Application of output-only modal method in monitoring of bridges using an instrumented vehicle

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ABSTRACT: In this paper, application of a well-known output-only modal analysis method called Frequency Domain Decomposition (FDD) method in monitoring of bridge frequency is presented. The obtained modal data can be used efficiently for bridge health monitoring. Three measurement strategies are suggested to collect the acceleration responses from vehicle axle for FDD method in a numerical study. It is shown that using multi vehicles leads to better results in compare to using one vehicle. The efficient effect of ambient excitation to all sides of the bridge is also discussed. In addition, application of FDD method in the case of closeness of vehicle frequency to bridge frequency is investigated. Finally, it is discussed that the FDD method can be an efficient alternative to FFT analysis which is common for analysing the vehicle measurement passing over the bridge.

KEY WORDS: Vehicle Bridge Interaction; FDD method; Instrumented vehicle; Health monitoring.

1 INTRODUCTION
Structural health monitoring of bridges is critical for their maintenance and for decisions on repair and rehabilitation. Many damage detection methods are based on the concept that damage involves changes in bridge structural properties such as stiffness, which cause detectable changes in modal properties of the structure. Therefore, doing a modal testing to identify modal properties of the structure before and after occurring damage is necessary in these methods [1]. For the structure such as bridge, output-only modal testing in which only response of the bridge are required, is more common rather than forced-vibration modal testing methods that require force history as well. However, with these methods, the structure should ideally be excited artificially at different points using a controlled or ambient force and responses should be measured at many points [2]. Although, these methods are currently being used practically, but they need many efforts, operators, equipments and also analysing huge amount of data.

Recently, the possibility of using bridge-vehicle interaction data measured on a vehicle to estimate the dynamic properties of a bridge has gained considerable interest [3]. The method is based on installing sensors on the vehicle instead of the bridge which is a low cost alternative to conventional structural health monitoring [3, 4]. This method is based on the Fast Fourier Transform (FFT) of the measured response that is the basic frequency analysis tool for such a short segment of response. It means natural frequencies of the bridge can be obtained using FFT spectrum of the measured response from travelling vehicle. Inspired by Yang’s work, several works have been proposed for bridge health monitoring based on the dynamic response of the passing vehicle over the bridge [5, 6, 7 and 8].

As in such these methods, the modal properties of the bridge are extracted only from bridge response that appeared in vehicle response, therefore, it can be considered as output-only modal analysis. In output-only modal analysis (or ambient modal analysis) modal data can be obtain using only measured responses, i.e. ,there is no need to measure or control the input forces [2]. While using output-only data simplifies the damage detection setup, it is still required to attach sensors to the bridge structure and analyse a large quantity of data.

In this paper, the concept of bridge monitoring based on output-only modal analysis methods is investigated using the response from sensor installed on the vehicle. Vehicle bridge dynamic interaction is modelled using finite element method (FEM). Frequency Domain Decomposition (FDD) method which is one of the most well-known output-only methods in frequency domain [9], is used in this paper. FDD method is applied to the vehicle acceleration response during passing over the bridge. Three strategies are suggested to provide data for application of FDD method. At the first strategy, measured acceleration response from one passing instrumented vehicle is used as a single measurement channel. At the second strategy, obtained vehicle responses from repeating same simulation of passing vehicle over the bridge in different ambient excitation force is used. For the third strategy, two connected quarter-cars equipped with accelerometers are simulated crossing the bridge. For all strategies frequency analysis using FFT and FDD methods are applied to the measured response of vehicle to monitor first natural frequency of the bridge. The obtained results show that application of the FDD method for monitoring bridge frequency has many advantages in comparing the classical FFT method.
2 VEHICLE BRIDGE INTERACTION MODEL

In order to assess application of FDD method, the dynamic response of the instrumented vehicle will be generated using numerical simulation (FEM). The FE model and procedure of calculating the vehicle dynamic responses are explained following.

The two-dimensional model of vehicle bridge interaction, which is used in this study, is shown in Fig. 1. A sprung mass $m_i$ of single degree of freedom (DOF) supported with an elastic spring $k_i$ is used to model passing vehicle. The bridge is modeled as a simply supported beam of length $L$, flexural rigidity $EI$ and mass per unit length $m^*$. A road profile is included in simulations and the irregularities of the profile are randomly generated according to the ISO standard [10] for a road class ‘A’ (very good profile, as expected in a well maintained highway).

A FE vehicle bridge interaction procedure that is explained in the literature [11] is used to model travelling of the vehicle over the bridge. The dynamic equations of motion of the VBI system are solved by Wilson-$\theta$ method step by step. The optimal value of the parameter $\theta = 1.42$ is used for unconditional stability in the integration schemes [12].

3 OUTPUT-ONLY MODAL METHODS

In the past few years, out-put only modal analysis (or operational modal analysis) has received a considerable attention. The most important advantage of this methods is, only the response signals are measured and the test structure is excited by the ambient forces [2]. Therefore, there is no need to measure the excitation forces. This method is mostly used for huge structures such as buildings, bridges and towers in which it is not possible to excite the structure artificially and also the measurement of excitation force is not possible [2].

3.1 Frequency Domain Decomposition (FDD) method

FDD is an output-only modal analysis method in the frequency domain that first proposed by Brincker et al. in [9]. It is assumed that the input force of the structure is a white signal. For low damping, the power spectral density matrix of outputs of the structure can be defined as $G_{yy}(j\omega)$. The dynamic behaviour of a structure is dominated by one of its mode close to the corresponding natural frequency. Therefore, the response of structure is similar to its mode shape close to each natural frequency. In order to obtain the natural frequencies and mode shapes, the power spectral density matrix of the response in each frequency is decomposed by taking Singular Value Decomposition (SVD) of the matrix as [9]:

$$\hat{G}_{yy}(j\omega) = [U][\Sigma][U]^H$$

where the matrix $[U] = \{[u]_1, [u]_2, \ldots, [u]_m\}$ is the unitary matrix including the singular vectors $[u]_{ij}$ and $[\Sigma]$ is a diagonal matrix including singular values $\sigma_{ij}$. Therefore, if singular values obtained from outputs of the structure plotted, dominant peaks are belonged to natural frequencies of the structure and corresponding singular vectors are mode shapes of the structure. If an improved version of FDD called Enhanced FDD (EFDD) is used, damping ratios of the structure can be estimated as well [9].

4 NUMERICAL ASSESSMENT

In this section, frequency analysis of the bridge using vehicle response is investigated using both FFT and FDD methods. The case study of the simulation is defined using vehicle and bridge properties. Following three strategies are suggested to use for FDD method.

4.1 Case study

A VBI system is shown in Fig. 1 is used in this study. The bridge properties are listed in Table 1.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Symbol</th>
<th>Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>m</td>
<td>$L$</td>
<td>30</td>
</tr>
<tr>
<td>Mass per unit</td>
<td>Kg</td>
<td>$m^*$</td>
<td>2000</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>GPa</td>
<td>$E$</td>
<td>27.5</td>
</tr>
<tr>
<td>Second moment of area</td>
<td>m^4</td>
<td>$I$</td>
<td>0.175</td>
</tr>
</tbody>
</table>

First three natural frequencies of the bridge obtained from FE method are given in Table 2.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.70</td>
</tr>
<tr>
<td>2</td>
<td>10.82</td>
</tr>
<tr>
<td>3</td>
<td>24.36</td>
</tr>
</tbody>
</table>

A typical vehicle model of sprung mass is used in this study to represent the behaviour of the vehicle interacting with the bridge. The vehicle properties are listed in Table 3.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Symbol</th>
<th>Vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Kg</td>
<td>$m_v$</td>
<td>1000</td>
</tr>
<tr>
<td>Stiffness</td>
<td>N/m</td>
<td>$k_v$</td>
<td>170×10^4</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hz</td>
<td>$\omega_1$</td>
<td>6.56</td>
</tr>
</tbody>
</table>
When road profile is used in the simulation, the vehicle frequency is excited and consequently is dominant to the measured acceleration response of the vehicle [13]. Therefore, sometimes it is hard to detect the bridge frequency from the vehicle response. As it was suggested by [13], in order to improve the bridge response portion to the measured response of the vehicle, random excitation are applied to other sites of the bridge that deals to improving the bridge frequency at the vehicle acceleration.

The simulation carried out using FE method in MATLAB software. The sampling time interval is selected to be 0.0005 to provide enough data for accurate identification. Therefore, total length of the acceleration response of the vehicle is 15 s. The simulated acceleration response of the vehicle during its travelling over the bridge is shown in Fig. 2 versus the vehicle distance from left side of the bridge.

![Figure 2. Acceleration response of the vehicle.](image)

In order to analysis the vehicle response, the fast Fourier transform (FFT) is performed to the simulated vehicle response based on Yang's idea [3]. The obtained FFT spectrum of the vehicle response is shown in Fig. 3.

![Figure 3. FFT spectrum of the vehicle acceleration response.](image)

As it was expected [13], the dominant peak of the spectrum corresponds to the vehicle frequency around 6.56 Hz that is mentioned in Table 3. In addition, a peak that is corresponded to the first natural frequency of the bridge is detectable around 2.7 Hz.

As it is observed in Fig. 3, when the raw amplitude of the spectrum is used for plotting the spectrum as y axle values, it is hard to pick the peak that is corresponded to the bridge frequency. Therefore, it is suggested to use logarithmic unit for plotting the spectrum. The FFT spectrum of the vehicle response is plotted again with new logarithmic values of the amplitude in Fig. 4. It can be seen that peak picking of the frequencies is obviously simpler in this view. Against the Fig. 3 in which a small peak is available for the bridge frequency, a sharp and clearly detectable peak is appeared for the bridge frequency in the logarithmic view of the spectrum.

![Figure 4. FFT spectrum of the vehicle acceleration response in db.](image)

4.2 **Strategy 1: Application of FDD method using single vehicle measurement**

In this section, FDD method is performed on the same simulated signal which is shown in Fig. 2.

As it was explained in section 3.1, first power spectral density matrix should be constructed from the simulated response. At the first strategy, FDD method is applied on the same simulated vehicle response that was used in section 4.1. Then sigular values can be extracted using SVD of the power spectral density. Obtained SVD diagram is shown in Fig. 5.

![Figure 5. SVD diagram of the vehicle acceleration response.](image)
It is shown in Fig. 5 that both vehicle and bridge frequency can be detected using FDD method. By comparing figures 4 and 5, it can be concluded, although bridge and vehicle frequencies are detectable in both diagram, but FDD method presents a very smooth and explicit curve in which aimed peaks are clearer than FFT method. Therefore, it is recommended to use FDD method instead of using FFT in identification of bridge frequency from measured vehicle response.

4.3 Strategy 2: Application of FDD method using multiple vehicle measurements from repeating the simulation

As explained before, FDD is an output-only modal analysis method. This method usually is used as a multi-channel method and rarely used using only one signal. It means in classical use of the method, several responses that are measured from installed sensors to several points of the structure, would be used as input of the FDD method. Therefore, it is possible to use more than one response in FDD method.

In this section, the explained test of section 4.2 are repeated for four times and the response of the vehicle is measured in each time separately. The only difference of these four simulations is that in each simulation different random excitation is applied to the bridge. Using multiple signals of the vehicle leads to some advantages. First, more data can be provided which causes to more accurate results. Particularly, when the speed of the vehicle is high and consequently the length of the time history is not large enough for a successful analysis. In addition, using multi-channel signals for the FDD method, gives multi SVD diagram that includes peaks at the natural frequencies. These extra peaks can be very useful in the case that operator is not sure about existence of frequency at the special peak. It means if SVD peak is occurred in all SVD curves, that peak can be detected as the frequency of the system with high assurance. Moreover, in the case that the vehicle frequency is close to the bridge natural frequency, availability of more data can be helpful for accurate identification of the frequencies.

The obtained SVD diagram from performing FDD method on the four times simulated responses is shown in Fig. 6. It can be observed in Fig. 6 that SVD diagram of this section has four curves which all of them have peaks at the vehicle and bridge frequencies.

It can be concluded that using multiple measurements leads to more data that is useful in the more complicated situations.

4.4 Strategy 3: Application of FDD method using measurements from two connected vehicles

As it was mentioned in the previous section, the obtained responses from repeating same simulations, are approximately same except in their random ambient excitations. Although, four different responses are obtained from repeating same simulation, but it would be more desired if totally different signals can be used.

![Figure 7. Two connected vehicle travelling over the bridge.](image)

In order to improve the efficiency of using multiple responses for FDD method, in this section, measured signals from two connected vehicles with same properties is used inspired by work of Yang et al. [14]. Therefore, two connected vehicles with same properties and constant distance $s=6$ are travelling over the bridge as shown in Fig. 7. Using
different vehicles causes to two different vehicle frequencies that makes harder to investigate the spectrum. Thus, in this section, properties of the vehicles are considered to be same as what is given in Table 3.

It is assumed that the simulation starts when vehicle 2 is at the left side of the bridge and finishes when the vehicle 1 arrives at the right side of the bridge. Therefore, the total path that each vehicle travelled in the simulation is $L = 24m$ and time length of each response is 12 s. The obtained simulated responses of two vehicles are shown in Fig. 8.

The FDD method is applied to the vehicle responses. Obtained SVD diagram is shown in Fig. 9. It is observed that there are two SVD curves which both have peaks at bridge and vehicle frequencies. Focusing on the second SVD curve in this strategy, it can be found that level of second curve is improved at the frequency peaks in compared with Fig. 6. This is because of using totally different data in FDD method. Therefore, the second SVD diagram is also able to help the operator during analysis of the frequencies.

A new vehicle with new properties is used in this section to decrease the frequency of the vehicle. Properties of the new vehicle are selected in the way that vehicle frequency be close to the bridge frequency (Given in Table 4). The strategy 3 using two connected vehicles is used in this simulation.

The FFT spectrum of the measured response of first vehicle is shown in Fig. 10. It is obvious that closeness of vehicle and bridge frequencies results an unsmooth and jagged spectrum curve and it is hard to detect proper peaks of the spectrum. It means that FFT analysis has weakness in investigating closely-spaced modes.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Symbol</th>
<th>Vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Kg</td>
<td>$m_v$</td>
<td>4000</td>
</tr>
<tr>
<td>Stiffness</td>
<td>N/m</td>
<td>$k_v$</td>
<td>$170\times10^4$</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hz</td>
<td>$\omega_1$</td>
<td>3.28</td>
</tr>
</tbody>
</table>

Table 4. Properties of the new vehicle.

In order to take advantages of the FDD method, it is recommended to use strategy 3. This strategy can be extended to use more than two vehicles or using multi-axle vehicle that is more practical in compared with connected vehicles.

4.5 Application of FDD in case of closeness of vehicle frequency to the bridge frequency

One challenge of estimation of bridge frequency using passing vehicle response is existence of vehicle frequency [15]. It means that vehicle frequency usually appears dominant to the spectrum. When, the frequency of the vehicle is close to the bridge frequency, although it causes to productive excitation of bridge frequency in the spectrum, but, several peaks would be appeared to the spectrum that makes it hard to process it and identify the frequencies.

On the other hand, FDD method has advantage of dealing with closely-spaced modes [16]. It means this method showed an efficient results in the case of closely-spaced modes. Therefore, it is suggested in this section, that use of the FDD method can be helpful in the case of closeness of the vehicle frequency to the bridge frequency.
The simulated responses obtained from two connected vehicles are used in FDD method similar to the procedure suggested in section 4.4. The SVD diagram is obtained using FDD method shown in Fig. 11. It is clear that the FDD method reveals a smooth diagram even in the case of closeness of vehicle and bridge frequencies. Therefore, in this special case, the FDD method is more effective in compared with FFT analysis.

5 CONCLUSION

In this paper potential of application of the FDD method in identification of bridge frequency using response of passing vehicle was investigated. Several strategies have been suggested to take advantage of using multi-channel response in FDD method. It was shown that using measured responses from different vehicles or from different axles could lead to more efficient results in comparing with repeating same simulations with different random excitation of bridge. In addition, efficiency of the FDD method in the case of closeness of bridge and vehicle frequencies was investigated. It was shown that the FDD method has advantage of identifying both bridge and vehicle frequencies in this case, while classical FFT analysis is disable to reveal the frequencies clearly. Therefore, it can be concluded that the FDD method can be an efficient alternative to the FFT for bridge identification using vehicle response.

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This optional section contains acknowledgments.

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