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A Note on the Size Distribution of Irish Mortgages.

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Abstract

Using Department of Environment data on number of mortgages by size category we find that the Weibull distribution accurately models the distribution of loans under €300,000 but severely underestimates the number of larger loans. We therefore use a a Pareto distribution for loans above this level. We estimate that from 2006 to 2008 there were fewer than 2,000 loans over €1 million with total value of €3 billion; and that there were 11,000 loans over €500,000 with estimated value of €9 billion. While the number of people taking out these mortgages is unknown, the conjecture that the largest 10,000 mortgage borrowers owe around €10 billion, largely for buy to let mortgages, does not appear implausible given these results. More tentatively, an ecological inference procedure suggests that interest only mortgages went almost exclusively to property investors.

1 Introduction.

The insolvency of the Irish banks and Irish state as a consequence of bad loans to property developers raises the obvious question of how much Irish banks stand to lose on other categories of loans such as mortgages, and in particular on large mortgages given out to property investors. In this note we attempt to calculate the exposure of Irish banks to large mortgages.

Between 2005 and 2008 Irish banks lent €25 billion to investment buyers, compared with €28 billion to first time buyers, and €36 billion to movers. The average loan taken out by investors was €325,000 compared with only €240,000 for first time buyers and €270,000 for movers.[1] Similarly, of new housing units

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[1]IBF/PwC Mortgage Market Profile.

value	100	150	200	250	300	400
CDF	0.08	0.20	0.40	0.59	0.75	0.91

Table 1: Proportion of mortgages below differerent values (in thousand Euro), 2007.

sold from 2004 to 2008, less than 45 per cent went to first time buyers, suggesting that this market was dominated by investors.

So what is the exposure of Irish banks to large, investor mortgages? The Department of the Environment publishes statistics on the number and total value of mortgages paid each year, and on the percentage of these mortgages in different value categories, up to €400,000.[2]

For example in 2007 banks paid out 84,000 mortgages, worth €22.5 billion. The proportion below different values, in thousand Euro, is shown in Table 1.

We want to estimate what proportion of the value of mortgages is accounted for the 9 per cent of mortgages over €400,000, and the values of larger categories of mortgages. We can see immediately that these data have a long right tail, inconsistent with normality: the mean mortgage was €268,000 while nearly 60 per cent of mortgages were less than €250,000.

In this note, we start with the standard assumption for data with a long right tail that the data follow a Weibull distribution. We show below that the Weibull distribution gives a good fit to the value of mortgages given out from 2005 to 2008 under €300,000. However the sharply rising hazard rate of the Weibull distribution causes it badly to underestimate the fraction of larger mortgages and, as a consequence, the mean value of all mortgages.

In Section 3 we therefore model the distribution of mortgages over €300,000 with a power law or Pareto distribution. This estimates that only 1,750 mortgages were given out over €1 million, but with a total value of €3 billion; while the estimated 11,000 mortgages over €0.5 million were worth €9 billion. Given that some individuals took out more than one mortgage, these results suggest that the estimate that the 10,000 borrowers with the largest mortgages owe around €10 billion is not implausible.

These large mortgages appear to have gone almost exclusively to property investors. In Section 4 we apply ecological inference to see the relationship between investor mortgages and interest only loans. We find that interest only

[2]Latest House Prices, Loans and Profile of Borrowers Statistics at http://www.environ.ie/en/Publications/StatisticsandRegularPublications/HousingStatistics/. All statistics are taken from this source unless otherwise stated.

loans appear to be overwhelmingly concentrated among property investors, but the limited data makes this finding conjectural.

The heavy right tail of mortgages means that simple default numbers can be misleading. For 2007, half of mortgages were for €228,000 or less; and the mean value of these loans is €155,000. By contrast, for the 4 per cent of mortgages over €625,000, the mean value was €1 million. It follows that one default in this upper tail, is equivalent to over six defaults in the lower half of the distribution.

2 The Weibull Distribution.

A standard way to model data with a long right tail is to use a two parameter Weibull distribution (Johnson, Kotz and Balakrishnan, 1995, 628–722).

$$f(x;\lambda,k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k} \qquad x \geq 0 \tag{1}$$

with associated cumulative distribution function

$$F(x) = 1 - e^{-(x/\lambda)^k}. \tag{2}$$

An appealing feature of the Weibull is the linear relationship between the doubly log transformed quantiles and $\log x$

$$\log\left(-\log\left(1 - F\left(x\right)\right)\right) = -k\log\lambda + k\log x \tag{3}$$

Plotting $\log\left(-\log\left(1 - F\left(x\right)\right)\right)$ against $\log x$ therefore does two things. First, it allows us to assess the goodness of the Weibull fit: the closer the points to a straight line, the better the fit of the Weibull distribution.

Figure 1 plots these for the Irish mortgages in 2007 shown in the first table. It can be seen that the relationship is almost linear, consistent with a Weibull distribution, apart from the largest value corresponding to mortgages below €400,000 which lies above the the predicted Weibull value. In other words, the Weibull distribution is conservative, assigning fewer mortgages to the highest category than was actually the case.

The second use of equation (3) is to estimate the parameters k and λ. An OLS regression of $\log\left(-\log\left(1 - F\left(x\right)\right)\right)$ on $\log x$ has slope k and intercept $-k\log\lambda$.

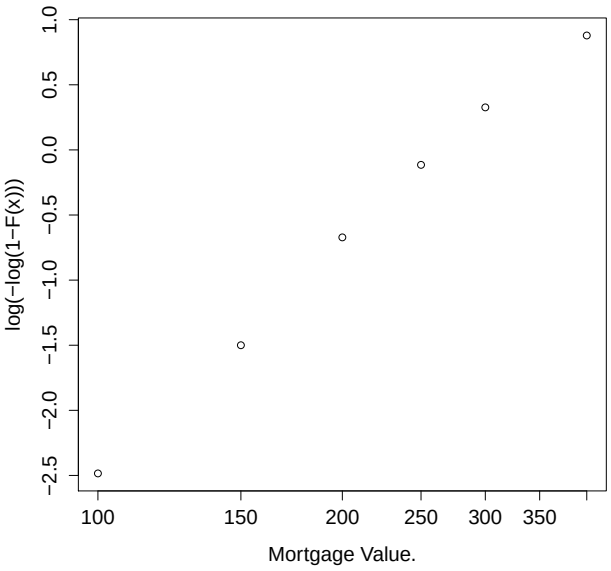

Figure 1: Relationship between doubly log transformed empirical cumulative distribution of Irish mortgages in 2007 and values of mortgages.

The expected value of a Weibull random variable is

$$E(x) = \lambda\Gamma\left(1 + \frac{1}{k}\right) \tag{4}$$

where Γ is the gamma function. The average value of mortgages below value M is

$$E(x|x < M) = \frac{\int_0^M x\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}\,dx}{1 - e^{-(M/\lambda)^k}}. \tag{5}$$

The numerator is not analytically integrable but, given estimates of k and λ, it is straightforward to integrate numerically.

Applying regression (3) to the 2007 data in Table 1, gives an estimate of $k = 2.48, \lambda = 269$. From (4), the predicted mean value of mortgages is €239,000 in 2007, about ten per cent lower than the actual average of €268,000, consistent with the underestimation of the upper tail of mortgages in Figure 1.

This underestimation of the upper tail is severe: the predicted proportion of mortgages over €400,000 is 7 per cent, only slightly less than the true value of 9 per cent. However, the tail shrinks rapidly consistent with the sharply rising

hazard function of the Weibull: only 5 per cent of mortgages are predicted to be over €420,000, and only 0.1 per cent over €500,000.

By contrast, the fit up to €300,000 is close as Figure 1 suggests. Omitting the last observation when estimating λ and k, the predicted proportion of mortgages under €300,000 is 75 per cent, exactly the same as the actual value.

3 Pareto Distribution.

We have seen that the Weibull distribution gives a good fit to mortgages under €300,000 but significantly underestimates the number of larger mortgages. To fit this upper tail, we make the standard assumption that the distribution of higher observations follows a power law or Pareto distribution (Johnson, Kotz and Balakrishnan, 1995, 573–627).

For values of the random variable above a minimum value L the Pareto has density function

$$f(x;\alpha) = \alpha L x^{-(1+\alpha)} \qquad x \geq L \tag{6}$$

and distribution function

$$F(x) = 1 - \left(\frac{L}{x}\right)^{\alpha}. \tag{7}$$

The conditional mean of x given $x \geq M \geq L$ is

$$E(x|x \geq M) = \frac{\alpha}{\alpha - 1} M \qquad \alpha > 1. \tag{8}$$

To estimate the parameter α we could estimate the slope of a regression of $-\log(1 - F(x))$ on $\log x$ as we did earlier for the Weibull distribution, but we clearly lack enough observations in the upper tail of mortgages.

Instead we start with the distribution of mortgages below €300,000; fit a Weibull distribution to this using (3) and then estimate the mean value of mortgages in this range from (5). For 2007, the mean value of mortgages under €300,000 in 2007 is estimated to be €191,000. Given that the mean value of all mortgages that year was €266,000, it follows that the mean value of the 25 per cent of mortgages over €300,000 was €491,000. In other words, we constrain the total value of mortgages given out to equal the actual value, and use the Pareto to estimate the distribution of loans among value categories.

	α	num 400k	val 400k	num 500k	val 500k	num 1mil	val 1mil
2008	2.37	6,518	4.5	3,841	3.3	743	1.3
2007	2.57	10,070	6.6	5,679	4.7	959	1.6
2006	5.03	5,756	2.9	1,873	1.2	57	0.1
Total		22,344	14.0	11,393	9.2	1,759	3.0

Table 2: Number and value (in billion Euro) of mortgages over €400,000, €500,000, and €1 million; for 2006–08. Calculations assume a Pareto distribution with parameter α of mortgages over €300,000.

From equation (8) it follows that $\alpha = 2.57$, so that the number of mortgages given out falls more than cubically with their value: there will be only one twelfth as many mortgages of value $2M$ as of value M, while the mean value of mortgages over amount M is $1.6M$. From (7), the Pareto slightly overestimates the fraction of mortgages over €400,000 as 11.9 per cent compared with the true value of 9.1 per cent.

Table 2 gives the estimated number and total value (in billions) of mortgages paid out from 2006 to 2008. Large mortgages for 2005 appear negligible: the Pareto distribution estimates that there were only 80 of them. There are an estimated 22,000 mortgages over €400,000 given out, close to the actual figure of 21,000. The estimated value of these is €14 billion. 11,000 mortgages over €0.5 million account of €9 billion, while 1,750 mortgages over €1 million account for €3 billion.

It is notable that, although the total number and value of mortgages fell between 2006 and 2008 [3], the share of mortgages going to large loans rose which is shown by the falling parameter α.

The data are therefore consistent with a small number of mortgages accounting for a large share of total loans. What the data do not indicate is how many people took out these mortgages. It would appear that few of these large loans were used for purchasing the borrower's own home—one of the largest mortgage providers has claimed that it made only 4 mortgages over €1 million for this purpose during the boom—and that nearly all were used for property investment.

[3] 111,000 mortgages worth €25.4 billion in 2006; 84,000 worth €22.4 billion in 2007; 54,000 worth €14.5 billion in 2008.

	Interest only	Annuity	
Investor	?	?	13
Other	?	?	87
	20	80	

Table 3: Proportion of interest only and investor loans, third quarter 2007.

4 Interest Only Loans.

Given that most of these large mortgages were taken out by property investors, what percentage were interest only loans?

The Department of the Environment gives the proportion of loans (by number) that were interest only and the Irish Banking Federation the proportion (again by number) that were for property investment. For instance, in the last quarter of 2007, 20 per cent of loans approved were interest only, while investors accounted for 13 per cent of mortgages paid. We are interested in what fraction of investor loans were interest only.

In terms of Table 3, we know the row and column sums, and want to know the entries in the four inner boxes. To estimate this we apply ecological inference, using the row and column sums for the 24 quarters from 2005 to 2010. Because of the small number of observations and potential instability during this period, our results are necessarily tentative. Estimation involves calculating a bivariate logit for the proportion of interest only loans to investors and other borrowers. We apply the nonparametric Bayesian procedure of Imai, Lu and Strauss (2011), implemented in the R package eco.

The results indicate that 67 per cent of mortgages given to investors were interest only, with a 95 per cent credible interval from $[0.47, 0.79]$. For other mortgages, the estimated proportion of interest only loans is only 4 per cent with a credible interval $[0.02, 0.07]$. In other words, interest only mortgages appear to have gone predominantly to investors. The rise in interest only loans during 2007 coincided with the making of large loans to investors noted above, suggesting that interest only loans went to those taking the largest investment mortgages.

5 Conclusions.

The starting point for this analysis was the claim in the *Irish Times* of 6 May 2010 in a story entitled "Interest-only mortgages depended on a rising market

and now many in the small-time landlord sector smell trouble" that "the interest-only candy pot was also raided by an estimated 10,000 accountants, lawyers and other professionals who spent between €1 million and €2 million each on buying homes with a typical loan-to-value mortgage of 80 per cent." Assuming an average purchase price of €1.4 million (the geometric mean of the end points); this suggests total borrowings of €11 billion.

Our analysis here has shown that instead of 10,000 million plus mortgages, there were probably fewer than 2,000 and these were mostly for investment rather than own home mortgages. However, looking at the 11,000 largest mortgages from the bubble peak of 2006-08, we find that the total is €9 billion. We do not know how many people held more than one mortgage, but it does not seem implausible that the total indebtedness of the 10,000 people with the largest mortgage debt is in the region of €10 billion.

The heavy right tail in mortgage lending suggests that considerable caution is needed in looking at default rates on mortgages: a single default on a few large mortgages can exceed the impact of many defaults on mortgages close to or below the median.

The extent of losses on these large investor loans are, of course, unknowable at this stage. In Kelly (2010) I showed how the value of Irish property was driven by the flow of new mortgages. Given that the value of new investor mortgages has fallen by 99 per cent between the second quarters of 2007 and 2011 suggests that recovery rates on defaulted mortgages could be extremely low.

References

Imai, Kosuke, Ying Lu and Aaron Strauss. 2011. "eco: R Package for Ecological Inference in 2 x 2 Tables." *Journal of Statistical Software* 42(5):1–23.

Johnson, Norman Lloyd, Samuel Kotz and Narayanaswamy Balakrishnan. 1995. *Continuous univariate distributions.* 2. ed. ed. New York: Wiley.

Kelly, Morgan. 2010. The Irish Credit Bubble. Working paper University College Dublin.

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