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<th>Risk-Averse Preventive Voltage Control of AC/DC Power Systems Including Wind Power Generation</th>
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<td>Rabiee, Abbas; Soroudi, Alireza; Keane, Andrew</td>
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Abstract—Preventive voltage control (PVC) deals with the alert state of power systems, where the system operates in a stable regime but loading margin (LM) is insufficient or some operational constraints have been violated. Hence, the aim of PVC is to ensure a desired LM (i.e., restoration of normal operation state), while minimizing the corresponding control costs. This paper proposes a new stochastic PVC (SPVC) model for power systems operation, taking into account the uncertainties of wind power generation. The uncertainty of wind power generation is handled at risk. Voltage set-points of generation units, active power reduction of predetermined generating units, load reduction of a predetermined load buses, along with the intermittent wind power generation, are employed as control measures in the proposed SPVC approach. Line-commutated converter high-voltage DC (LCC-HVDC) link constraints, doubly fed induction generators’ (DFIGs) capability curves are also considered in the proposed SPVC approach. To illustrate the effectiveness of the proposed approach, it is applied on the IEEE 39-bus test system. The obtained results substantiate the applicability of the proposed SPVC model to ensure secure operation of AC/DC power systems with high penetration of offshore wind farms.

Index Terms—Conditional value at risk (CVaR), high voltage direct current (HVDC), preventive voltage control (PVC), scenario based modeling, uncertainty, wind power.

NOMENCLATURE

Sets:
- \( NB \) Set of system buses
- \( NG \) Set of generators
- \( NB_G \) Set of buses with generation units
- \( NG_b \) Set of generators connected to bus \( b \)
- \( NG_PV\) Set of generators participating in PVC
- \( NB_PV \) Set of load buses participating in PVC
- \( N_{B_{EXT}} \) Set of buses connected to external network

Indecises:
- \( s \) Scenario index
- \( b \) Bus index
- \( i \) Generator index
- \( m \) Rectifier\((m = r)\)/inverter\((m = i)\)

AC network’s variables and parameters:
- \( P_{G_i}/Q_{G_i} \) Active/reactive power generation by \( i^{th} \) thermal generation unit
- \( P_{bus}/Q_{bus} \) Active/reactive load in bus \( b \)
- \( P_{G_s}/Q_{G_s} \) Active/reactive power generation by \( s^{th} \) thermal generation unit in loadability limit point
- \( P_{fr}^{r}/Q_{fr}^{r} \) Active/reactive load in bus \( b \) in loadability limit point
- \( P_w/Q_w \) Active/reactive power output of wind farm
- \( \mu_{bus}/Q_{bus} \) Cost of decreasing active/reactive demand at bus \( b \)
- \( \mu_{G_i}/Q_{G_i} \) Cost of active/reactive power purchased from external network at bus \( b \)
- \( P_{fr}^{s}/Q_{fr}^{s} \) Magnitude/angle of \( b_j^{th} \) element of \( \omega_{w,s} \)
- \( \mu_{b}^{s}/Q_{b}^{s} \) Rated active power output of wind farm
- \( \pi_{l,s}/Q_{l-s} \) Re-dispatch cost of increasing/decreasing the generator’s active power output in bus \( i \)
- \( \pi_{G_i}/Q_{G_i} \) Re-dispatch cost of increasing/decreasing the generator’s reactive power output in bus \( i \)
- \( \pi_{w,s} \) Ratio of actual active/reactive power demand to the corresponding forecasted value in scenario \( s \)
- \( \pi_{w,s} \) Ratio of available wind power generation capacity to its rated capacity in scenario \( s \)
- \( \lambda_{G_i}/Q_{G_i}^{sch} \) Scheduled active/reactive power generation by \( i^{th} \) thermal unit
- \( \phi_{b} \) Voltage magnitude/angle in bus \( b \)
- \( \phi_{b} \) Voltage magnitude/angle of bus \( b \) at loadability limit point.

HVDC variables:
- \( P_{dc} \) Active power flowing through HVDC link
- \( \varphi_{m} \) Angle difference between the fundamental line current and line-to-neutral AC voltage
- \( R_{cm} \) Commutation resistances
- \( V_{dc} \) DC voltages at the HVDC terminals
- \( I_{dc} \) DC current carried by the HVDC link
- \( \alpha_{m} \) Ignition angle
- \( V_{in} \) Ideal no-load voltage at the terminals
- \( B_{m} \) Number of series-connected bridges in a terminal
- \( R_{L,d} \) Resistance of HVDC cable
- \( Q_{dc} \) Reactive power flowing into HVDC link
- \( B_{sh,m} \) Susceptance of HVDC shunt filters
- \( T_{m} \) Tap ratio of HVDC’s transformer
- \( V_{m} \) Voltage magnitudes of the AC terminals of HVDC terminals
- \( Q_{sh,m} \) VAR compensations at HVDC terminals

Risk associated variables:
- \( \eta \) Auxiliary variable to define CVaR
- \( \beta \) Conditional value at risk (CVaR)
- \( \pi_{s} \) Expected value operator
- \( \pi_{s} \) Probability of occurrence in scenario \( s \)
- \( \beta \) Weighting factor to indicate the importance of expected value \( \beta = 1 \) compared to risk value \( \beta = 0 \)

I. INTRODUCTION

Voltage stability is defined as the ability of a power system in maintaining proper voltages at load buses in normal operation and emergency conditions. This issue has been recently at the central point of attention due to various reasons such as growth of electrical energy demand, economic...
and environmental constraints in expanding generation and transmission capacities, and market pressure to reduce operating costs. Moreover, the recent trends toward smart grids and also increasing share of renewable energy resources in many power systems, due to the environmental concerns and low marginal operating costs, have intensified the need for powerful approaches for power system security enhancement [1], [2]. Under such circumstances, there is possibility of voltage instability occurrence and therefore, it has to be considered as an integral part of power systems operation and planning studies [3]. It can be avoided by preventive voltage control (PVC) or by post-contingency corrective voltage control (CVC). The post-contingency CVC aims to restore voltage stability by directing the system into a new secure equilibrium point shortly after a severe contingency, such as the outage of a heavily loaded transmission line or transformer [4]. But, PVC is initiated in the condition that the system’s loading margin (LM), is less than a desired value [5]. In this situation, PVC measures modify the operating point to an equilibrium point with a LM greater than the desired value, and hence, voltage security of the system ensured [5]. LM is defined as the distance from current operating point to the voltage collapse (or loadability limit) point [6].

A powerful tool is needed in order to handle the uncertainties of wind and demand values and quantify the potential risk at the first step. On the second step the system operator should try to minimize these undesired impacts simultaneously.

### B. Literature Review

Nowadays offshore wind farms are developing in many countries, because the best locations for onshore wind farms are already developed. The offshore wind farms are commonly located far away from the onshore grid. If the distance is long or if the offshore wind farm is connected to a weak AC onshore grid, a high-voltage dc (HVDC) transmission system may be a more suitable choice than the conventional high-voltage AC transmission [7].

A common phenomenon which is likely to occur in practical power systems, is the shortage of LM during the peak load period due to heavy loading condition. Hence, dependable PVC actions such as re-adjustment of voltage controllers’ set points, active/reactive generation re-dispatch, load curtailment in more vulnerable nodes are vital to restore sufficient LM. One of the main barriers in proper implementation of PVC, is the uncertainties associated with the forecasted system parameters. By increasing penetration of volatile renewable energy resources such as wind power generation, the most important uncertain parameters which directly affect the LM as well as the total costs are wind power generation and demand values [8], [9]. These uncertainties impose technical and economic risks on the system operator [10]. For example [8] addresses a methodology based on PV and QV analysis for probabilistic risk of voltage collapse. The uncertainty in the amount of generation provided by the renewable resources is considered by scenario based modelling. LM is considered as a stochastic variable in [9] and the impact of uncertain power injections by wind farms on the LM is investigated. In [11]

an approach is proposed to increase wind power penetration by placing new wind generation at the strong buses from the voltage stability viewpoint. Voltage stability is evaluated by modal and QV analyses. In [12] a probabilistic framework is proposed which evaluates voltage/rotor angle stability in the presence of HVDC connected wind farms. The authors in [13] proposed a methodology for coordinating reactive output of wind generators with other reactive sources for voltage stability enhancement. They show that significant improvement in voltage stability margin can be obtained if the reactive power output of wind farms can be properly coordinated with other reactive power resources in the system.

It could be observed from the above literature survey that in the presence of severe uncertainties characterizing the voltage stability of the system, both technically and economically, is a critical issue. Since PVC actions impose significant costs to the system, reduction of these costs is desirable for system operator. Hence, it is necessary to characterize the voltage stability issue precisely, to make realistic decisions in which both technical and economic concerns are considered simultaneously.

<table>
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<th>Reference</th>
<th>HVDC</th>
<th>Uncertainty</th>
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The proposed method tries to analyze the HVDC connected wind turbines considering the uncertainty and risk hedging issues. The voltage stability is enhanced using a stochastic programming technique.

### C. Contributions

Table I summarizes a taxonomy of proposed methodologies for wind power integration in OPF. Hence, in this paper a new stochastic PVC (SPVC) model is proposed for operation of AC/DC power systems considering the uncertainty of wind power generation and demand values. Scenario based modeling is employed to properly handle these uncertainties. The aim of the proposed SPVC is to modify the operation point of the system in a way that for all probable scenarios, the system has sufficient LM and the cost of PVC controls is minimized. Also, it is assumed that wind power is supplied from an offshore site which is connected via a HVDC link to the AC onshore grid. In order to provide a SPVC which is both technically and economically efficient, conditional value at risk (CVaR) index is employed to determine the best strategy for system operators to cover the power demand uncertainties from different options in a secure way, especially form the LCC-HVDC connected uncertain offshore wind forms. Considering the CVaR in SPVC ensures that at in all probable scenarios, the required LM is ensured as well as the cost of PVC controls is reasonable. Hence, the proposed SPVC model gives a technical-economic decision making tool for power system operators.

The main contributions of this paper are listed as follows:
1) It provides a methodology to characterize both technical and economic issues of voltage stability. 
2) The proposed method aims to reduce the associated technical and economic risks of PVC via CVaR index. 
3) It considers the main sources of uncertainties, i.e. wind power generation and demand values, simultaneously. Hence, the proposed SPVC approach provides a realistic and practical decision making tool for system operator. 
4) Since the new wind farms are constructed at the offshore sites and connected to the system via HVDC links, the proposed SPVC scheme considers the steady state model of such technology and formulate the voltage stability problem in the presence of offshore and HVDC-connected wind farms. 
5) The proposed SPVC scheme, considers the LM as a stochastic variable which precisely models the system trajectory from the modified operation point to the collapse point in different probable scenarios. This treatment of system-wide voltage stability problem in the presence of uncertainties is not only considers the worst-case scenario but also other operating states.

**D. Paper Organization**

The rest of this paper is set out as follows: Section II presents a basic description of PVC. Section III deals with the uncertainty modeling and risk implementation in stochastic problems. Section IV presents formulation of the proposed SPVC problem. Case study and numerical results are given in Section V and finally, Section VI summarizes the findings and concludes the paper.

**II. DESCRIPTION OF PVC**

According to the classification given in [16], operational states of practical power systems are as follows: Normal, Alert, Emergency, In Extremist and Restorative modes. The system is in Alert state if the operational constraints such as voltage/line-flow limits are violated or the security level of the system decreases below a certain threshold. In such a condition, preventive controls are activated manually or automatically, to restore the Normal state. One of the main criteria for addressing security of power systems, is the distance in MW (or MVar or MVA) from the current operating point to the maximum transmittable power point or voltage collapse point [17]-[18], which is called loading margin (LM). Voltage collapse occurs following a large disturbance or suddenly load increase in a heavily loaded power system. Control actions taken to prevent voltage instability prior to occurrence of contingencies or subject to the daily load changes are called preventive voltage control (PVC). For secure operation of a system, it is suggested to preserve specified LMs for both pre- and post-contingency states [18]. PVC deals with the situation in which the system operating point is stable but LM is less than the desired value. Accordingly, PVC actions are taken to ensure a desired LM. The control measures employed in PVC includes [5]: 

- Reactive power re-dispatch of dynamic VAR sources including generators, synchronous condensers, and FACTS controllers, 
- Determination of on/off positions of capacitors/reactors, 
- Determination of transformers tap and phase shifters set point, 
- Load curtailment.

In Fig. 1, the base-case operating point \( A \), (with the corresponding load \( P^0 \)), is located on curve (1) with the corresponding loadability limit point \( A' \). This point is stable, but with insufficient LM (i.e. \( \lambda_0 < \lambda_{des} \)). Following the PVC, loci of system’s operation points will be modified to curve (2), and new secure operating point \( B \) with \( \lambda = \lambda_1 \geq \lambda_{des} \) is obtained. \( \lambda_1 \) is calculated subject to the corresponding loadability limit point, i.e. \( B' \).

![Fig. 1. The operation point evolution by execution of PVC](image)

**III. UNCERTAINTY AND RISK MODELING**

In every engineering problem the decision maker tries to solve an optimization problem which contains some objective functions and some constraints. Usually the input parameters of such an optimization problem are subject to uncertainty. Depending on the nature of uncertainty, there are several techniques to model them such as Information gap decision theory [19], [20], fuzzy logic [21], robust optimization [22] and stochastic techniques. The stochastic techniques usually require the historic data of uncertain parameters. One of the well-known methods in stochastic techniques is the scenario based modeling. It is assumed that the probability density functions (PDF) of uncertain parameters are available. In scenario based modeling, the uncertain outcome set for uncertain parameter is discretized using its PDF. Each discrete event \( (A_s) \) is called a scenario with the known probability of occurrence \( (\pi_s) \). The conventional way of dealing with uncertainty using scenario based method is optimizing the expected value of \( (A_s) \) over the scenario set as follows.

\[
\min \text{ / } \max \sum \pi_s f(\bar{U}, A_s)
\]

\[
\Xi_f(\bar{U}, A_s) = \sum \pi_s f(\bar{U}, A_s)
\]

\[
H_i(\bar{U}) \leq 0, i \in \Gamma
\]

\[
\bar{U} = \{\Psi, \Omega_s\}
\]

\( \Gamma \) is the set of all constraints. In this framework, the decision variables include two sets of decisions \( \Psi, \Omega_s \). The first variable set (\( \Psi \)), represents the “Here and now” or “First
stage” decisions. These variables are set prior to realization of uncertain parameters and are independent of scenarios. The second variable set (Ωs), represents the “Wait and see” or “Second stage” decisions. These variables are set posterior to realization of uncertain parameters and are dependent of scenarios. The whole decision making procedure is called the two stage decision making. Optimizing the expected value is not the best way to deal with uncertainties because without considering the risks of objective functions the decision maker may face undesired situations. There are several risk measures to handle the risks in scenario based uncertainty modeling and decision making like variance, shortfall probability, expected shortfall, value at risk and conditional value at risk (CVaR). In this paper, CVaR is chosen as the risk measure (ℜ) to be optimized along with expected value because of its significant advantages over other risk measures, numerical efficiency and stability of calculations [23]. The procedure for maximization/minimization is as follows [24]:

\[
\begin{align*}
\max_{\Omega} OF &= \beta \Xi_f(\bar{U}, \Lambda_s) + (1 - \beta) \Re \\
\Re &= \eta - \frac{1}{\epsilon} \sum_s \pi_s \max(\eta - f(\bar{U}, \Lambda_s), 0) \\
\min_{\Omega} OF &= \beta \Xi_f(\bar{U}, \Lambda_s) + (1 - \beta) \Re \\
\Re &= \eta + \frac{1}{\epsilon} \sum_s \pi_s \max(f(\bar{U}, \Lambda_s) - \eta, 0)
\end{align*}
\]  

where \(\Re\) is computed as the expected profit in the \((1 - \epsilon)100\%\) worst scenarios. \(\epsilon\) is indicating the upper/lower tail of the cost/benefit distribution in \(\min / \max (7)/(5)\). \(\Xi_f\) is the expected value of objective function over scenarios. \(\eta\) is the value at risk which is a lower \(\alpha\) - percentile of random variable \(\Lambda_s\).

IV. PROBLEM FORMULATION

The goal of the system operator is optimizing the expected values of two objective functions namely, minimizing the cost of preventive voltage control and maximizing the loading margin of each scenario while satisfying network’s equality and inequality operational constraints. The assumptions, objectives, decision variables and constraints are described as follows.

It is worth mentioning that the content of sections IV-B, IV-C and IV-D are derived from well-established load flow and HVDC models which are given in [4], [15], [16], [25]–[28].

A. Objective functions

There are two objective functions (OFs) in the proposed probabilistic PVC framework, namely the expected value of loading margin (i.e., \(\Xi_{LM}\)), and the expected value of total cost of active/reactive power generation/consumption re-dispatch along with energy procurement cost from external network (or pool market), (i.e. \(\Xi_{TC}\)). These OFs are characterized as follows.

\[
\Xi_{LM} = \sum_s \pi_s \lambda_s
\]  
\[
\Xi_{TC} = \sum_s \pi_s TC(s)
\]  
\[
TC(s) = \sum_{i \in NG_{PVPC}} \left( \mu_t^p \Delta P_{Gi,s}^{up} + \mu_t^q \Delta Q_{Gi,s}^{up} \right)
+ \sum_{i \in NG} \left( \mu_s^p \Delta P_{Gi,s}^{dn} + \mu_s^q \Delta Q_{Gi,s}^{dn} \right)
+ \sum_{i \in NB_{PVPC}} \left( \mu_s^p \Delta P_{Li,s}^{up} + \mu_s^q \Delta Q_{Li,s}^{up} \right)
+ \sum_{i \in NB_{hex}} \left( \mu_s^p \Delta P_{ex}^{ex} + \mu_s^q \Delta Q_{ex}^{ex} \right)
\]  

Equation (9) is the expected value of LM which indicates that the average LM in all scenarios should be maximized which ensures sufficient distance from the voltage collapse point in all scenarios. Also, (10) is the expected cost of PVC actions which should be minimized. The PVC cost associated with each scenario is given in (11), where the first and second lines in the right hand side of (11) are active and reactive power generation re-dispatch costs, the third line is the cost of active and reactive load re-adjustment, and the last line is the cost of energy procurement from the external network to make an instantaneous balance between the generation and demand.

B. AC power flow constraints

The standard AC power flow constraints and operational limits are as follows [4], [25].

\[
\sum_{i=1}^{NG} (P_{Gi,s} - (P_{b,s} - \Delta P_{b,s})) = 0
\]  
\[
V_{b,s} \sum_{j} V_{i,s} y_{ij} \cos(\theta_{ij} - \theta_{j,s} - \gamma_{b,j}), \forall b \in NB
\]  
\[
\sum_{i=1}^{NG} (Q_{Gi,s} - (Q_{b,s} - \Delta Q_{b,s})) = 0
\]  
\[
V_{b,s} \sum_{j} V_{i,s} y_{ij} \sin(\theta_{ij} - \theta_{j,s} - \gamma_{b,j}), \forall b \in NB
\]  
\[
P_{Gi,s} = \begin{cases} 
P_{ch}^{Gi} \
P_{Gi}^{ch} + \Delta P_{Gi,s}^{up} - \Delta P_{Gi,s}^{dn} \
\end{cases}, \forall i \in NG_{PVPC}
\]  
\[
Q_{Gi,s} = \begin{cases} 
Q_{ch}^{Gi} \
Q_{Gi}^{ch} + \Delta Q_{Gi,s}^{up} - \Delta Q_{Gi,s}^{dn} 
\end{cases}, \forall i \in NG_{PVPC}
\]  
\[
P_{Gi,\min} \leq P_{Gi,s} \leq P_{Gi,\max} 
\]  
\[
Q_{Gi,\min} \leq Q_{Gi,s} \leq Q_{Gi,\max} 
\]  
\[
0 \leq \Delta P_{Gi,s}^{up} \leq \Delta P_{Gi,s}^{dn} \leq \Delta P_{Gi,s}^{max} 
\]  
\[
0 \leq \Delta Q_{Gi,s}^{up} \leq \Delta Q_{Gi,s}^{dn} \leq \Delta Q_{Gi,s}^{max} 
\]  
\[
P_{ex,\min} \leq P_{ex}^{ex} \leq P_{ex,\max} 
\]  
\[
Q_{ex,\min} \leq Q_{ex}^{ex} \leq Q_{ex,\max} 
\]  
\[
\delta_{th,\min} \leq \delta_{s,t} \leq \delta_{th,\max} 
\]  

where, constraints (12), (13) are load flow equations of secure operation point (i.e. point \(B\) in Fig. 1), equations (14)-(21)
shows the re-dispatch possibility of active/reactive power generations and the corresponding limits, constraints (22), (23) are the limits on the exchanged active/reactive power between the system and the external network. Also, the limits on the voltage magnitudes and line flows are given by (24), (25).

In order to precisely characterize the LM, it is necessary to consider the power flow equations and operational limits for the corresponding loadability limit point (point \( B' \) in Fig. 1) as follows \([26],[28]\).

\[
\forall b \in NB, \forall s:\n\begin{align*}
\sum_{i=1}^{NG_b} \hat{P}_{G_{i,s}} + P^c_{b,s} - \hat{P}_{L_{b,s}} &= \quad \text{(26)} \\
\hat{V}_{b,s} \sum_j \hat{V}_j,s \hat{V}_{ij,s} \cos(\hat{\theta}_{b,s} - \hat{\theta}_{j,s} - \gamma_{ij}) &= \quad \text{(27)}
\end{align*}
\]

where, (26), (27) are load flow equations of loadability limit point, equations (28)-(29) are active/reactive loads at the loadability limit point, constraints (30)-(32) are active/reactive power generation at the loadability limit point and their corresponding limits. Also, the limits on the voltage magnitudes at the loadability limit point are given by (33). Besides, the constraints (34)-(37) ensure feasibility of the post-PVC operating point (point \( B \) in Fig. 1) and a trajectory from point \( B \) leading to point \( B' \) when the loads increment \([26],[28]\). Also, (38) guarantees a positive LM for all scenarios.

\( i: \) Inverter:

\[
\begin{align*}
V_{d0m,s} &= \frac{3\sqrt{3}}{\pi} B_m T_m V_{m,s} \quad \text{(39)} \\
V_{d_{m,s}} &= V_{d0m,s} \cos(\alpha_{m,s}) - B_m R_{c,m} I_{d,s} \quad \text{(40)} \\
I_{d,s} &= \frac{V_{d_{m,s}} - V_{d,s}}{R_{l,d}} \quad \text{(41)} \\
cos(\varphi_{m,s}) &= \frac{V_{d_{m,s}}}{V_{d0m,s}} \quad \text{(42)} \\
P_{d_{m,s}} &= V_{d_{m,s}} I_{d,s} \quad \text{(43)} \\
Q_{m,s} &= P_{d_{m,s}} \tan(\varphi_{m,s}) \quad \text{(44)}
\end{align*}
\]

where (39) gives the relationship between ideal no-load voltage at the DC sides of the LCC-HVDC link, and the AC sides voltages. Equation (40) is the actual voltages at both DC terminals due to the commutation overlap, and (41) is the DC current flowing through HVDC. Also, (42) is the power factors at the HV bases of HVDC link’s AC sides. Constraints (43), (44) are the DC active powers (which are equal to AC active powers), and the reactive power absorbed by the HVDC link’s AC terminals By neglecting the converters’ losses, respectively.

\( D. \) Load Flow Equations at the Interface of AC/DC Networks

According to Fig. 2, at the inverter side of the HVDC link, the power balance equations of AC/DC networks are as follows \([15],[27]\).

\[
\begin{align*}
P_{G_{i,s}} + P_{d_{i,s}} - P_{L_{i,s}} &= \quad \text{(45)} \\
V_{i,s} \sum_j V_{j,s} V_{ij,s} \cos(\theta_{i,s} - \theta_{j,s} - \gamma_{ij}) &= \quad \text{(46)}
\end{align*}
\]

From Fig.2, at the rectifier side, by neglecting power losses of transformers connecting the wind farm to the HVDC terminal, the power balance equations of the AC/DC systems are as follows:

\[
\begin{align*}
P_{d_{r,s}} &= P_{w,s} \quad \text{(47)} \\
Q_{d_{r,s}} &= Q_{w,s} + B_{sh,r} V_{r,s}^2 + Q_{sh_{r,s}} \quad \text{(48)}
\end{align*}
\]

where, for \( m = r, i \):

\[
\begin{align*}
Q_{sh_{m,s}} &\leq Q_{sh_{m,s}} \leq Q_{sh_{m,s}}^{max} \quad \text{(49)}
\end{align*}
\]

Also, the DFIG-based wind farm power production limits are as follows:

\[
\begin{align*}
0 &\leq P_{w,s} \leq P_{w,s}^{vol} \quad \text{(50)} \\
Q_{w,s} &\leq Q_{w,s} \leq Q_{w,s}^{max} \quad \text{(51)}
\end{align*}
\]

for connection of offshore DFIG-based wind farms to onshore AC power grids, installation of synchronous reactive power compensator at both Rectifier terminals of LCC-HVDC links is essential, because AC voltage source is necessary for proper commutation of thyristor valves. The compensator utilized for this aim is STATCOM \([29]\), which can produce suitable three phase AC voltage at the Rectifier terminals, both in start-up and steady state operation of offshore wind
farms. If the offshore AC grid voltage is kept constant by STATCOM control, then the output active and reactive power of DFIGs can be set for particular rotor (wind) speed to affect a standard variable speed energy capture. In this paper, the reactive power compensator at both terminals of LCC-HVDC link is considered. Referring to Fig. 2 and equations. (46) and (48), it is evidently observed that the compensator is modelled as an AC voltage source which can produce \( Q_{sh_{w,s}} \) at rectifier and inverter AC terminal. Hence, three phase AC voltage source is properly modelled.

E. Scenario based Uncertainty Modelling of Demand and Wind Power Generation

In this paper, a scenario based modeling is proposed to handle the uncertainty of demand and wind power generation [30]. In one hand, the system operator intended to perform PVC with the least imposed cost, and on the other hand, it is necessary to preserve a desired LM. Besides, since different scenarios of wind and demand are considered, it is possible to attain very high PVC cost or very small LM in some scenarios (i.e. risky decisions). Hence, the risk associated with each objective function (i.e. PVC cost and LM) should be considered properly in the proposed SPVC model. In order to do this, CVaR is utilized in this paper as follows

\[
\begin{align*}
\text{min } OF_1 &= \beta \Xi_f(X, \Lambda_s) + (1 - \beta) \Re_c \tag{52} \\
f(X, \Lambda_s) &= TC(s) \tag{53} \\
\Re_c &= \eta_c + \frac{1}{1 - \epsilon} \sum_s \pi_s \zeta_s \tag{54} \\
f(X, \Lambda_s) - \eta_c &\leq \zeta_s \tag{55} \\
0 &\leq \zeta_s \tag{56}
\end{align*}
\]

The equations (52) to (56) are formulated and interpreted based on (7)-(8). In (52), \( OF_1 \) consists of two terms, where the first term is the expected cost of PVC (\( \Xi_f(X, \Lambda_s) \)) in all possible scenarios, and the second term is the corresponding CVaR value (\( \Re_c \)) which is the index of risk. Finally, \( \eta_c \) is the value at risk for cost function.

\[
\begin{align*}
\text{max } OF_2 &= \beta \Xi_g(X, \Lambda_s) + (1 - \beta) \Re_{LM} \tag{57} \\
g(X, \Lambda_s) &= \lambda_s \tag{58} \\
\Re_{LM} &= \eta_{LM} - \frac{1}{1 - \epsilon} \sum_s \pi_s \zeta_{LM} \tag{59} \\
\eta_{LM} - g(X, \Lambda_s) &\leq \zeta_{LM} \tag{60} \\
0 &\leq \zeta_{LM} \tag{61}
\end{align*}
\]

The equation (57) is formulated and interpreted based on (5)-(6). Also, \( OF_2 \) in (57) composed of two terms, in which the first term is the expected value of LM in all possible scenarios (\( \Xi_g(X, \Lambda_s) \)), and the second term (i.e. \( \Re_{LM} \)) is its corresponding CVaR value which is the measure of risk. Besides, \( \eta_{LM} \) in (60) is the value at risk for LM.

Hence, the overall OF to be minimized is defined as the weighted sum of the above risk constrained OFs as follows.

\[
\begin{align*}
OF &= w_1 \frac{OF_1 - OF_{1\max}}{OF_{1\max} - OF_{1\min}} + (1 - w_1) \frac{OF_2 - OF_{2\max}}{OF_{2\max} - OF_{2\min}} \tag{62}
\end{align*}
\]

It is worth noting that \( OF_1 \) should be minimized contrarily to \( OF_2 \). Hence, (62) is formulated in a way that minimizing the overall \( OF \) corresponds to minimization of \( OF_1 \) and maximization of \( OF_2 \), simultaneously.

F. Decision Variables

The decision variables of the proposed probabilistic PVC model include: Active power re-dispatch of pre-determined set of generation units, terminal voltage set point of all generator buses, tap position of load tap changers, active power generation of wind farm, reactive power injections at both terminals along with the tap settings of on-load tap changers of HVDC links. The proposed robust decision making framework finds the optimal values for these variables considering the uncertainty of wind power generation outputs. The sets of control, state and dependent variables are described as follows.

\[
\begin{align*}
\lambda_s &= \begin{cases} \lambda_s & \forall s \\ V_b, \nu_{dn, \nu_{up}} & \forall b \in NB_G \\ T_{m,s} & \forall m = r, i \\ P_{w,s} & \forall s \\ Q_{sh_{w,s}} & \forall m = r, i, \forall s \\ \Delta P_{G_{i,s}}, \Delta P_{D_{i,s}} & \forall i \in NG_{PVC}, \forall s \\ \Delta P_{L_{b,s}}, \Delta P_{D_{i,s}} & \forall b \in NB_{PVC}, \forall s \\ \hat{U} &= \begin{bmatrix} \lambda_s \\ V_b, \nu_{dn, \nu_{up}} \\ T_{m,s} \\ P_{w,s} \\ Q_{sh_{w,s}} \\ \Delta P_{G_{i,s}}, \Delta P_{D_{i,s}} \\ \Delta P_{L_{b,s}}, \Delta P_{D_{i,s}} \end{bmatrix} \tag{63} \\
\hat{X} &= \begin{bmatrix} V_b, \nu_{dn, \nu_{up}} \\ T_{m,s} \\ P_{w,s} \\ I_{d,s} \\ \nu_{dn, \nu_{up}} \\ \nu_{dn, \nu_{up}} \end{bmatrix} \tag{64} \\
\hat{Y} &= \begin{bmatrix} Q_{G_{i}}, \hat{P}_{G_{i}}, \hat{Q}_{G_{i}} \\ S_{t}(V, \theta, s) \end{bmatrix} \tag{65}
\end{cases}
\end{align*}
\]

V. CASE STUDY AND NUMERICAL RESULTS

The proposed probabilistic PVC model is implemented and examined on the New-England 39 bus test systems. The single line diagram of this system is depicted in Fig. 2, and its data are presented in [31]. The initial operating point is also given in [31]. It is assumed that the very short term load forecast is 8% higher than the base-case load level given in [31]. For this load level, LM is very low and the system is in alert operation state. Thus, the PVC is activated to restore normal operation condition, by means of available control measures. In order to determine the LM in each scenario, the loads are increased evenly with \( k_{Lb} = 1 \). Also, active power output of the generators, not hitting their upper generation limits in the base-case, are also increased evenly with \( k_{Gi} = 1 \).

The cost coefficients of generators and demands (in $/MW) to alter their scheduled productions and consumptions have been selected close to the corresponding locational marginal prices (LMPs) values [32]. It is assumed that adjusting the power production up is slightly more expensive than adjusting down the power production for generators [32]. Also, since only adjusting down is allowed in this model for loads, the corresponding cost is assumed to be slightly higher than the corresponding LMP.

The LMPs for this system in different scenarios are given in Tables X and XI (in the Appendix) for nodal active and reactive power injections. These values have been obtained
by MATPOWER [31]. Hence the cost of adjusting up for generators and adjusting down for loads (given in eq. (11)) are assumed to be 5% higher that the corresponding LMP. Also, the cost of power purchased from external network is also treated as the adjusting up cost of generators. Similarly, for adjusting down of power generation, the costs are assumed to be 0.95% of the corresponding LMP.

The generation units located at the following buses are selected by ISO for active power re-dispatch in PVC scenarios: 30, 31, 35, 37, 39. Besides, the following buses are permitted to utmost 20% active/reactive power demand reduction: 4, 8, 15, 16, 20, 21, 29, 39. Also, it is assumed that according to a contract between the ISO and an external network, the maximum amount of active and reactive power import to the network is 600 MW and 400 MVar, respectively. The system is connected to the external network through bus 25, as depicted in Fig. 2.

In this study, an offshore wind farms is considered, with the capacity of 1000 MW. As it is depicted in Fig 2, it is connected to the AC system through bus 16, via a 24-pulse LCC-HVDC link. HVDC link is bipolar with the rating of 1000 MW and 250 kV. The data of this DC links derived from [27].

Using the technique described in [22], the PDF of wind speed is divided into several intervals, and the probability of falling into each interval is calculated. A mean value is also assigned for each interval which is indicator of the corresponding interval. Demand values are also modeled using a normal PDF with a known mean and variance (which is available from load forecasting unit). It is assumed that the load and wind power generation scenarios are independent so the scenarios are combined to construct the whole set of scenarios as follows.

$$
\pi_s = \pi_l \times \pi_w
$$

(66)

where \(\pi_l\) and \(\pi_w\) are the probabilities of \(l\)-th load and \(w\)-th wind scenarios, respectively. Hence, total number of scenarios, is \(\pi_l \times \pi_w\), where \(\pi_l\) and \(\pi_w\) are the number of individual load and wind scenarios. If the number of scenarios are too high then some scenario reduction techniques [33] can be applied to the problem.

In this work, three scenarios are considered for load by using the normal distribution. The mean value of load in each bus, is its forecasted value and the standard deviation is assumed to be 2% of the corresponding mean value. Also, three scenarios are considered for wind power generation, which the probability of all individual scenarios are given in Table II. It should be noted that finding the scenarios which describe all states of wind power generation depends on the wind farm location. Some techniques have been reported in the literature to find these states and the associated probabilities (which are based on the historical data and measurements) [34]. In other words, there is no general table for describing the wind power states (or scenarios). By incorporating these scenarios using (66), 9 mixed wind-load scenarios are attained which are given in Table III.

Active and reactive loads in bus \(b\), as well as the available wind power at scenario \(s\) are calculated as follows.

$$
P_{L_{b,s}} = p_{l,s} \times P_{L_{b}}^{f_r}
$$

(67)

$$
Q_{L_{b,s}} = q_{l,s} \times Q_{L_{b}}^{f_r}
$$

(68)

$$
P_{w,s}^{fr} = p_{w,s} \times P_{w}^{f_r}
$$

(69)

It is also assumed that in scenario \(s\), \(q_{l,s} = p_{l,s}\).

The proposed probabilistic PVC model is implemented in General Algebraic Modeling System (GAMS) environment, and solved by SNOPT solver [35]. The probabilistic PVC is solved for different risk levels and weighting factors of the aforementioned objective functions. Table IV gives the obtained results for different values of \(\beta\) and \(w\). In this study two extreme cases are investigated in detail, namely risk neutral strategy (without concerning about risk, i.e. for \(\beta = 1\)), and risk averse strategy (with fully concerning about risk, i.e. for \(\beta = 0\)). In both cases, the optimal settings of both “here and now” and “wait and see” control variables are presented.

A. Risk neutral (RN) strategy (when \(\beta = 1\))

Using the technique described in [36], the satisfaction level of each objective function is calculated and described in Table V. It is observed from this table that the best compromise solution in this strategy is Sol23. For this optimal solution, the expected total cost and the expected LM are $4345.783 and
0.105, respectively. Also, the optimal re-dispatch of generation units are given in Fig. 3 for this solution. Besides, the values of active power generation re-dispatch of the generation units participating in PVC are presented in Fig. 4.

Active and reactive power outputs of the wind farm in all scenarios ($s_1 - s_9$) are given in Fig. 5. Besides, Fig. 6 shows reactive power purchased from external network (e.g. pool market) in all scenarios for this solution. It is observed from this figure that the purchased power in Scenario $s_9$ is much higher than other scenarios, because in this scenario the demand is 2% greater than the expected value and the available capacity of wind farm is 75% of its nominal value. Also, optimal schedules of active/reactive load reduction at the buses participating in PVC program are depicted in Fig. 7. Besides, the optimal schedule of the HVDC connection for all scenarios are summarized in Table VI. The optimal tap ratios of transformers at both rectifier and inverter sides of the HVDC are as follows: $T_r = 0.515pu$ and $T_i = 0.483pu$.

Table V

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$w$</th>
<th>$OF_1$ ($\text{$/kA$/}$)</th>
<th>$OF_2$</th>
<th>$\mu_{OF_1}$</th>
<th>$\mu_{OF_2}$</th>
<th>$\text{min}(\mu_{OF_1}, \mu_{OF_2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol1</td>
<td>0.00</td>
<td>2842.930</td>
<td>0.265</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Sol2</td>
<td>0.25</td>
<td>28305.435</td>
<td>0.265</td>
<td>0.999</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Sol3</td>
<td>0.50</td>
<td>4345.783</td>
<td>0.106</td>
<td>0.999</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td>Sol4</td>
<td>0.75</td>
<td>680.892</td>
<td>0.042</td>
<td>0.977</td>
<td>0.132</td>
<td>0.132</td>
</tr>
<tr>
<td>Sol5</td>
<td>1.00</td>
<td>59.055</td>
<td>0.012</td>
<td>0.999</td>
<td>0.015</td>
<td>0.015</td>
</tr>
</tbody>
</table>

B. Risk averse (RA) strategy (when $\beta = 0$)

In this case, the risks associated with both objective functions are fully considered. Table VII gives the satisfaction level of each objective function for different values of weights (i.e. $w$). It is evidently observed from this table that in this case solution $\text{Sol}_3$ is dominated and it is the optimal compromise solution with the expected total cost and the expected LM equal to $18108.682$ and $0.219$, respectively. Therefore, in this section the optimal settings of control variables are given for this solution. The optimal settings of generator bus voltages are given in Fig. 8. Also Fig. 9 illustrates active power re-dispatch of generators participate in the PVC.

Active and reactive power outputs of the wind farm connected to bus 16 for different scenarios are also shown in Fig. 10. Besides, Fig. 6 shows reactive power purchased from external network, (e.g. pool market), in all scenarios. Active/reactive load reduction at the selected buses in all scenarios are depicted in Fig. 11. Table VIII gives the optimal schedule of HVDC link in all scenarios for this solution. Also, in this case, the optimal values of tap ratios at both AC sides of the HVDC link are as follows: $T_r = 0.523pu$ and $T_i = 0.491pu$.

Table VII

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$w$</th>
<th>$OF_1$ ($\text{$/kA$/}$)</th>
<th>$OF_2$</th>
<th>$\mu_{OF_1}$</th>
<th>$\mu_{OF_2}$</th>
<th>$\text{min}(\mu_{OF_1}, \mu_{OF_2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol1</td>
<td>0.00</td>
<td>35533.831</td>
<td>0.265</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Sol2</td>
<td>0.25</td>
<td>2826.079</td>
<td>0.265</td>
<td>0.999</td>
<td>0.108</td>
<td>0.108</td>
</tr>
<tr>
<td>Sol3</td>
<td>0.50</td>
<td>18108.682</td>
<td>0.219</td>
<td>0.999</td>
<td>0.829</td>
<td>0.829</td>
</tr>
<tr>
<td>Sol4</td>
<td>0.75</td>
<td>679.361</td>
<td>0.043</td>
<td>0.978</td>
<td>0.161</td>
<td>0.161</td>
</tr>
<tr>
<td>Sol5</td>
<td>1.00</td>
<td>34.080</td>
<td>0.013</td>
<td>0.998</td>
<td>0.049</td>
<td>0.049</td>
</tr>
</tbody>
</table>
C. Discussion on the obtained results

At the above two sections the optimal values of control variables have been proposed in terms of “First stage” control variables (such as generator bus voltages, tap ratios of HVDC’s transformers), and “Second stage” control variables (such as wind farm’s active/reactive power output, load reduction, active power generation re-dispatch and power purchased from external network).

In RN strategy, the probabilistic PVC is solved without concerning about the risks associated with each objective function, and hence it is a risky decision making strategy. In RA strategy, the corresponding risk of each objective function is considered and hence the results obtained from PVC are risk averse. Table IX summarizes the expected values of LM, TC, active/reactive load reduction and power purchased from the external network, along with the power output of wind farm. The following results are deductible from this table.

- By comparing RN and RA strategies, it is observed that in the RN solution, the expected TC is much less than the corresponding value in the RA attitude. It means that considering risk in decision making process, leads to conservative decision with higher control cost.
- At the RA strategy, the expected LM is much higher than the corresponding value at the RN strategy, which is due to the inclusion of the risk through CVaR constraints.
- It is observed that in RA strategy a slightly less wind power output is achieved, which leads to a higher expected load reduction.

![Active/reactive power output (in MW/MVar) of wind farm, in RN strategy.](image1)

![Active (MW) and reactive (MVar) load reduction, in RN strategy.](image2)

![Purchased reactive power (in MVar) from external network, in RN/RA strategy.](image3)

![Active (MW) and reactive (MVar) load reduction, in RN strategy.](image4)
10

Fig. 8. Generator bus voltages (in pu), in RA strategy.

Fig. 9. Active power generation re-dispatch (in MW), in RA strategy. The horizontal axis shows the scenarios.

power generation is scheduled in comparison with the RN strategy, which means that considering the risk does not affect the utilization of renewable energies. Besides, it is observed that in RA strategy more load curtailment is required.

Finally, it is observed that inclusion of risk in PVC leads to a solution with higher LM level (more voltage stability margin), but with the expense of higher control costs.

Table IX

<table>
<thead>
<tr>
<th>Variable</th>
<th>RN</th>
<th>RA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_W$ (MW)</td>
<td>855.000</td>
<td>852.370</td>
</tr>
<tr>
<td>$Q_W$ (MVAR)</td>
<td>436.084</td>
<td>439.092</td>
</tr>
<tr>
<td>$P_{rd}$ (MW)</td>
<td>534.215</td>
<td>534.215</td>
</tr>
<tr>
<td>$Q_{rd}$ (MVAR)</td>
<td>137.578</td>
<td>137.578</td>
</tr>
<tr>
<td>$P_{ex}$ (MW)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$Q_{ex}$ (MVAR)</td>
<td>1.469</td>
<td>1.469</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper a stochastic model is proposed for preventive voltage control (PVC) in AC/DC power systems. The LCC-HVDC link is employed to connect offshore wind farm to the rest of AC IEEE 39-bus test system. The main conclusions are summarized as follows:

- The uncertainties of wind power generation and demand are handled using scenario based uncertainty modelling approach. This would allow the decision maker to avoid unwanted circumstances (increasing cost / decreasing the LM). The conditional value at risk (CVaR) is used as the risk measure.
- Various control variables such as generator bus voltages, tap ratios of HVDC connecting transformers, active/reactive power generation re-Dispatch, load curtailment, power generation of wind farm along with power purchased from external network are considered in the proposed stochastic model of PVC. Some of these variables are "first stage" variables (do not change in various scenarios) and the rest of them are "second stage" decision variables (change in different scenarios).
- The expected LM is maximized while minimizing the corresponding control costs for different levels of risk associated with each objective function.
- The risk averse strategy leads to a solution with higher LM level (i.e. more distance to voltage collapse), but with
the expense of higher corresponding PVC costs. Hence, the proposed model offers an optimal decision making tool for system operators, which makes a compromise between preserving a secure operating point and the imposed control costs.

References


Abbas Rabiee (M14) Received the B.Sc. degree in electrical engineering from Iran University of Science and Technology (IUST), Tehran, in 2006, and the M.Sc. and Ph.D. degrees in electrical power engineering from Sharif University of Technology (SUT), Tehran, Iran, in 2008 and 2013, respectively. Currently, he is an Assistant Professor at the Department of Electrical Engineering, Faculty of Engineering, University of Zanjan, Zanjan, Iran. His research interests include power system operation and security, renewable energies, and the application of optimization techniques in power system operation.

Alireza Soroudi (M14) Received the B.Sc. and M.Sc. degrees from Sharif University of Technology, Tehran, Iran, in 2002 and 2004, respectively, both in electrical engineering, and Ph.D. degree from Grenoble Institute of Technology (Grenoble-INP), Grenoble, France, in 2011. He is the winner of the ENRE Young Researcher Prize at the INFORMS 2013. He is currently a senior researcher with the School of Electrical, Electronic, and Mechanical Engineering, University College Dublin with research interests in uncertainty modeling and optimization techniques applied to Smart grids, power system planning and operation.

Andrew Keane Andrew Keane (S04M07-SM’14) received the B.E. and Ph.D. degrees in electrical engineering from University College Dublin, Ireland, in 2003 and 2007, respectively. He is currently a Senior Lecturer with the School of Electrical, Electronic, and Communications Engineering, University College Dublin. He has previously worked with ESB Networks, the Irish Distribution System Operator. His research interests include power systems planning and operation, distributed energy resources, and distribution networks.