<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Allowing for a rocking datum in the analysis of drive-by bridge inspections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Authors(s)</strong></td>
<td>Keenahan, Jennifer; O'Brien, Eugene J.</td>
</tr>
<tr>
<td><strong>Publication date</strong></td>
<td>2014-08-29</td>
</tr>
<tr>
<td><strong>Conference details</strong></td>
<td>Civil Engineering Research in Ireland, Belfast, UK, 28 - 29 August, 2014</td>
</tr>
<tr>
<td><strong>Link to online version</strong></td>
<td><a href="http://www.cerai.net/">http://www.cerai.net/</a></td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/6930">http://hdl.handle.net/10197/6930</a></td>
</tr>
</tbody>
</table>
Allowing for a Rocking Datum in the Analysis of Drive-by Bridge Inspections

J. KEENAHAN & E.J. O'BRIEN

Dept. of Civil, Structural and Environmental Engineering, University College Dublin, Ireland

Abstract

‘Drive-By’ damage detection is the concept of using sensors on a passing vehicle to detect damage in a bridge. At highway speeds, the vehicle spends a short amount of time on the bridge: it may not even go through a full cycle of vibration, resulting in only a partial signal of the bridge motion being detected. Given that the spectral resolution of standard signal processing techniques depends on the length of data in the signal, they cannot be used to identify the bridge frequency accurately. In addition, the nonlinear and non-stationary nature of the vehicle-bridge interaction system poses challenges. The aim of this study is to model a ‘drive-by’ bridge inspection approach using a beam in free vibration. An optimisation approach is proposed in numerical simulations as an alternative to standard signal processing techniques to overcome the challenges of short signals and the nonlinear nature of the drive-by system.

Keywords: bridge, damage, drive-by, inspection, SHM, short

1. Introduction

‘Drive-by’ bridge inspection (Kim and Kawatani, 2009) involves the instrumentation of a vehicle, rather than the bridge, in order to assess bridge condition. A significant limitation to date has been the need for the vehicle to traverse the bridge at low speeds (Lin and Yang, 2005; Toshinami et al., 2010). At highway speeds, the vehicle spends a short amount of time on the bridge which may not go through a full oscillation during the time that the vehicle is on it. As a result only a partial signal of the bridge motion is detected.

Many bridge damage detection methods use Fourier analysis as the principal signal-processing tool (Staszewski and Robertson, 2007). However, Fourier analysis has several shortcomings; it is unable to accurately represent non-periodic functions (Pines and Salvino, 2006), non-stationary functions (Qian and Chen, 1999) and it requires linearity. This is a challenge as available data in the ‘drive-by’ context is from a nonlinear system (Huang et al. 1998). Measured signals produced resulting from structural damage are of a non-stationary nature (Staszewski and Robertson, 2007) and the signals are short in duration (Kim and Melhem, 2004).

The Short Time Fourier Transform (STFT) was later developed for processing non-stationary signals. The width of the window remains constant throughout the analysis and therefore represents a compromise between time and frequency-based views of a signal. Cerda et al. (2012) examine the ‘drive-by’ concept. Vertical acceleration signals from the vehicle are processed using the STFT, and results show that low vehicle speeds are needed to accurately identify changes in bridge natural frequency.

The Wavelet Transform represents the next logical step in the development of signal processing methods: a windowing technique with variable size windows. Other basis functions, such as the Haar, Mexican hat, Coiflet, Daubechies and Morlet, can be
used (Kim and Melhem, 2004). However, the Wavelet Transform still suffers from the convolution of the signal with an a priori basis function (Ayenu-Prah and Attoh-Okine, 2009) as available wavelet dictionaries are often not appropriate for analysing the nonlinear behaviour of many structural systems (Salvino et al. 2005).

The Hilbert Transform has long been proposed as an advanced signal processing technique (Huang et al. 1998). It is the convolution of the signal with the function \( \frac{1}{\pi t} \). However, a drawback in the ‘drive-by’ context is that it requires a signal where the instantaneous frequency does not change with time, and intra-wave frequency modulation is typical of nonlinear systems (Cantero and OBrien, 2013). A more recently developed method, the Hilbert-Huang Transform (HHT), has been proposed to overcome this challenge. The HHT uses Empirical Mode Decomposition (EMD) to decompose the signal into functions where the frequency does not change with time and then applies the Hilbert Transform. However, authors that have used the HHT with EMD in a moving load context have found that the speed of the vehicle negatively impacts the success of damage detection (Bradley et al. 2010).

This limitation of vehicle-bridge interaction in detecting the modal properties of a bridge cannot be overcome by increasing the scanning frequency of the sensors (Keenahan et al., 2014). The underlying problem is that often the bridge has not undergone a full period of its fundamental frequency in the time it takes the vehicle to cross the bridge. Therefore, the only practical solution, if standard signal processing is to be used, is to decrease the velocity of the vehicle to capture more oscillations. In this paper, an optimisation approach is proposed as an alternative to standard signal processing techniques to overcome the challenges of short signals, an example of which is the nonlinear system of ‘drive-by’ monitoring at highway speeds.

2. Optimisation as an Alternative to Standard Signal Processing Techniques

The concept of using an optimisation approach to determine the bridge frequency is based on minimising the sum of the squares of the differences between measured data and a theoretical expression, with frequency as the decision variable. Initially, the equation for a beam in free vibration is used to generate the measured data as this is comparable to the motion of a bridge exposed to the passing of a vehicle of much less mass:

\[
u(x,t) = C \cos(\omega t - \alpha) \sin \left( \frac{n\pi x}{L} \right) \tag{1}\]

where \( u(x,t) \) is the displacement at position, \( x \), and time, \( t \), \( C \) is a constant representing the amplitude, \( \omega \) is the frequency of vibration, \( \alpha \) is a phase angle, \( n \) is the mode number and \( L \) is the length of the beam. The frequency of vibration, in rad s\(^{-1}\), is determined from:

\[
\omega = \left( \frac{n\pi}{L} \right)^2 k \tag{2}
\]

where \( k \) is the square root of the ration of beam stiffness to beam mass. Taking values of \( C = 1, \alpha = 0, k = 50, L = 10 \) and \( n = 1 \), the frequency is analytically determined from equation (2) to be 4.9348 rad s\(^{-1}\). Beam displacements are determined using equation (1) for \( x \) varying in 0.1 m increments, and \( t \) varying in 0.005 s increments,
corresponding to a speed of 20 m s\(^{-1}\). The value for \(C = 1\) is chosen so that the displacements are normalised.

### 3. One mode of vibration

Initially, a single sensor is visualised on a vehicle model, that can measure absolute vertical translation of the bridge. For the purpose of determining the data that this sensor would read, it was envisaged that the vehicle model would traverse the beam, but with no vehicle-bridge interaction. Figure 1 illustrates the data that would be read by the sensor if it were present on the vehicle.

**Figure 1:** Bridge displacements read by the sensor on the vehicle model: (a) vehicle at start of bridge; (b) vehicle at 0.2\(L\); (c) vehicle at 0.4\(L\); (d) vehicle at 0.6\(L\)

In Figure 1(a), the sensor on the vehicle model reads the displacement at position \(x = 0\) m at time \(t = 0\). In Figure 1(b), the vehicle has moved 0.2\(L\) along the bridge. The displacement read by the sensor is from the second time curve (triangular data points) at \(t = 0.2\), enclosed by the dark circle. In Figure 1(c), the vehicle has moved even further along the bridge and the displacement is read from the third time curve (square data points) at \(t = 0.4\) and at position \(x = 0.4L\). Scans of the bridge deflections are taken by the moving vehicle model at a spacing step of 0.1 m and a time step of 0.005 s, and are shown in Figure 2.
An optimisation approach, which seeks to minimise the difference between measured data and a theoretical expression, is used. The stream of displacements in Figure 2 is taken as the measured data and equation (1) as the theoretical expression. The optimisation approach correctly determines the unknown bridge frequency in equation (1) to be 4.9348 rad s\(^{-1}\).

Global damage is simulated as a change in the ‘\(k\)’ parameter of equation (2). The actual frequencies for each damage level are determined analytically from equation (2). These frequencies, along with those determined by the optimisation approach, are presented in Table 1 and are identical at five places of decimal.

**Table 1** - Actual and inferred frequencies for a range of damage levels

<table>
<thead>
<tr>
<th>(k)</th>
<th>Actual Frequency (rad s(^{-1}))</th>
<th>Frequency from optimisation (rad s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4.9348</td>
<td>4.9348</td>
</tr>
<tr>
<td>49</td>
<td>4.8361</td>
<td>4.8361</td>
</tr>
<tr>
<td>48</td>
<td>4.7374</td>
<td>4.7374</td>
</tr>
<tr>
<td>47</td>
<td>4.6387</td>
<td>4.6387</td>
</tr>
<tr>
<td>46</td>
<td>4.5400</td>
<td>4.5400</td>
</tr>
</tbody>
</table>

Figure 3 illustrates the scanned bridge deflections from the moving vehicle model for different damage levels. There is a clear distinction between the curves, even for small changes in damage.
4. Two modes of vibration

The inclusion of a second mode of vibration results in a more complex expression for the displacement of the beam in free vibration:

\[ u(x, t) = C_1 \cos(\omega_1 t - \alpha_1) \sin\left(\frac{1\pi x}{L}\right) + C_2 \cos(\omega_2 t - \alpha_2) \sin\left(\frac{2\pi x}{L}\right) \]  

(3)

where \( C_1 \) and \( C_2 \) represent the amplitudes of the first and second mode respectively, \( \omega_1 \) and \( \omega_2 \) are the frequencies and \( \alpha_1 \) and \( \alpha_2 \) are the phase angles. Using the parameter values of Table 2, Figure 4 gives the deformed shape of the bridge at a range of points in time. As before, the value of \( C_1 = 1 \) is chosen such that the mode one displacements are normalised, and the amplitude of mode two is taken to be one tenth of the amplitude of mode one.

Table 2 - Parameter values for two modes of vibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0</td>
<td>0.7854</td>
</tr>
<tr>
<td>( n )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( k )</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>( L )</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
As before, a vehicle containing one sensor is considered to be traversing the beam at a speed of 20 m s\(^{-1}\) and scanning deflections every 0.005 s. Global damage is again simulated as a change in the \(k\) parameter of equation (2). Figure 5 illustrates a scan of the bridge deflections from the vehicle for different levels of damage.

It clearly illustrates that damage can be detected from a scan of bridge deflections read by a sensor on the vehicle model. A similar optimisation approach is used in this example where there are now five unknown variables; two frequencies, two amplitudes and the difference in phase angle between the modes. It is assumed here that the phase angle of the first mode is zero. The variables, correctly determined from the optimisation procedure, are given in the Table 3 and exactly match the actual values in the number of significant digits shown.

**Figure 4:** Beam displacements for a beam in free vibration for two modes

**Figure 5:** Bridge deflections from the moving vehicle for different levels of damage for two modes of vibration

**Table 3** - Inferred frequencies, amplitudes and phase difference for a range of damage levels
To gain an understanding of the conditioning of the system, a plot of the contours of the objective function (considering only the two frequencies as variables) is shown in Figure 6, where the objective function is the sum of the squares of the differences between measured and theoretical data.

The figure shows that the optimisation approach can find the global minimum in the range of frequencies from 0 to 20 rad s\(^{-1}\). The minimum is indicated in the figure by the white cross. The objective function is highly sensitive to the first natural frequency but insensitive to the second. However, it should be noted that it is the frequency of the first mode that will be used for damage detection and so the accuracy of the inferred second natural frequency is unimportant.

5. Conclusions

This paper investigates the feasibility of using an instrumented vehicle model to detect damage in a bridge. An optimisation approach is proposed as an alternative to standard signal processing techniques because of the brevity of the signal. Simulations show that modest losses of stiffness in the bridge can be detected using the vehicle measurements. Overall the results presented in the paper indicate that the method has the potential to be developed as an effective tool for the monitoring of bridge damage.

References


