Title	The non-stationarity of apparent bridge natural frequencies during vehicle crossing events
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Publication date	2013-12
Publication information	Cantero, Daniel, and Eugene J. O'Brien. "The Non-Stationarity of Apparent Bridge Natural Frequencies during Vehicle Crossing Events" 41, no. 4 (December, 2013).
Publisher	Faculty of Mechanical Engineering
Item record/more information	http://hdl.handle.net/10197/7077

The non-stationarity of apparent bridge natural frequencies during vehicle crossing events

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In this paper, it is shown numerically how the natural frequencies of a bridge change during the crossing of a vehicle. An Euler-Bernoulli beam is modelled traversed by a single DOF vehicle. The use of such a simple Vehicle-Bridge interaction model is justified by the objective of providing insight into the structural dynamics of a moving load interacting with a bridge. The numerical results indicate that the variations in natural frequencies depend greatly on vehicle-to-structure frequency ratio and mass ratio. In some conditions, significant variations in modal properties are observed. Additionally, it can be analysed from the passing vehicle response. Time-frequency signal analysis of the vehicle's vertical acceleration clearly shows how the frequencies evolve during the event. The frequency localization properties of the Wavelet transform (Modified Littlewood-Paley) are exploited in analysing the signal and highlighting the relevant results.

Keywords: Vehicle; bridge; natural frequency; wavelet; dynamic; vibration; non-stationary.

1. INTRODUCTION

Bridge natural frequencies have long been used as indicators of bridge health. In addition, the dynamic increment of moving bridge load has been the subject of many numerical studies since the pioneering theoretical work of Frýba [1] in the 1960's and 1970's. Dynamic Vehicle-Bridge Interaction (VBI) is a complex problem that has been investigated extensively. Field measurements, experimental testing, new mathematical tools and higher computing capabilities have helped to improve the understanding in this field. However, there is still some additional insight to be gained into the fundamentals of VBI. This paper aims to clarify one aspect of this problem.

In recent decades Structural Health Monitoring (SHM) has become an important area of research. There are many approaches using a wide range of damage indicators. Methods based on vibrations are of particular interest as they provide the potential to detect damage at points remote from the location of the sensor. For bridge assessment, a promising indirect technique has emerged over the last decade, namely, 'drive-by' bridge inspection [2]. It has shown that an analysis of the accelerations of a passing vehicle can provide information about the bridge's dynamic properties. This idea is first presented and validated by [3] and has since then been investigated by many research groups [2, 4-7]. These studies show the potential of this new idea which offers many advantages, including reduced cost, the potential for more frequent monitoring, ease of sensors' power supply and little traffic disruption.

However, most of the research fails to recognize the time-variant nature of the VBI problem. In this study, by means of a simple model, it is shown how the apparent bridge frequencies change with changing vehicle position. It is also proposed that the extraction of frequencies should be performed by means of an adequate time-frequency signal analysis. In particular this paper briefly presents the Wavelet Transform and uses it to track the evolution of frequencies for various positions of a vehicle.

Results show a clear correspondence with previous drive-by bridge inspection studies. A reduced velocity of the vehicle is considered, which Gonzalez et al. [8] suggest is necessary to provide results with sufficient accuracy. Additionally, for clarity, only the first natural frequency of the structure is considered since its contribution to the dynamic response is generally the most important. However the implications of the presented results apply to all VBI problems and for any degree of complexity.

First, the paper starts with a brief description of the numerical model used. Then it presents the problem of time-varying frequencies in VBI problems. Section 4 analyses the acceleration signal of the traversing vehicle, first using the standard Fourier analysis and then using a Wavelet transform.

2. NUMERICAL MODEL

The numerical model developed for this study represents a simply supported bridge traversed by a moving vehicle (Figure 1). The structure is modelled as an Euler-Bernoulli beam using a Finite Element formulation. The vehicle is approximated as a 1-DOF oscillator connected to the profile by a spring and

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dashpot system. The dynamic interaction between systems is solved in an iterative manner, and the particular numerical values used are listed in Table 1. Note that the vehicle's mass m and suspension stiffness k are adapted in subsequent sections. Additional information on the numerical solution and model properties can be found in [9].

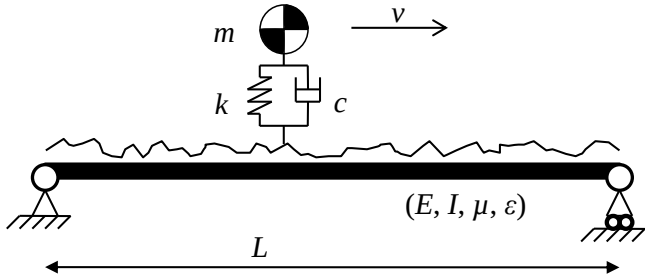


Figure 1. Sketch of Beam and moving 1-DOF oscillating mass

Table 1. Constant model properties

Model	Variable	Description	Value	Unit
Beam	L	Span	25	m
	E	Young's modulus	35×10^9	N/m^2
	I	Second moment of area	1.3901	m^4
	μ	Mass per unit length	18 358	kg
	ε	Damping	3	%
Vehicle	v	Velocity	5	m/s
	c	Viscous damping	10 000	N·s/m

Eqs. (1) and (2) present the natural frequencies of the vehicle and the fundamental frequency of the beam respectively, both in Hz. The structure, due to its continuous nature, has an infinite number of natural frequencies, but only the first (fundamental) frequency will be studied here, which for the particular values defined in Table 1, is 4.09 Hz.

$$f_v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

$$f_b = \frac{\pi}{2L^2}\sqrt{\frac{EI}{\mu}} \tag{2}$$

The equation of motion of the vehicle is presented in Eq. (3) where y_v represents the vertical displacement of the vehicle's mass, $y(x,t)$ is the bridge deflection, and x_v indicates the longitudinal position of the vehicle.

$$m\ddot{y}_v + k\big(y_v - y(x_v,t)\big) = 0 \tag{3}$$

The bridge's equation of motion is given in Eq. (4).

$$EI\frac{\partial^4 y}{\partial x^4} + \rho\frac{\partial^2 y}{\partial t^2} = -k(y - y_v)\delta(x - x_v) \tag{4}$$

This partial differential equation can be expressed as a set of ordinary differential equations considering the solution as the sum of the product of mode shapes and generalized coordinates (η). In particular in this paper only one mode is considered and thus the beam deflection is approximated for the case of a simply supported beam in Eq. (5).

$$y(x,t) \approx \sin\left(\frac{\pi}{L}x\right)\eta(t) \tag{5}$$

And thus Eq. (4) can be rewritten as in Eq. (6). The coupled Vehicle-Bridge system is defined by the system of Equations (3) and (6). Following section performs a numerical eigenvalue analysis of this system of equations.

$$\ddot{\eta}(t) + \left((2\pi f_b)^2 + \frac{k}{\mu L}\sin\left(\frac{\pi}{L}x_v\right)^2\right)\eta(t) - \frac{k}{\mu L}\sin\left(\frac{\pi}{L}x_v\right)y_v = 0 \tag{6}$$

Two non-dimensional parameters used throughout this study are defined in Eqs. (7) and (8). The Mass Ratio (MR) relates the vehicle mass to the structure's mass and the Frequency Ratio (FR) is the ratio between vehicle and bridge frequencies.

$$MR = \frac{m}{\mu L} \tag{7}$$

$$FR = f_v/f_b \tag{8}$$

3. COUPLED SYSTEM FREQUENCIES

The complexity of the VBI problem arises mainly due to the time-variant nature of the equations of motion. When the vehicle is on the bridge, their responses are coupled together and they cannot be analysed independently. The system matrices are different for every position of the vehicle on the bridge. Thus, the natural frequencies of the combined system also vary during the vehicle crossing.

Based on the model described in section 2, the coupled equations of motion are obtained for various locations of the vehicle along the bridge span. Performing an eigenvalue analysis on these equations it is possible to extract the system's natural frequencies and their evolution with the vehicle's position.

Figure 2 shows the first two systems frequencies for four distinct situations, where the MR has been fixed to 0.5. When FR is 0.5 (Figure 2a) there are only small variations in the vehicle and bridge apparent frequencies for changing vehicle locations. However, for FR values closer to the unit the system frequencies change significantly. In Figure 2b the frequency ratio is 0.95 and the maximum difference between the bridge's original and apparent frequencies is 2.29 Hz. It is evident that while the vehicle is near the supports the frequencies correspond to those of the independent vehicle and bridge systems (Eqs. (1) and (2)). However, when the vehicle is located elsewhere on the bridge, a significant shift of the frequencies is observed and the maximum difference occurs when the sprung mass is located at mid-span.

Figures 2c and 2d show situations where the traversing vehicle's original frequency is greater that the bridge's frequency (FR>1). In such cases the apparent bridge frequency reduces compared to its original.

It is convenient to normalize the frequency shifts with respect to the bridge's fundamental one, as seen in Eqs. (9) and (10). While it is not shown here, this normalization generalizes the results, making them

applicable to any vehicle and bridge properties, including different span length.

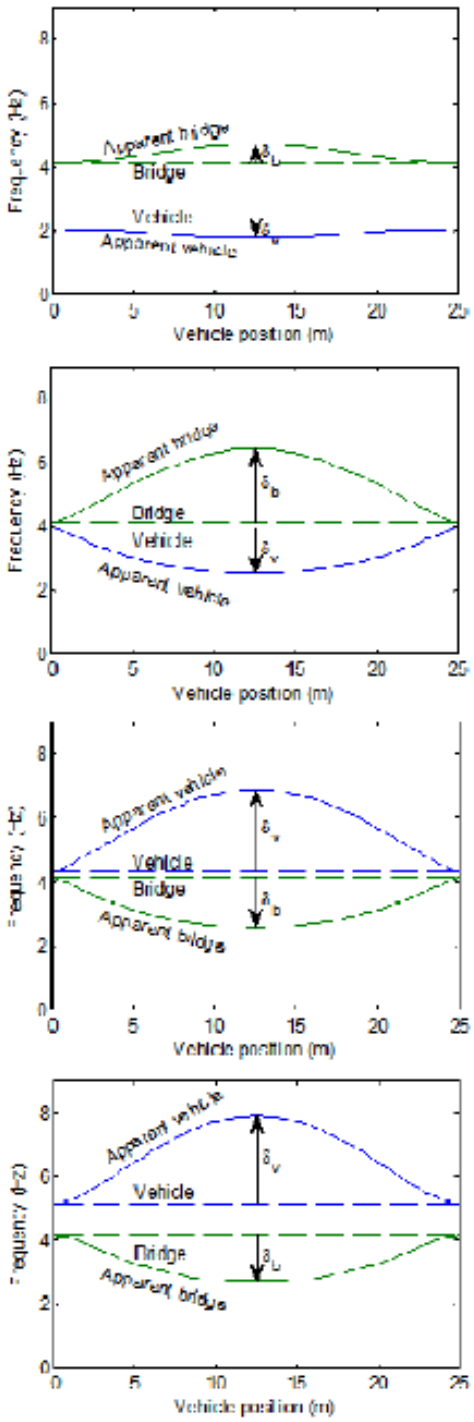


Figure 2. Apparent (solid) and uncoupled (dashed) vehicle and bridge fundamental frequencies system for MR = 0.5 and (a) FR = 0.5; (b) FR = 0.95; (c) FR = 1.05; (d) FR = 1.2

$$\delta_v^N = \delta_v / f_b \quad (9)$$
$$\delta_b^N = \delta_b / f_b \quad (10)$$

In the context of SHM, the shift in bridge frequency that occurs when the vehicle is exactly at mid-span is of greatest interest. Figure 3 presents the variation of δ_b^N for a range of mass and frequency ratios. It shows that, in general, higher MR values produce greater frequency shifts. For instance, in the situation where MR = FR = 0.75, Figure 3 gives a δ_b^N= 0.46. This means that the fundamental frequency of the beam increases by a factor of 1+0.46 when the appropriate vehicle is at mid-span.

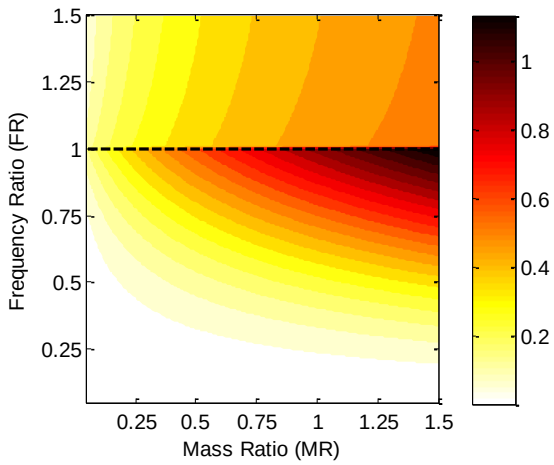


Figure 3. Normalized beam's frequency shift (δ_b^N), in absolute value, for vehicles located at mid-span

Figure 3 shows a clear distinction for cases above or below the FR = 1 line. When FR < 1 the bridge frequencies increase in value (shifts up as in Figure 2), whereas for FR > 1 the beam frequencies reduce (shift down) following a different pattern. However, the most important fact about the results presented in Figure 3 is that δ_b^N is never zero. This means that, in every VBI problem, the presence of a vehicle on the bridge will produce some change in the bridge's fundamental frequency.

4 SIGNAL PROCESSING

The frequency content analysis of the vehicle's vertical accelerations should give information about both the vehicle and the traversed structure. For this purpose the dynamic interaction model presented in Section 2 is solved for mass and frequency ratios MR = 0.1 and FR = 0.9. This represents a situation of a 46t vehicle crossing the 25 m span bridge specified in Table 1. Figure 4 presents the vehicle´s vertical acceleration traversing the beam moving at 5 m/s over a perfectly smooth profile.

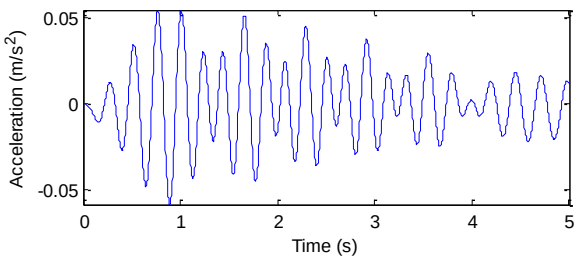


Figure 4. Vertical acceleration of vehicle while traversing the bridge

The standard approach for analysing signals in the frequency domain is the Fast Fourier Transform (FFT). This provides excellent information for stationary signals by means of a very efficient algorithm. The disadvantage of this tool is that the information obtained is for the whole signal and localized variations in time

vanish within the results. Figure 5 gives the result from applying the FFT to the acceleration signal and presents it in terms of Power Spectral Density (PSD). Clearly the theoretical system frequencies and those extracted from the signal analysis differ significantly. This difference can be explained by the shift in frequencies due to changing position of the vehicle on the bridge. This example highlights the need for a time-frequency analysis.

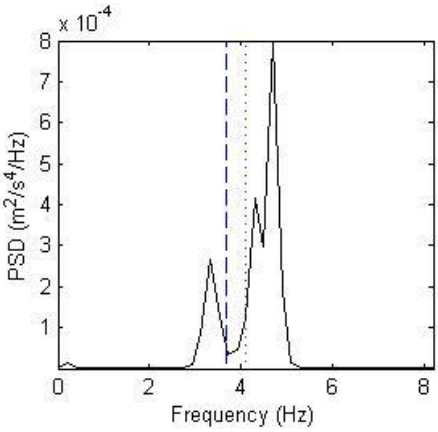
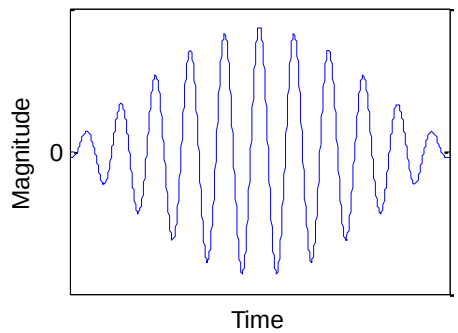


Figure 5. Power spectral density of acceleration signal. Vertical lines are vehicle (dotted) and bridge (dashed) frequencies

An excellent time-frequency tool is the Continuous Wavelet Transform (CWT) that has been developed over the last three decades [10]. This transform decomposes the analysed signal into a set of coefficients in two dimensions, shift and scale, where scale is inversely proportional to frequency. A basis function is translated (shift) and stretched (scale) and compared against the signal. High coefficients indicate a good match between signal and wavelet at a particular instant in time and associated frequency. This tool offers variable resolution and provides a map of energy content of the signal in time and frequency. A mathematical description of the CWT has been presented in many publications, and the reader can refer to [11] amongst many others.

To obtain the maximum benefit of the CWT it is important to select the most suitable basis for each particular problem. There are many possibilities and here the orthogonal wavelet basis called Modified Littlewood-Paley (MLP) is selected to study the frequency evolution of VBI problems. The authors considered alternative wavelet bases, including, Haar, Morlet, Mexican Hat and Littlewood Paley amongst others. It was found that MLP provides clearest frequency evolution results. This basis, proposed by [12], features excellent frequency localization properties. Figure 6 presents the MLP basis in the time and frequency domains. For a full description of this wavelet basis, refer to [12].

a)



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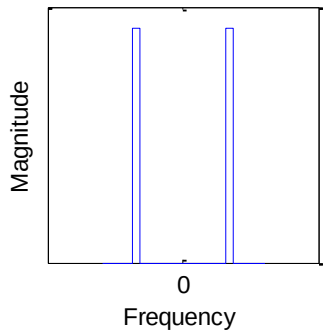


Figure 6. Modified Littlewood-Paley wavelet basis in a) time domain; b) two-sided power spectra density

Figure 7 shows the results of the CWT on the vehicle's accelerations (Figure 4) for four different wavelet bases, namely Morlet, Mexican hat, Littlewood-Paley and MLP. The time-varying theoretical values of the system frequencies are superimposed on the wavelet coefficients. Only in Figure 7d it is possible to clearly identify the evolution of the frequencies while the vehicle traverses the bridge.

4.1 Non-smooth profile example

The same configuration is studied again considering now some profile roughness. A Class A profile [13] has been included in the VBI model. A 100 m approach was incorporated to allow the vehicle to reach dynamic equilibrium before traversing the structure.

Figure 8(a) shows the PSD of the vehicle's vertical acceleration while traversing the bridge. A high energy concentration is observed near the vehicle's frequency due to the profile excitation that dominates the dynamic response of the system. However there is a small peak near the bridge's frequency that could be associated with the structure. As in Figure 5, theoretical and measured frequencies differ significantly. When the same signal is analysed using CWT with the MLP basis, Figure 8(b) shows that the frequency evolution becomes clear again. Now high energy concentrations are observed near the vehicle's frequency, but most importantly, the evolution in time can be clearly appreciated. It is also possible to identify the bridge's time-varying fundamental frequency.

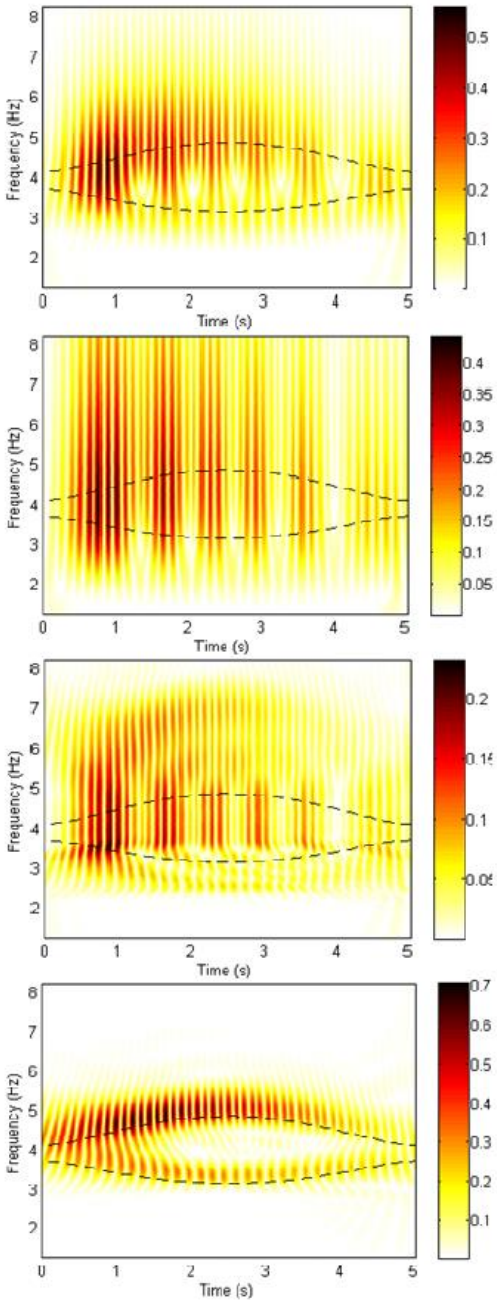


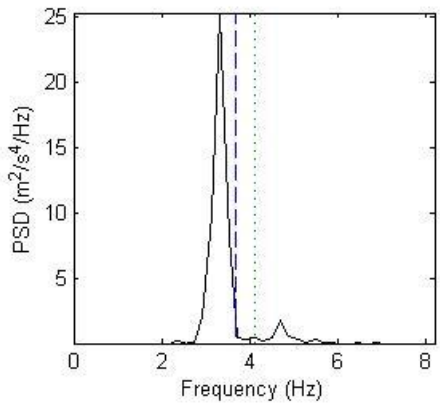
Figure 7. Wavelet transform coefficients of vehicle acceleration signal and theoretical frequency values (Dashed lines) for various basis; a) Morlet; b) Mexican hat; c) Littlewood-Paley; c) Modified Littlewood-Paley

5. DISCUSSION

The results presented in this paper are very relevant for the correct interpretation of the VBI phenomenon. For road traffic, the vehicle's mass is an order of magnitude less than the bridge's total weight, resulting in small MR that leads to small frequency shifts. However, there may be situations where higher frequency shifts occur because the structure's mass is small (steel bridges), the vehicle's mass is high (railway bridge) or multiple road vehicles are present on the bridge.

It is important to note that the presented study focuses only on the fundamental frequency of the structure. This allowed for a clear presentation of the idea, providing additional insight to the fundamentals of VBI. However, frequency shifts occur for higher modes too, particularly if the vehicle frequency matches any other of the natural frequencies of the bridge. In reality, a vehicle has several frequencies, and when positioned along different locations of the span, it will affect (shift) many of the structure's frequencies in a complex manner.

a)



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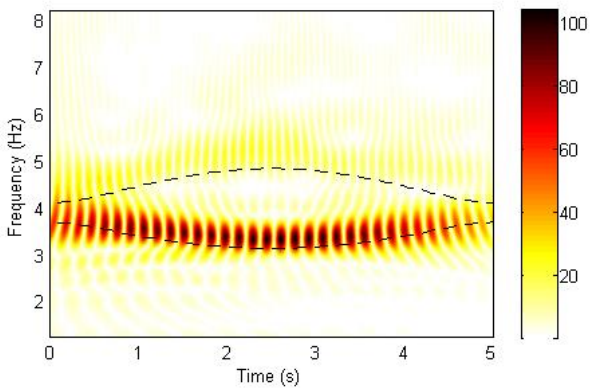

Figure 8. Vehicle vertical acceleration for vehicle traversing bridge on a Class A profile a) Power Spectral Density; b) Continuous Wavelet Transform with MLP basis

The change of system frequencies with vehicle position is an intrinsic feature of VBI that affects the response of the whole system, not only the vehicle's acceleration. Bridges when excited by passing vehicles will vibrate at varying frequencies which will be reflected in the frequency content of strain, displacements and accelerations of the structure. This is of relevance for direct SHM systems that use traversing vehicles as the source of excitation.

6. CONCLUSIONS

This paper has highlighted the time-varying nature of the VBI problem and its implications for SHM. The presence of a vehicle on the bridge will inevitably change the natural frequencies of the system. Using a simple numerical model, the shift in the bridge's fundamental frequency is found and presented using a non-dimensional representation. Then the vertical acceleration signal of a traversing vehicle is studied which exposes the need for an appropriate time-frequency analysis tool. The paper proposes the use of the Wavelet transform in combination with the MLP

basis to accurately track and extract the structure's frequencies.

ACKNOWLEDGMENTS

The research leading to these results has received funding from the European Union's Seventh Framework Programme (FP7/2007-2013) under the Marie Curie, Industry and Academia Partnerships and Pathways (IAPP) call, Grant agreement no. 286276.

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