant modulus is similar or close to the strain using time-variant modulus. A detailed study of the effect of load magnitude and velocity on modulus of elasticity for different cross-sections along the beam can be found in Aied and González [14].

Strain calculations for a number of nodal points along the beam are plotted against time using a time-variant modulus (Figure 9). Behaviour of the strain is such to produce a sharp ascending and descending slope when the load is close to the reference node. These peaks are caused by a time-variant modulus similar to the constant modulus when the load is near the reference point (mid-span).
It can be concluded that a proportional relationship exists between the maximum difference of the strains and the loading velocity. In addition, it can be seen that, slow velocities (5 m/s and 10 m/s) higher peak responses occur for strains using time-variant modulus but as the velocity increases (15 m/s and 20 m/s) the contrary starts to take place.

The strain is taken at the lowest fibre of the cross-section where tension is highest (i.e., 0.325 m from centroid). Some authors argue that the modulus of elasticity in compression is more affected by high strain rates than in tension [3]. Therefore, in the compression region the modulus of elasticity has more influence. This indicates that, although the strain in the compression region is lower than in tension, greater rate of increase in the modulus of elasticity is likely to occur due to high strain rates.

3.3 Strain at varying velocity

To investigating influence of moving velocity on time-variant modulus of elasticity, various speeds involving 5 m/s, 10 m/s, 15 m/2 and 20 m/s (with interval speed of 5 m/s for range from 5 m/s to 25 m/s) were used at mid-span cross-section. The results are shown in Figure 9. At the slowest velocity of 5 m/s the strain based on time-variant modulus is smaller than strain based on the constant modulus but at the maximum point the strains are similar. A maximum difference of 3.7% is found at 5 m/s but when the load is on the reference node the strains are similar. At faster velocities of 10 m/s and 15 m/s a similar pattern in the strain measurements is observed but at 15 m/s the strain based on the time-variant modulus is 5.6% lower than the strain based on the constant modulus which is at 0.35 s just after the load crosses the mid-span point. The fastest velocity of 20 m/s produces 7.6% maximum change in the strain using the time-variant modulus.

The impact of velocity on the displacement response is equivalent to that obtained for strain. When using a constant modulus, there are no constricting factors that limit the increase in displacement due to higher dynamic effects. However, faster loads cause the time-variant modulus to reach
higher values, therefore displacement is reduced following the stiffer response of the structure.

3.4 Strain for different beam lengths

Earlier, a 10m long bridge has been used for analysing the dynamic response of a beam. Two different beam spans of 20m and 30m are simulated with cross-sectional depths of 1m and 1.5m respectively (Figure 11). The two spans have a constant modulus of elasticity of 35x10^9 N/m^2 and a density of 2400 kg/m^3. Frequencies of 2.8 Hz and 4.3 Hz for 20m and 30m long span bridges respectively resulted from the adopted beam properties. The number of discretized elements is 100 and the time step of simulation is 0.001 s for both beam spans with a load of 100 kN travelling at 25 m/s. Simulated measurements are obtained at the mid-span point of the beam.

Results of strain for 20m long span (Figure 11a) give very small variations between the two strain calculations and at the peak (mid-span) the strain is almost the same with maximum difference occurring close to the mid-span (at 0.39 s and 0.42 s). Longer beam span of 30m strain difference is even smaller with most visible difference near the mid-span. Therefore, the longer the span length of a beam the lower the impact of time-variant modulus for midpoint of the beam is observed.

![Figure 11. Strain based on constant or time-variant modulus](image)

4 IMPACT ON DYNAMIC AMPLIFICATION

Dynamics on the bridge are primarily affected by vehicle velocity, matching bridge and vehicle natural frequency, and roughness of approach road and the bridge surfaces [15]. The bridge responses are typically characterized in the codes through the use of static response and a dynamic amplification factor (DAF) [calculated by a ratio of maximum total response (static + dynamic) over maximum static response] [13]. A number of authors [16-18] have found that as the velocity of the vehicle increases the DAF alternates from high to low in a pattern characterized by a series of peaks and troughs that increase with speed. The pattern depends on the frequency of the bridge and a pseudo-frequency that depends on the load velocity and bridge length.

In Figure 11, the DAF using constant and time-variant modulus is plotted for a range of load velocities for a bridge length of 10m at mid-span. Significant difference between the DAF from the time-variant modulus and ones from the constant modulus is noticed at a loading velocity of 25 m/s. At low velocities, it is noticeable how the DAF based on a time-variant modulus is generally below one. This is a result of the structure responding in a stiffer manner under loads at speed than under a static load. Therefore, these loads at low speed are not able to generate sufficient large dynamics and strains as to produce DAFs above one. When using a time-variant modulus, it is necessary to reach speeds above 45 m/s to enter a region where DAFs are consistently above one.

![Figure 10. DAF versus velocity](image)

DAF is affected to a large extent by the variation in the modulus. However, these results are based on Equation (1) which might be exaggerated for a true bridge. A more accurate picture of how DAF can be affected by strain rate for a particular bridge can be experimentally obtained by measuring the response of the bridge to a number of vehicles travelling with different loading and velocity conditions.

5 SUMMARY AND CONCLUSIONS

The response of a one dimensional finite element beam to a point force is simulated to calculate the change in strain and displacement due to a time-variant modulus. For this purpose, the relationship between strain rate and time-variant modulus provided by CEB-FIP Model Code is used in the analysis. The velocity of the vehicle has a high impact on the response given that as the force travels faster, the modulus rises and restricts the increase in the response. Responses at high velocities lead to lower maximum strain and displacement peaks using a time-variant modulus than using a constant modulus of elasticity. These differences between using a constant or time-variant modulus are clearly observed when investigating DAF. If the assumption of time-variant modulus is adopted, very high speeds are necessary to identify DAF.
values of 1.1, i.e., 50 m/s. However, if a constant modulus is employed, a DAF of 1.1 can be reached at 25 m/s. These results are based on the application of the CEB-FIP equation and a simplistic point load model on a one-dimensional beam. Therefore, a site-specific equation will need to be calibrated (i.e., using a traffic population in the case of a bridge) to draw conclusions for other scenarios.

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REFERENCES


