Spatially variable assessment of lifetime maximum load effect distribution in bridges

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ABSTRACT: Bridge structures are key components of highway infrastructure and their safety is clearly of great importance. Safety assessment of highway bridges requires accurate prediction of the extreme load effects, taking account of spatial variability through the bridge width and length. This concept of spatial variability is also known as random field analysis. Reliability-based bridge assessment permits the inclusion of uncertainty in all parameters and models associated with the deterioration process. Random field analysis takes account of the probability that two points near each other on a bridge will have correlated properties. This method incorporates spatial variability which results in a more accurate reliability assessment.

This paper presents an integrated model for spatial reliability analysis of reinforced concrete bridges that considers both the bridge capacity and traffic load. A sophisticated simulation model of two-directional traffic is used to determine accurate annual maximum distributions of load effect. To generate the bridge loading scenarios, an extensive Weigh-in-Motion (WIM) database, from five European countries, is used. For this, statistical distributions for vehicle weights, inter-vehicle gaps and other characteristics are derived from the measurements, and are used as the basis for a Monte Carlo simulation of traffic. Results are presented for bidirectional traffic, with one lane in each direction, with a total flow of approximately 2000 trucks per day.

KEY WORDS: Random field, spatial variability, traffic, load, bridge, probabilistic, safety, assessment.

1 INTRODUCTION

The response of reinforced concrete (RC) bridge structures depends on many uncertain factors such as traffic load, environmental condition, material properties (concrete strength, concrete density), concrete cover and geometric dimensions. The response is considered satisfactory when the desired limit state (such as load carrying capacity) is fulfilled with an acceptable degree of certainty.

Probabilistic assessment of limit state violations at any stage during a structure’s lifetime is often done using reliability analysis. The reliability assessment of a bridge is a complex problem requiring the modelling of aleatory and epistemic uncertainties existing in both load and response to the load. These uncertainties fluctuate within the length, width and depth of the structure and the response of the bridge is accordingly affected by these fluctuations. Random field analysis is a means of mathematical representation of the spatial variability which takes account of the correlation between any two individual elements.

Random field analysis offers significant advantages such as rational assessment of bridge deck safety that can produce a whole design space instead of just a one point result. It accounts for the contribution of spatial variability of load and resistance to the overall reliability of the bridge structure.

Methods of evaluating the spatial capacity of bridges has received a great deal of attention (Karimi et al., 2005, Keshal, 2009, Li et al., 2004, Vu, 2003). However, the uncertainty associated with the load model has not been considered extensively with the exception of Keshal (2009) who provides an insight on methods used to generate the load effects from specific traffic data obtained using Weigh in Motion (WIM).

WIM is the process of measuring the weights of trucks travelling at full highway speed. The WIM traffic records in this study are used to calibrate a comprehensive model of traffic loading. In conventional approaches statistical distributions such as those from the Extreme Value (EV) family are fit to measured or simulated block-maximum load effects. One day is widely used as a block size in EV distributions such as the Weibull or Type 2. A Monte Carlo (MC) simulation approach is used here to develop a very efficient algorithm simulating different bridge loading scenarios, allowing many lifetimes – thousands of years of traffic – to be simulated. The resulting lifetime maximum distribution can be expected to be considerably more accurate than that calculated using the conventional approach.

This paper presents a method of probabilistic assessment of safety which considers an entire 2-dimensional bridge deck, including an allowance for the spatial variability of load effect. The bridge deck is divided into rectangular segments with properties assumed to be statistically independent. Once load effect and resistance distributions are found for each individual area, the theorem of total probability is used to determine the probability of failure. The failure criteria used in this paper is defined as that of load effect exceeding resistance.

2 RANDOM FIELD ANALYSIS

Structural safety assessment depends on a number of structural properties and operating conditions. Furthermore, these properties and conditions exhibit random variation across the surface of a structure. There can be variation due to the quality of concrete properties, concrete cover,
inconsistency in traffic load, environmental condition, workmanship, etc. (Kenshal, 2009, Vu, 2003). Traditionally, these random variables are modelled deterministically with a single characteristic value which is unable to account for their variations (Vu, 2003). Homogeneous random variables, with associated probability density functions, is another conventional approach to modelling. This takes account of the inherent variability of properties (Melchers, 1999). Hence, spatial variability of variables across the length and width of a structure is simply ignored (Kenshal, 2009, Vanmarcke & Grigoriu, 1983, Vu, 2003, Vu & Stewart, 2005). Neglecting such sources of uncertainty has a significant impact on the evaluated safety (Kenshal, 2009). Thus, it is important to include the spatial variability of parameters in a safety assessment.

Random field analysis takes account of the probability that any pair of points corresponding to any locations within the structure will have correlated properties. This is achieved by discretising the corresponding random fields into a set of spatially correlated random variables. Several methods have been proposed for the discretisation of random fields into random variables. The midpoint method, which represents the random field by the value at the centroid, is widely used in the literature. This method is relatively easy to implement, provides numerically stable results and is applicable to Gaussian and non-Gaussian random fields.

The statistical correlation between any pair of elements can be modelled through the use of a mathematical function, termed the autocorrelation function. This function is based on the correlation characteristic of the corresponding random field. The autocorrelation function represents the correlation coefficient between two elements separated by the interval $\xi$. Different models may be used to define the spatial correlation between two elements within the given structure. The exponential autocorrelation function has been used widely to represent the spatial variability of material properties and loading in other fields of engineering (Hisada & Nakagiri, 1985, Kersner et al., 1998, Liu & Der Kiureghian, 1989, Mahadevan & Haldar, 1991, Yamazaki, 1988). For a 2-dimensional random field, it is defined as:

$$
\rho(\xi) = \exp\left(-\frac{(|\xi|)^2}{d_x^2}\right) - \left(\frac{|\xi|}{d_x^2}\right)
$$

where $d_x$ and $d_y$ are the correlation lengths for a 2-dimensional random field in the $x$ and $y$ directions respectively. The distances between the centroid of elements $i$ and $j$ are $\xi_x = x_i - x_j$ and $\xi_x = y_i - y_j$ in the $x$ and $y$ directions respectively.

It should be noted that some parameters, such as steel strength, have low spatial variability due to quite strict quality control whereas adopting certain quality control measurement for other parameters does not have significant influence, e.g., concrete strength and cover. As a result, it is not necessary to consider all parameters as spatially random. Sensitivity analysis can be used to determine how the uncertainty in the probability of failure can be influenced by the spatial variability of different parameters.

This general method of random field analysis has been adopted by various authors. Vu & Stewart (2005) divide the structural element into $N$ statistically independent areas (i.e., with no correlation between these areas). Each independent area is considered as a random field (see Figure 1). The optimal size of the discretised elements needs to be specified for random field analysis. This size of element should not be too large to avoid underestimation of spatial variability (Vu, 2003). On the other hand, numerical problems associated with the decomposition of a large covariance matrix will result from choosing very small elements.

![Figure 1. Independent areas and correlated elements](image)

For this study, the authors adopt an approach similar to that of Vu & Stewart (2005), combining the spatial variability of resistance with spatial variability of loading across a bridge deck. The main focus of this paper is the spatial variability of the load model and to highlight the significant influence of this variability on the result.

### 3 SAFETY ASSESSMENT

Structural reliability consists of the following steps. The relevant limit state is first identified and the structure broken into $N$ independent elements. The load and resistance model is developed for each element and the autocorrelation function specified. For each element, the probability density functions for the load effect and resistance of each element are defined. Quantify the probability of failure for each element. Finally, the theorem of total probability is used to find the marginal probability of failure of whole deck.

An example will be used to illustrate the approach. It consists of a simply supported reinforced concrete solid bridge deck of length 40 m and width 8m.

#### 3.1 Traffic load model

Non-permit traffic load, sometimes termed ‘normal’ is distinguished from special vehicles which require permits to carry weights above the usual legal limits (Minervino et al., 2004, EC1, 2003). As permit vehicles are a critical factor in bridge loading (Moses, 2001, Sivakumar, 2007), the model developed for this study includes all vehicles, whether permit or non-permit, that likely to cross a bridge at full highway speed in its lifetime.
Monte Carlo simulation is used to generate bridge traffic loading scenarios for a number of years. Statistical distributions for vehicle properties such as weights and inter-vehicle gaps are taken from Weigh-in-Motion (WIM) measurements.

This study focuses on short to medium span bridges, where the combination of static load effect and dynamic amplification governs over congestion (Flint & Jacob, 1996). WIM data was collected for 290 weekdays over a 19 month period in 2005/2006 from the two slow lanes of the 4 lane D1(E50) highway near Levoča in Slovakia. A small percentage of vehicles – 13 327 out of a total of 761 665 – were removed in a series of data quality assurance checks. Of the remaining 748 338 vehicle, 349 606 were traveling in one direction and 398 732 in the other. The average daily flows were 1031 truck per day in one direction and 1168 in the other. The maximum number of axles in a vehicle observed at this site was 11.

There were 78 vehicles having gross vehicle weight (GVW) exceeding 70 t, 8 vehicles in excess of 100 t and a maximum observed GVW of 117.1 t. In the Monte Carlo (MC) simulation, the GVW and number of axles for each truck are generated using the 'semi-parametric' approach (OBrien et al., 2010). This involves the use of an empirical bivariate distribution (i.e., bootstrapping) for GVW and number of axles up to a specified GVW threshold, chosen here as 64 t. Above this threshold, a parametric fit is used, not only to smooth the trend but also to generate vehicles with weights and numbers of axles greater than those observed. A truncated Normal distribution is used for this purpose.

The MC simulation generates streams of traffic in each direction with GVW, axle spacing and inter vehicle spacing consistent with the measured site data. Bending moment is calculated at all elements in the bridge for each increment in time during each truck(s) crossing event. Two extreme loading scenarios are illustrated in Figure 2. The first includes a very large individual vehicle, in this case 162 t on 12 axles. The second involves a large vehicle meeting a common vehicle type, in this case, 152 t on 11 axles meets a 31 t vehicle on 5 axles.

![Figure 2. Typical extreme loading scenarios](image)

Two hundred years of traffic is simulated and the load effects calculated for each element of bridge deck. The Weibull extreme value distribution is found to fit well to the data. The load effect distributions for three elements are shown in Figure 3a and 3b. The elements are labelled as $A_{x,y}$, counting from element $A_{1,1}$ in the South West corner (front left in Figure 3a). The elements considered are $A_{5,1}$, $A_{7,1}$ and $A_{6,2}$, corresponding to coordinates $(x,y) = (5,1), (7,1)$ and $(5,2)$ respectively.

![Figure 3. Maximum-in-life load effect distributions](image)

### 3.2 Resistance model

Design or assessment codes can be used to derive models for the capacity of members to resist load effects such as bending moment or shear force. For the purposes of this paper, a lognormal distribution is assumed for moment resistance with a mean of $6.95 \times 10^7$ and a standard deviation of $5.92 \times 10^8$ for all individual elements.

### 4 RESULTS AND DISCUSSION

When the limit state function is not time-dependent, it can be defined as

$$z = G(R, S) = R - S$$

where $z$ is limit state margin, $G$ is limit state function, $R$ is random variable representing the capacity and $S$ is the random variable representing the corresponding load effect. The limit
state histograms for three elements (discussed before), A_{1,5}, A_{1,7}, A_{2,5} are illustrated in Figure 3.

![Figure 3. Limit state histogram](image)

The probability of limit state violation is used here as a proxy for the probability of failure:

$$P_f = P(G(x) \leq 0) = \int_{G(x) \leq 0} f_x dx$$  \hspace{1cm} (3)

Using the defined resistance and load models, the probability of failure is calculated for each element. The results are presented in Table 1.

<table>
<thead>
<tr>
<th>i/j</th>
<th>A(:,1)</th>
<th>A(:,2)</th>
<th>A(:,3)</th>
<th>A(:,4)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{16}</td>
</tr>
<tr>
<td>2</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{16}</td>
</tr>
<tr>
<td>3</td>
<td>7.77 \times 10^{16}</td>
<td>6.42 \times 10^{13}</td>
<td>2.66 \times 10^{15}</td>
<td>1.55 \times 10^{11}</td>
</tr>
<tr>
<td>4</td>
<td>3.65 \times 10^{09}</td>
<td>1.35 \times 10^{07}</td>
<td>3.92 \times 10^{08}</td>
<td>6.00 \times 10^{08}</td>
</tr>
<tr>
<td>5</td>
<td>9.83 \times 10^{09}</td>
<td>8.04 \times 10^{07}</td>
<td>2.13 \times 10^{06}</td>
<td>1.06 \times 10^{06}</td>
</tr>
<tr>
<td>6</td>
<td>1.80 \times 10^{07}</td>
<td>1.91 \times 10^{06}</td>
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<td>2.52 \times 10^{06}</td>
</tr>
<tr>
<td>7</td>
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<td>1.18 \times 10^{06}</td>
<td>2.59 \times 10^{06}</td>
<td>2.73 \times 10^{06}</td>
</tr>
<tr>
<td>8</td>
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<td>1.72 \times 10^{09}</td>
<td>4.68 \times 10^{11}</td>
<td>5.48 \times 10^{10}</td>
</tr>
<tr>
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<td>7.77 \times 10^{15}</td>
</tr>
<tr>
<td>10</td>
<td>7.77 \times 10^{16}</td>
<td>7.77 \times 10^{15}</td>
<td>7.77 \times 10^{15}</td>
<td>7.77 \times 10^{15}</td>
</tr>
</tbody>
</table>

Using the equation $\beta = -\phi^{-1}(P_f)$ the corresponding reliability index for each element is calculated for all elements. The results are illustrated in Figure 4.

![Figure 4. Reliability index at midpoint of each element](image)

As resistance was assumed to have the same distribution throughout the bridge length, the lowest reliability index corresponds to mid-span, where the load effect of bending moment is greatest. Reliability index variation across the width arises from a difference in the intensity of traffic within the two directions (349 606 vehicles in one direction as opposed to 398 732 in the other). There is also an edge effect due to greater bending moment at the edges transversely.

The theorem of total probability is used to calculate the failure probability of the entire structure based on the calculated probability of failure for each element.

$$Total \ Probability \ of \ Failure = \sum_{a}^{N} P_{fa} \times \prod_{a=1}^{N} (1 - P_{fb})$$ \hspace{1cm} (4)

Where N is total number of elements and $P_{fa}$ is probability of failure of element a with renumbering elements from 1 to N.

The spatial approach to the calculation of failure probability gives $2.281 \times 10^{-05}$ which is about 2.3 times greater than the conventional approach where each element of the bridge is considered in isolation. This indicates that the conventional approach may be highly conservative.

5 CONCLUSION

A framework is presented for the reliability assessment of a two-dimensional solid slab bridge deck. The bridge is divided into independent elements and the probability of failure and reliability index calculated for each element. Probability density functions are assumed for resistance and are calculated using Monte Carlo simulation for load effect. Random field analysis is proposed as a method of modelling the spatially variable resistance.

For the example considered, the conventional approach of reliability evaluation, considering individual elements in isolation, is inaccurate on the conservative side.
REFERENCES


