Learning to Tax - Interjurisdictional Tax Competition under Incomplete Information

Johannes Becker, University of Münster
Ronald B Davies, University College Dublin

WP15/19
October 2015
Learning to Tax –
Interjurisdictional Tax Competition under
Incomplete Information

Johannes Becker
University of Münster
Institute of Public Economics
Wilmergasse 6-8
48143 Münster
Germany
johannes.becker@wiwi.uni-muenster.de

Ronald B. Davies
University College Dublin
G215 Newman Building
Belfield
Dublin 4
Ireland
ronbdavies@gmail.com

September 22, 2015

*We thank Karolien De Bruyne, Andreas Haußer, Vilen Lipatov, Ben Lockwood and seminar and conference participants at the Universities of Ghent and Siegen, the 2013 Oxford Tax Conference, and the Trade and Networks Conference in Leuven. All errors are our own.
Abstract

We present a multi-period model in which countries set source-based taxes without having precise information how their and their neighbours’ tax rates affect the tax base. Countries can learn from past experience and from observing their neighbours’ outcomes and/or tax policy choices. We consider the sequence of Markov perfect equilibria and show that the beliefs become more precise over time and, eventually, correct. The precision of beliefs in a given period increases in the number of observed countries. In equilibrium, tax rates are inefficiently low if the value of learning is positive and the pace of learning increases in the level of tax rates (because higher tax rates trigger larger tax base effects which helps learning); in the presence of fiscal externalities, tax rates are too homogeneous (because variance in tax policies enhances learning). If, due to fiscal externalities, the value of learning is negative, the opposite may be true. From the viewpoint of empirical measurement, the model generates time patterns that look as if countries react to each other even if there are no fiscal externalities. We conclude that the existing evidence may therefore be inconclusive with regard to the existence of tax competition.

JEL classification: H25, H32, H87

Keywords: social learning, policy diffusion, tax competition
1 Introduction

Optimal tax policy in open economies is a trade-off between the benefits of taxation (e.g. public goods, redistribution) and its cost (e.g. loss in tax base, distortion of economic activity). The latter crucially depends on the elasticity of the tax base. The existing literature mostly approaches tax competition by assuming that the elasticity of the tax base is common knowledge. However, it seems more than plausible that policy-makers have no better access to information than do researchers and – at best – have to measure elasticity as they must: based on past experience, both their own and their neighbours'.

In this paper, we present a model where governments gradually learn about the true elasticity of the tax base and demonstrate that this affects the nature and welfare properties of tax competition. We consider an infinite horizon model with multiple countries. Each country sets its source-based tax rate on business profits in each period. There is an a priori belief on the true state of nature (which determines the tax base elasticity). By observing its own and its neighbouring countries' outcomes, a country updates its belief function. The model allows for fiscal externalities (i.e. tax base leaving a given country and being taxed by some other country), but also for their absence (e.g. tax base reductions due to decreasing entrepreneurial effort). We restrict the analysis to Markov perfect equilibria with the information set being the only aspect that is transferred from one period to the next.

Our results show that learning will almost always be perfect in the long run and, thus, the classical tax competition results will prevail. However, incomplete knowledge of the tax base elasticity substantially affects tax competition in the short and medium term. First, even in the absence of resource externalities (e.g. due to internationally mobile capital), tax rates are internationally correlated. The reason is that countries learn from each other and update their beliefs in the same direction. This implies that evidence on internationally correlated tax rates is not necessarily evidence in favor of tax

---

2 See Feldstein (1999) and Saez (2001) for the notion that the elasticity of the tax base in a sufficient statistic for optimal tax policy.
competition. Second, uncoordinated tax policy choices are inefficient even if there are no cross-border resource flows. The reason is that an individual government ignores the information externality on other governments’ learning behavior. For instance, if high tax rates trigger large tax base effects and, thus, help learning about the underlying fundamentals, tax rates may be inefficiently low – provided that the value of learning is positive (which is not generally the case). In the presence of fiscal externalities, tax rates may then be inefficiently homogeneous since tax rate variance helps learning. Third, since internationally mobile capital creates negative externalities in taxation, learning may be inefficiently fast because it exacerbates the fiscal externality. Fourth, in specific circumstances, a ‘non-learning equilibrium’ may occur in which countries cannot learn about the true tax base elasticity. A curious side finding is that this is the case if incomplete knowledge is added to the classical symmetric tax competition model (Zodrow and Mieszkowski 1986). Fifth, as a policy implication, tax coordination prevents countries from learning which, depending on the fundamentals, may be good or bad.

We also consider a modified version of the model in which the neighbours’ outcomes are unobservable and only their policy choices are common knowledge. Countries can infer their neighbours’ experience from their policy choices. This implies that information spreads with a lag. Tax rate changes in one country subsequently ‘trigger’ tax rate changes in neighbouring countries. Existing empirical studies that use the time lag of the neighbours’ tax rate measures can be reinterpreted as being in line with this model. The special case of a symmetric equilibrium in which learning breaks down ceases to exist; otherwise all other results remain robust.

Finally, we relax the assumption of a common state of nature and allow for country heterogeneity. The possibility of learning from the neighbours’ choices now depends on the joint distribution of states of nature. If states of nature are correlated (i.e. if the covariance is non-zero), observing neigh-

---

3 Other models, including Davies (2005), provide additional settings where inter-jurisdictional tax competition can be welfare improving.

4 In studies using contemporaneous tax rates, the same can occur due to the correlation between previous year taxes (which are often omitted) and current taxes.
bours may be beneficial. An implication is that updates are stronger when more similar neighbours are observed (similar in the sense that their state of nature is strongly related on one’s own). We thus rationalize the use of spatial lags and similarity weights in empirical analysis of tax competition.

International (or interjurisdictional) interdependence of tax rate choices has been documented in a number of empirical studies, see Devereux et al. (2008), Overesch and Rincke (2009, 2011), Heinemann et al. (2010). In the literature, there are two predominant approaches to explain this interdependence. The first is 'tax competition' in the tradition of Zodrow and Mieszkowski (1986) and Wilson (1986). Because taxes abroad affect the domestic endowment with mobile resources, a change in tax in a neighbouring country may trigger a change in domestic policy. A well-known prediction of this approach is the race to the bottom, i.e. a sharp reduction in source-based taxes on mobile entities due to international competition. The second explanatory approach is the 'yardstick competition' of Besley and Case (1995) in which voters judge the performance of local policy makers by comparing their choices to the policies implemented elsewhere. As such, by mimicking other countries’ policies, politicians can improve their chances of reelection. Our model represents a third alternative in which tax rate interdependence occurs even if cross-border flows of mobile resources are negligible and political competition is not subject to asymmetric information. Countries observe their own and their neighbours’ experience with tax policy and adjust their beliefs on some shared state of nature which in our model is the tax base elasticity. This learning results in convergence of beliefs and therefore policies, not because policy choices elsewhere affect the payoffs of a given jurisdiction’s policy, but because learning results in more precise information on some underlying fundamentals. Furthermore, our model provides predictions consistent both with the empirical evidence

---

5 Although our discussion focuses on international policy diffusion, our theory equally applies to the literature on policy competition between jurisdictions within a nation. Empirical work here includes Fredricksson, List, and Millimet (2003), Levinson (2003), Heyndels and Vuchelen (1998), Besley and Case (1995), and Mintz and Smart (2004).

6 See Salmon (1987) for an initial application to taxes and Brueckner (2003) for an overview.
that is usually interpreted in favor of either resource competition of yardstick competition.

Our approach builds on the theory of social experimentation (see e.g. Bala and Goyal 1998) which assumes that agents learn from each other by observing their neighbours’ actions and their outcomes. Agents form networks which are defined by information flows. If all agents in a group can observe each others’ actions and outcomes, a network is 'fully connected'. In such a network, beliefs and actions converge over time and, eventually, the 'optimal' action is found and adopted. Set in the context of international tax setting, we consider a fully connected network of countries. As the theory of social learning predicts, the beliefs on the true tax base elasticity converge over time and, eventually, the optimal tax rate is found. Since actions (or: policy choices) affect the size and quality of information, actions have information externalities and the pace of learning is, at least in many cases, inefficient (see also Rose-Ackerman 1980, for the argument that federations yield inefficiently low levels of experimentation). This is reflected in our model that tax rates can be expected to be inefficiently high or low (depending on how the level of tax rates affect the pace of learning). We add to this theory by considering real economic externalities (i.e. fiscal externalities) between the learning agents (i.e. the countries) which, for instance, give rise to a non-learning equilibrium. Moreover, these externalities crucially shape the welfare consequences of learning. With negative externalities, learning may be inefficiently fast. In a later part of the paper, we modify the model by assuming that only actions (i.e. tax rate setting) can be observed. This builds on the social learning literature (Gale 1996, Gale and Kariv 2003, Vives 1996) which assumes that agents can observe their neighbours’ actions but not their outcomes. Then, agents have to induce

\footnote{Note that we are not the first to apply this theory to public policy. Buera et al. (2011) consider social experimentation of countries which decide between market-oriented and state-oriented economic policies. Their approach differs from ours in a number of aspects. For example, the government in Buera et al. (2011) makes dichotomic choice between market-oriented and state-oriented policies whereas the choice of tax rate in our model is continuous. Most importantly, they abstract from real economic externalities (fiscal externalities in our model) which are at the heart of tax competition theory.}
their neighbours’ experience (or outcomes) by observing their actions. In our context, an individual country may induce its neighbouring country’s tax base effects of tax policy just by observing its tax rate choice. Then, a tax cut abroad may trigger a tax cut at home due to an update of beliefs on the true elasticity of the tax base. This information externality creates an interdependence of tax rates even in the absence of real economic externalities.

The remainder of the paper is organized as follows. Section 2 presents the model and derives the main results and discusses some extensions. Section 3 outlines an alternative version of the model using social learning model instead of social experimentation. Section 4 discusses heterogeneity in the unobservable state of nature. Section 5 reinterprets the existing evidence from the viewpoint of our model. Section 6 concludes.

2 Model

2.1 Setup

We consider an infinite horizon model with \( N \geq 2 \) countries, indexed by \( i \) (where \( N \) is assumed finite). In each country, there is a representative firm which uses an input good \( k \) to produce output \( g^s (k_{i,t}, x_i) \) where \( t \) denotes the period and \( x_i \) is a vector of country-specific, time-invariant characteristics. The input good may be tradable across borders (e.g. mobile capital) or may be completely immobile internationally (e.g. entrepreneurial effort). We assume that \( g^s_k > 0 \) and \( g^s_{kk} < 0 \). The exact shape of the output function depends on the state of nature \( s \) which is initially unobserved. It is drawn in period 0 from a cumulated distribution function \( F_s (s) \) where \( f_s (s) = F'_s (s) \) denotes the density function. Let \( s^* \) be the actual state of nature and \( g^* (.) \) the actual output function. For later use, we make the following assumption (where, for the moment, we drop time subscripts to simplify presentation).

\( g^* (.) \) can be interpreted, for example, as either a decreasing returns production function with a constant price or a constant returns production with a declining price of output. For simplicity, we adopt the former nomenclature.
Assumption 1 a) The output elasticity $\frac{g_s(k_i,x_i)k_i}{g^s(k_i,x_i)}$ increases in $s$ for all $k_i$.

b) The concavity of the production function, measured by $\left| \frac{g_s(k_i,x_i)k_i}{g^s(k_i,x_i)} \right|$ decreases in $s$ for all $k_i$.

An example for an output function that is compatible with Assumption 1 is $g^s(k,x_i) = (k)^s$ where $s \in (0,1)$. If $g^s(.,.)$ is interpreted as a revenue function, i.e. the product of price and quantity, an example that is compatible with Assumption 1 is $g^s(k,x_i) = (A - \frac{1}{s}k)k$ where the price of output is given by $(A - \frac{1}{s}k)$, with $A > 0$ a constant, and the quantity by $k$ (assuming a linear production function where one unit of $k$ produces one unit of output good).

The input is purchased at a cost of $r_i$ (in the case of freely tradable input goods, the law of one price would result in $r_i = r$). Let $\tau_i$ be the source-based tax in country $i$. We assume that the input cost cannot be deducted from the tax base (which, if $k$ is interpreted as capital, is in line with the usual tax treatment of equity financing). Net-of-tax profits are given by:

$$(1 - \tau_i) g^s_i(k_i,x_i) - r_i k_i.$$ (1)

Assuming that the firm takes $r_i$ as given, profit maximization implies $g^s_i(k_i,x_i) = \frac{r_i}{1 - \tau_i}$. The equilibrium input price $r_i$ may be affected by demand in other countries (e.g. the interest rate for capital could be determined on the world capital market with the capital supply being fixed, like in Zodrow & Mieszkowski 1986), therefore $r_i$ can be expressed as a function of all tax rates and the state of nature, i.e. $r^s_i = r^s_i(\tau)$ where $\tau = (\tau_1, ..., \tau_i, ..., \tau_N)$ denotes the vector of tax rates. Input demand can thus be expressed as $k^s_i(\tau)$.

Each country is ruled by a government which maximizes tax revenue. We assume that the government cannot directly observe $s^*$, nor can it observe the level of input demand $k^s_i(\tau)$ or the input price $r^s_i = r^s_i(\tau)$. It does, however, observe tax revenues, which for $i$ are given by

$$\pi_{i,t} = \tau_{i,t} (g^s_i(\tau_t) + \varepsilon_{i,t})$$ (2)

9 If $g^s(.,.)$ is interpreted as a revenue function, i.e. the product of price and quantity, an example that is compatible with Assumption 1 is $g^s(k,x_i) = (A - \frac{1}{s}k)k$ where the price of output is given by $(A - \frac{1}{s}k)$, with $A > 0$ a constant, and the quantity by $k$ (assuming a linear production function where one unit of $k$ produces one unit of output good).

10 It is straightforward to introduce tax deductibility of input cost into the model as long as we rule out full deductibility (which would eliminate any tax effects); however, it adds no further insights.
where $g^\delta_1 (\tau)$ abbreviates $g^\delta (k_i (\tau), x_i)$ and $\varepsilon_{i,t}$ is an idiosyncratic shock (a measurement error) with mean of zero which is drawn from a cumulated distribution function $F_{\varepsilon} (\varepsilon)$, where $f_{\varepsilon} (\varepsilon) = F'_{\varepsilon} (\varepsilon)$ denotes the density function. $F_{\varepsilon} (\varepsilon)$ is common knowledge. Note that, by observing tax revenue and the tax rate, the government can calculate the tax base (both its own and its neighbours’). This does not, however, directly identify $s^*$ due to the white noise arising from the shocks $\varepsilon_{i,t}$. Note further that the variance of the tax base, $g^\delta_1 (\tau_t) + \varepsilon_{i,t}$, is only affected by the tax rate via the input use.

In each period $t \geq 1$, the government chooses its own tax rate $\tau_i$. The government in $i$ estimates its expected revenue in period $t$, for a given tax environment $\tau_t$, to be

$$T_{i,t} (\tau_t) = \tau_{i,t} \int g^\delta_i (\tau_t) f (\tilde{s} | I_{i,t}) d\tilde{s}$$

(3)

where $\tilde{s}$ denotes potential states of nature and $I_{i,t}$ the set of information in period $t$ and $f (\tilde{s} | I_{i,t})$ the density of $\tilde{s}$ conditional on $I_{i,t}$.

In line with the literature on social experimentation (Bala and Goyal 1998, Buera et al. 2011), we will assume that the government sets $\tau_{i,t}$ in each period to maximize (3). That is, it does not willingly experiment in order to gain information. This assumption is crucial and can be justified as follows. First, political cycles and short election terms (not modelled here) may force the incumbents towards a certain degree of myopia. Second, governments may not have the analytical power to solve for optimal experimentation strategies. Third, allowing for forward-looking experimentation makes the solution incredibly difficult and, in many instances, infeasible.

Let $\tau_{-i}$ denote the vector of tax rates of all countries except for $i$.

**Lemma 1** For each environment $s$ and $\tau_{-i,t}$, there is a unique optimal tax rate, denoted by $\tau^*_i (\tau_{-i,t})$, that strictly decreases in $s$.

**Proof.** The government’s first order condition in each period is given by

$$g^\delta_i (\tau_t) + \tau_{i,t} \left. \frac{dg^\delta_i (\tau_t)}{d\tau_{i,t}} \right|_{\tau_{i,t} = \tau^*_i (\tau_{-i,t})} = 0$$

and, thus, the tax rate by $\tau^*_i (\tau_{-i,t}) = -g^\delta_i (\tau_t) / \left. \frac{dg^\delta_i (\tau_t)}{d\tau_{i,t}} \right|_{\tau_{i,t} = \tau^*_i (\tau_{-i,t})}$. With (2) it follows $\tau^*_i (\tau_{-i,t}) = -\frac{g^\delta_i (\tau_t)}{g'_{i,k} (\tau_t) k'_{i,t}}$. With $k'_{i,t} = \frac{1}{1 - \tau_{i,t} g'_{i,k} (\tau_t)}$,
the optimal tax rate can be expressed as

$$\tau_{i,t} = \frac{1}{\tau_{i,t}} \left( \frac{\frac{\tau_{ik}(\tau_{i,t})}{\tau_{ik}(\tau_{i,t})}}{\tau_{ik}(\tau_{i,t})} \right).$$

With Assumption 1a), \( \frac{\tau_{ik}(\tau_{i,t})}{\tau_{ik}(\tau_{i,t})} \) decreases in \( s \) and with 1b) \( \frac{\tau_{ik}(\tau_{i,t})}{\tau_{ik}(\tau_{i,t})} \) decreases in \( s \) as well. It follows that \( \tau_{i,t} \) strictly decreases in \( s \).

Expected revenue maximization in each period implies the optimality condition

$$\int g^s_i(\tau_{i,t}) f(\bar{s}|I_{i,t}) d\bar{s} + \tau_{i,t} \int \frac{dg^s_i(\tau_{i,t})}{d\tau_{i,t}} f(\bar{s}|I_{i,t}) d\bar{s} = 0$$

in each \( t \). Thus, actual policy depends on the information set in each period.

The timing is as follows. In period 0, Nature draws \( s^* \) and each government \( i \) is exogenously endowed with an initial tax policy \( \tau_{i,0} \). After this, tax revenues are realized. In period 1, each government refines its belief about the true state of nature, based on the observation of \( \pi_0 = (\pi_{1,0}, ..., \pi_{i,0}, ..., \pi_{N,0}) \). They then simultaneously choose new tax rates, \( \tau_{i,1} \), and first period payoffs, \( \pi_{i,1} \), are received. Period 2 and all subsequent periods are equal to period 1, the only difference being that the information set (i.e. the amount of information available) becomes larger over time.

### 2.2 Learning

While \( s^* \) is unobservable, the functions \( F_s(.) \) and \( F_s(.) \) as well as the country-specific vectors \( x_i \) are common knowledge. Moreover, the history of tax rates and realized tax revenues for all \( i \) are known by all players (in the terms of the learning theory, we consider a model of social experimentation with a fully connected network). Observations of one’s own and the neighbours’ tax bases can be used to update the beliefs on the true state of nature.

How does updating work? In period 1, the information set of country \( l \)

---

11 This initial tax policy avoids the need to describe the choice of the initial tax rate when there is no information. Alternatively, the choice of \( \tau_{i,0} \) could be endogenized, with governments simply using the common knowledge distributions of \( s \).
consists of $2N$ elements, $I_{t,1} = \{ \pi_0, \tau_0 \}$. Given this,

$$f_s(\tilde{s}|I_{t,1}) = \frac{\prod_i f_\pi(\pi_{i,0}(\tau_0)|\tilde{s})}{\prod_i f_\pi(\pi_{i,0}(\tau_0))} \cdot f_s(\tilde{s})$$  \hspace{1cm} (5)$$

where $f_s(\tilde{s})$ is the unconditioned (a priori) density of $\tilde{s}$ and $f_\pi(\pi_{i,t}|s) = f_\pi(\pi_{i,t}/\tau_{i,t} - g_i^s(\tau_t))$ is the derived density of $\pi_{i,t}$. In period $t$, the information set has $2Nt$ elements and is given by $I_{i,t} = \{ \pi_t, \tau_t \}_{t=0,\ldots,t-1}$. The updated density is

$$f_s(\tilde{s}|I_{i,t}) = \frac{\prod_{l=0}^{t-1} \prod_i f_\pi(\pi_{i,l}(\tau_l)|\tilde{s})}{\prod_{l=0}^{t-1} \prod_i f_\pi(\pi_{i,l}(\tau_l))} \cdot f_s(\tilde{s})$$  \hspace{1cm} (6)$$

for $t \geq 2$.

Since countries can observe their own and their neighbours’ tax bases, the information sets are equal across countries.

### 2.3 Equilibrium

In the following, we will describe the sequence of equilibria over time. As is well-known, in an infinitely repeated game virtually all allocations can be sustained as an equilibrium. We will therefore constrain ourselves to Markov perfect equilibria, where the only feature that is transferred from period to period is the increasing size of the information set. With $x_i$ and $s^*$ being time-invariant, the only feature that changes across time is the beliefs expressed in $f(\tilde{s}|I_{i,t})$ due to changes in the size of the information set $I_{i,t}$.

**Definition 1** An equilibrium is defined as a vector $\tau$, which satisfies

$$\tau_{i,t} = \arg \max T_{i,t}(\tau_t) \quad \text{given} \quad \tau_{-i,t}$$

for all $i$. 


In the following, we will assume that there exists a unique Markov-perfect equilibrium. Before we turn to the welfare properties of the equilibrium, we can characterize the equilibrium.

The first feature of the equilibrium follows from the observation that information sets are equal across countries. This implies that changes in the information set will have similar consequences in each country (or equal consequences if the $x_i$ are identical). Since there is a unique optimal tax for each $s$ which monotonously increases in $s$, news will trigger either tax increases or tax decreases in all countries.

**Lemma 2 (Correlated tax changes)** Tax changes are correlated internationally. In the case of perfect symmetry, correlation is perfect, i.e. tax rates are equal in each period.

**Proof.** Omitted.

Note that the correlation of tax rates does not need to be triggered by cross-border flows of resources. It may actually be that $k$ is immobile across borders and that tax rates have no direct externalities (in terms of resource flows). However, since countries update in the same way, tax rates are correlated. We will discuss this further later on.

Countries learn about $s^*$ by observing their own history of experience and their neighbours’ policy outcomes. In a specific setting, this learning may not take place. This case is described in the following Lemma.

**Lemma 3 (Non-learning equilibrium)** Assume there is a set of $(s, \tau)$ vectors which are observationally equivalent, i.e. $g_i^s(\tau') = g_i^{s''}(\tau'')$ for all $(s', \tau'), (s'', \tau'')$ in that set. Then, a non-learning equilibrium may arise in which countries do not learn about $s^*$.

**Proof.** Omitted.

——

12 Proving the existence and the uniqueness of tax competition equilibria is inherently difficult. It is usually presumed that, if the objective function (here, tax revenue) is sufficiently concave, that a unique equilibrium exists. See Bayindir-Upmann and Ziad (2005) for a discussion.
Since learning only takes place by observing tax bases, observational equivalence implies that the true state of nature cannot be identified. False beliefs are 'confirmed' in the sense that the evidence does not contradict it. Observational equivalence does not imply, though, that an individual government is indifferent between two tax vectors \( \tau' \) and \( \tau'' \) from the observational equivalence set. To see this, consider the classical tax competition model (Zodrow & Mieszkowski 1986) with internationally mobile capital, but a fixed worldwide stock of capital. In case of perfect symmetry and given that all tax rates are equal, tax bases do not depend on the level of tax rates, i.e. the stock of capital in a given country is the same. Then, the level of tax rates depends on the beliefs on tax base elasticity. The larger the expected tax base elasticity, the lower the tax rates and, thus, revenue. Therefore, governments will prefer an equilibrium in which all countries believe that tax base elasticity is (close to) zero. In such a framework, cross-border capital flows, and thus learning, only occur if there are tax rate differences. Since countries do not willingly experiment, these differences do not occur under perfect symmetry. Learning is thus prevented. So, curiously, the classical tax competition model fails in reaching the equilibrium associated with the true state of nature once one adds uncertainty about capital demand elasticity and common priors. In this case, learning requires some heterogeneity in \( x_i \) or the endowed period 0 tax rates (which in the absence of \( x_i \) differences would be identical if chosen endogenously).

When this is not the case, we can establish the following Proposition which states that, in the long run, our model approaches the solution described in Zodrow & Mieszkowski (1986).

**Proposition 1** For \( t \to \infty \) and/or \( N \to \infty \), learning is perfect, except for in the case described in Lemma 3.

**Proof.** The proof is based on the observation that the first part on the right hand side of (6) approaches zero if \( \tilde{s} \neq s^* \) and the number of periods or countries approaches infinity. In contrast, if \( \tilde{s} = s^* \), this term approaches infinity.

\[ \text{\ding{182}} \]
The above Proposition reflects the general finding in Bala and Goyal (1998) that, in fully connected networks, beliefs and actions converge and that 'social conformism' is attained.

2.4 Welfare

We will now turn to the question whether learning is efficient in this framework. Before we do so, we have to define two measures, the precision of beliefs and the value of learning. First, the precision of beliefs measures the accuracy of current beliefs about the true state of nature $s^*$. We define the precision of beliefs as

$$\left( \int (\tilde{s} - s^*)^2 f_{\tilde{s}}(\tilde{s}|I_{i,t}) d\tilde{s} \right)^{-1}$$

(7)

It follows from Prop. 1 that, in the long term, the precision of beliefs approaches infinity. Since $s^*$ is unknown, the 'expected precision' is calculated by replacing $s^*$ with $\int \tilde{s} f_{\tilde{s}}(\tilde{s}|I_{i,t}) d\tilde{s}$. Since learning is unbiased, we have $\lim_{t \to \infty} \int \tilde{s} f_{\tilde{s}}(\tilde{s}|I_{i,t}) d\tilde{s} = s^*$ and the expected precision is an unbiased estimator of the precision.

Lemma 4 In expected terms, an increase in the number of observations (either over time or through an increase in observed countries) weakly increases the precision of beliefs.

Proof. In expected terms, new information reduces the first part on the right hand side of (6) if $\tilde{s} \neq s^*$ and increases it if $\tilde{s} = s^*$. ■

A more indirect implication of the above Lemma is that the learning curve is concave, i.e. the largest gains in belief precision occur at the beginning. Thus, to the degree that beliefs become more precise over time, one should expect tax changes to become less frequent and less pronounced over time.

How can the precision of beliefs be affected by tax policy itself? Before we answer this question, note that the effectiveness of learning depends on the signal-to-noise ratio (SNR) which is defined as the ratio of the variance
of the signal over the variance of the shock $\varepsilon$,

$$SNR = \frac{\int (g^s(\tau) - g^*(\tau))^2 f_s(\bar{s}|I_{i,t}) \, ds}{\sigma_\varepsilon} + 1$$

where $\sigma_\varepsilon$ denotes the variance of $\varepsilon$.

Taxes affect the SNR by changing the variance of the signal. The higher the SNR, the larger the increase in the precision of beliefs. Note that countries ignore the effect of their tax policy on learning since they only maximize current tax revenue. Moreover, it ignores the information externality on all other countries.

**Lemma 5** In equilibrium, a small tax rate change in country $i$ that increases the signal-to-noise ratio has a positive externality on the belief precision in all countries $\sim i$.

**Proof.** See the considerations above. ■

Whether the above Lemma implies that learning is inefficiently slow or not depends on the value of learning. Since welfare is here – by assumption – equal to tax revenue, the value of learning is positive when learning increases expected tax revenue and vice versa. Let $I'_{i,t}$ denote an information set that contains more information (i.e. more or better observations) than $I_{i,t}$. Let $\tau'_{i,t}$ denote a tax rate that is based on $I'_{i,t}$. The value of information can be calculated by comparing expected revenue with $I_{i,t}$ and $I'_{i,t}$. The difference for current tax revenue is

$$\Delta T_{i,t} = \tau'_{i,t} \int g^s(\tau_{i,t}, \tau_{-i,t}) f(\bar{s}|I_{i,t}) \, ds - \tau_{i,t} \int g^s(\tau_{i,t}, \tau_{-i,t}) f(\bar{s}|I_{i,t}) \, ds$$

which can be expressed as

$$\Delta T_{i,t} = \tau'_{i,t} \int g^s(\tau_{i,t}, \tau_{-i,t}) [f(\bar{s}|I'_{i,t}) - f(\bar{s}|I_{i,t})] \, ds$$

$$+ \int (\tau'_{i,t}g^s(\tau'_{i,t}, \tau_{-i,t}) - \tau_{i,t}g^s(\tau_{i,t}, \tau_{-i,t})) f(\bar{s}|I_{i,t}) \, ds$$

$$+ \tau'_{i,t} \int g^s(\tau'_{i,t}, \tau'_{-i,t}) - g^s(\tau_{i,t}, \tau_{-i,t}) f(\bar{s}|I'_{i,t}) \, ds$$

$$\text{(8)}$$

$$\text{(9)}$$
The first term on the right hand side captures the change in the expected value of tax revenue due to a change in beliefs; in expected terms this effect is zero. The second term depicts the change in expected revenue due to a change in a country’s own tax rate. This term is necessarily positive in expected terms as better information leads to better estimates (in expected terms) and the old tax rate is feasible as well (in other words, the country will only change its tax rate if it can increase its revenues). Finally, the third term shows the effect of adjusted neighbours’ tax rate on domestic revenue. This effect cannot be signed, as it depends on the direction of the adjustment.

Strictly speaking, the value of learning is the discounted expected difference $\sum_{t=1}^{\infty} \delta^{t-l} \Delta T_{i,t}$ with $\delta < 1$ being some strictly positive discount factor. Since the value of learning cannot be signed unambiguously, the following Proposition has to be conditioned on the value.

**Proposition 2** Assume that the economy is not in a non-learning equilibrium. Then, if the value of learning is positive (negative), learning in equilibrium is inefficiently slow (fast).

**Proof.** Omitted.

In the following two subsections, we consider two specific applications of the theory which allow for a deeper welfare analysis of the tax competition equilibrium.

### 2.5 Application 1: A model without fiscal externalities

Assume that the input good $k$ is entrepreneurial effort which is not tradable and, thus, immobile across borders. Then, there are no resource externalities since tax bases only depend on the entrepreneurial effort (we assume here that, if the entrepreneur reduces her effort, her leisure increases which, by assumption, has no externality on other individuals). If $s^*$ were known, tax policy in country $i$ would be completely independent of the tax policy in other countries. With an unknown state of nature, however, country $i$ has
an incentive to observe tax policy and revenues in other countries, since they reveal information on the common state of nature.

Due to the absence of fiscal externalities, \( g_i^s(\tau_{i,t}, \tau_{-i,t}) = g_i^s(\tau_{i,t}) \) and the value of learning is strictly positive. The reason is that learning reduces, in expectation, the difference between the current tax rate and the optimal tax rate. We can now establish the following.

**Proposition 3** Assume that \( k \) is not mobile across borders and there are no fiscal externalities. Then, learning is inefficiently slow. If the SNR increases in the tax rate, tax rates are inefficiently low, and vice versa.

**Proof.** In choosing an optimal tax rate, an individual government ignores that its tax policy reveals information to other governments. A small increase or decrease in the tax rate has no impact on welfare in \( i \), but increases the SNR and thus enhances learning (see Lemma 5). Thus, learning is inefficiently slow. If a small increase (decrease) in tax rates increases the SNR, such an increase (decrease) could unambiguously increase welfare. Thus, tax rates are either too low or too high.

The intuition behind this result is as follows. Tax policy has a positive information externality, i.e. an increase or a decrease in a country’s tax rate increases the precision of beliefs in all other countries. Since learning has an unambiguously positive value, tax rates are – at least in expectation – too low or too high.

### 2.6 Application 2: A classical tax competition model

Now, assume that the input good \( k \) is internationally mobile capital. Let the global capital supply be exogenously given as in Zodrow and Mieszkowski (1986). In the symmetric equilibrium, a coordinated increase in tax rates does not change the allocation of capital. This may give rise to a non-learning equilibrium as defined in Lemma 3. As such, countries may believe that capital is highly (barely) mobile and therefore set low (high) tax rates. Since all countries set the same tax rates based on the same beliefs, countries
do not learn about the true state of nature (provided that tax base functions are indeed observationally equivalent, i.e. \( g^{s'}(\tau) = g^{s''}(\tau) \) for \( a_{s'} \neq a_{s''} \)).

Generally, the pace of learning depends on the heterogeneity of tax rates. If tax rates are completely homogeneous, learning may break down. Thus, the more heterogeneous tax rates are, the faster the learning. As such, due to negative fiscal externalities, learning may have negative welfare effects since it can hasten the race to the bottom.

**Proposition 4** Assume that the economy is not in a non-learning equilibrium.

(i) If the value of learning is positive, learning is inefficiently slow and tax rates are inefficiently homogeneous. Tax rates are inefficiently low (high) if the SNR increases (decreases) in the tax rates.

(ii) If the value of learning is negative, learning is inefficiently fast and tax rates are inefficiently heterogeneous. Tax rates are inefficiently high (low) if the SNR increases (decreases) in the tax rates.

**Proof.** See the considerations above.

The above Proposition means that, for instance, Germany would profit from France setting a high tax on business profits, as it allows Germany to study the consequences and learn from it. This is the benefit of learning. However, if Germany observes that, in response to the French tax hike, a large amount of capital floods into Germany implying that capital mobility is high, future tax rates on both sides of the Rhine will be low, resulting in a much faster convergence to the equilibrium described by Zodrow and Mieszkowski (1986) in which tax rates are inefficiently low, representing the downside of learning.

### 3 Tax competition when neighbours’ outcomes are unobservable

The above model is based on the assumption that countries can observe their neighbours’ tax revenues. This may be considered a strong assumption
given that tax data is hard to collect, difficult to interpret and is usually only available after a lag of two or more years. We now change the above model by assuming that a country’s revenues cannot be immediately observed by its neighbours; tax rates, however, can be observed as before. Thus, we now consider a social learning model (see e.g. Gale and Kariv 2003).

Now, the timing of the model is as follows. Again, each government inherits an initial tax rate \( \tau_{i,0} \) before nature draws \( s^* \). Then, \( \pi_{i,0} \) is realized. In period 1, the government refines its belief about the true state of nature, based on the observation of \( \pi_{i,0} \) and decides on a tax rate \( \tau_{i,1} \). Then, the first period tax base, \( \pi_{i,1} \), is realized. In period 2, and all subsequent periods, country \( i \) can observe the other countries’ tax rate choice of the preceding period and may perfectly infer their tax bases from two periods before, i.e. \( \pi_{-i,0} \) for \( t = 2 \) and, generally, \( \pi_{-i,t-2} \) for period \( t \).

The crucial difference to the case of observable outcomes is that information sets are not equal across countries anymore. Each country has one piece of information (its own current outcome) that other countries lack. Since countries base their tax rate choice on all available information, tax rates can only be equal by chance. Once tax rates are set, other countries can infer the missing information from the tax rate choice.\(^\text{13}\)

That is, all information available from \( t - 2 \) is reflected in tax rates in \( t \).

**Lemma 6** A tax rate increase (decrease) in \( i \) in \( t - 1 \) causes a tax rate increase (decrease) in all other countries \(-i \) in \( t \).

\(^{13}\)To be precise, updating works as follows. In period 1, the information set consists of a single element, \( I_{i,1} = \{ \pi_{i,0}(\tau) \} \). Given this, \( f_s(\hat{s}|I_{i,1}) = \frac{f_s(\pi_{i,0}(\tau_0)|\hat{s})}{f_s(\pi_{i,0}(\tau_0))} \cdot f_s(\hat{s}) \). In period 2 and all subsequent periods, country \( i \) infers its neighbours’ payoffs from two periods before, i.e. \( \pi_{-i,0} \) for \( t = 2 \) and, generally, \( \pi_{-i,t-2} \) for period \( t \). In period \( t \), the information set has \( N(t-2) + 1 \) elements and is given by \( I_{i,t} = \{ \pi_{i,t-1}(\tau_{t-1}), \pi_{j,i}(\tau_i) \}_{j=1,...,N} \). The updated density is

\[
\begin{align*}
    f_{s,i}(\hat{s}|I_{i,t}) &= \prod_{l=0}^{t-2} \prod_{i=1}^N f_s(\tau_{i,l}(\tau_i)|\hat{s}) \cdot \frac{f_s(\pi_{i,t-1}(\tau_{t-1})|\hat{s})}{f_s(\pi_{i,t-1}(\tau_{t-1}))} \cdot f_s(\hat{s})
\end{align*}
\]

for \( t \geq 2 \), with the second term being country specific and the first one equal across country.
Note the difference to the case of observable outcomes where tax rates in the same period are correlated. There, observations in $t - 1$ cause tax rate changes in $t$; since observations are the same across countries, tax rates are correlated. Now, the tax rates themselves convey recent information. Therefore, tax rate changes in $t - 1$ cause neighbouring tax rate changes in $t$.

The optimization calculus of the individual country becomes somewhat more complicated since it has to account for potential news which is, currently, unobservable. To see this, consider some country $j$’s information set, with $j \neq i$, given by $I_{j,t} = \{ \pi_{j,t-1}(\tau_{t-1}), \pi_{t}(\tau_{t}) \}_{t=0,...,t-2}$. From the viewpoint of country $i$, $\pi_{j,t-1}(\tau_{t-1})$ is unknown and, for a given state of nature $s$, may be treated like a stochastic variable with mean $g_{j,t-1}(\tau_{t-1})$ and variance $\sigma_\varepsilon$. Let $G_{-i}(\varepsilon_{-i})$ denote the common distribution of shocks of all countries $-i$ and $\tau_{-i,t}(\varepsilon_{-i})$ denote the tax rate of country $-i$ that is realized for a certain realized vector of this shock. Now define $g_i^{\tilde{s},e}(\tau_{t}) \equiv \int g_i^{\tilde{s}}(\tau_{-i,t}(\varepsilon_{-i}), \tau_{i,t}) dG_{-i}(\varepsilon_{-i})$ which is the expected tax base for a given $\tilde{s}$, taking into account that $i$’s competitors ‘randomize’ over their strategies (which is how their behaviour can be described from the viewpoint of $i$).

The government’s optimization problem becomes,

$$\max_{\tau_{i,t}} \int g_i^{\tilde{s},e}(\tau_{t}) f(\tilde{s}|I_{i,t}) \, ds$$

An equilibrium is then defined as a situation where all countries set the tax rate that maximizes the ex-ante value of the above expression. We stick to the assumption that governments set tax rates as to maximize their contemporaneous tax revenue. That is, again, countries do not willingly experiment or attempt to manipulate their neighbours’ beliefs. We can then show the following.

**Lemma 7** A non-learning equilibrium cannot exist.

**Proof.** The non-learning equilibrium hinges on the property of equal information sets across countries. With unobservable tax revenues, information
sets generally differ and are only equal by chance. With \( t \to \infty \), the probability of equal information sets approaches zero. ■

All other results remain (at least qualitatively) the same: An increase in the number of observations increases the precision of beliefs. In the long run, learning is perfect (now even without the exception of a non-learning equilibrium). Depending on the scenario, tax rates are inefficiently low or high and learning is inefficiently slow or fast.

4 Spatially correlated states of nature

In this section, we will relax the (admittedly strong) assumption that the state of nature is equal across countries. Instead we will assume that country \( i \)'s and country \( j \)'s state of nature differ, for each \( i \neq j \). Let \( s = (s_1, ..., s_N) \) denote the vector of states of natures. The true vector, \( s^* \), is a realization of a common distribution \( H(s) \) with density function \( h_s(s) \).

As before, countries observe their own and their neighbours’ policy outcomes and update their beliefs. With \( \tilde{s} \) denoting a potential realization of \( s \), the update density is given by

\[
h_s(\tilde{s}|I_{i,t}) = \frac{\prod_{l=0}^{t-1} h_\pi(\pi_l(\tau_l)|\tilde{s})}{\prod_{l=0}^{t-1} h_\pi(\pi_l(\tau_l))} \cdot h_s(\tilde{s}) \tag{11}
\]

where \( h_\pi(\pi_l(\tau_l)|\tilde{s}) \) is the derived density of the outcome variable vector \( \pi_l(\tau_l) \).

Since an individual country is ultimately ultimately interested in its own payoffs, it sets payoffs to maximize expected revenue given by

\[
T_{i,t}(\tau) = \tau_{i,t} \int g^\pi(\tau_l) h_{s,i}(\tilde{s}|I_{i,t}) \, ds \tag{12}
\]

with \( h_{s,i}(\tilde{s}|I_{i,t}) = \int_{s_1} \int_{s_2} ... \int_{s_N} h_s(\tilde{s}|I_{i,t}) \, ds_1 ... ds_N \), i.e. the integral over all states of nature expect for \( s_i \).

How much country \( i \) may increase its precision of beliefs by observing
other countries’ policy outcomes depends on the degree to which updates in those countries’ beliefs yield information on country i’s state of nature. If states of natures are completely independent variables, updates do not include information. However, if the pairwise covariances in $H(s)$ is not strictly zero, observations from other countries yield valuable information for country i. The stronger the covariance the more information is implied.

**Lemma 8** (i) An observation of a neighbour’s policy outcome only increases the precision of beliefs if the covariance of the neighbour’s and one’s own state of nature exceeds zero.  
(ii) The increase in the belief precision increases in the covariance.

**Proof.** Omitted. ■

In simple words, it is only worth observing the neighbours’ policy when a country knows that high tax base mobility in the neighbouring country implies high tax base mobility at home. In contrast, if there is no a priori correlation between these two states of nature, observing the neighbours’ policy outcomes does not improve the own estimation of the state of nature.

Note, however, that independent draws of the state of nature do not imply that tax rates do not correlate. In the presence of fiscal externalities (like in the classical tax competition framework), tax rates will correlate even if states of nature are independent and different.

### 5 A reinterpretation of the evidence and policy implications

In our model framework, countries set policies as if they react to each other. More precisely, the time pattern of tax rates generated by the model suggests that tax competition for mobile resources takes place and that tax rates are strategic complements. However, this pattern occurs even in the absence of cross-border resource flows. In this section, we will reconsider the evidence in favor of tax competition and analyze whether the existing evidence allows for a clearcut identification of the underlying model.
Before we do so, we emphasize a methodological point. The empirical literature is, of course, interested in causation. The model above, in the version with observable outcomes, derives correlation: News incorporated in lagged outcomes cause both tax rates to change. Still, with the methods used in the empirical studies covered here, the pattern of tax rates must look like as if neighbouring tax changes cause tax changes at home. In econometric terms, there is a hidden variable (observation of outcomes in the preceding period) not controlled for in the empirical design. Indeed this is the point of the criticism of Gibbons and Overman (2012) who take the empirical tax competition literature to task over identification issues.

For instance, Devereux et al. (2008) regress statutory tax rates on a weighted average of neighbouring tax rates from the same period. In their preferred estimation, a one percentage point increase in the neighbours’ tax rate increases the domestic tax rate by 0.67 percentage points. This finding could be interpreted as being in line with our model. By observing outcomes, all countries adjust their tax rates in the same direction; not at the same rate (as country characteristics may differ and states of nature may be heterogeneous), but they strongly correlate. The method used by Devereux et al. (2008) identifies this as neighbouring taxes causing domestic taxes to change.\footnote{In order to identify evidence for tax competition instead of evidence for other theories, e.g. yardstick competition, Devereux et al. (2008) control for openness. They find some evidence that tax rate interactions are stronger in the absence of capital controls. However, Overesch and Rincke (2011) apply a different method and cannot find an impact of openness on tax rate interactions.}

Overesch and Rincke (2011) regress statutory tax rates on their own lag (since tax rates are sticky) and the neighbours’ lagged tax rate measure (an average weighted by distance). Their estimation results imply that the long-run results are quantitatively in line with those found by Devereux et al. (2008). Interestingly, though, it is the lagged variables that show up to have significant impact. In the framework of our model, this could be in line with a version in which outcomes are unobservable and countries interpret each others’ tax rate changes.

Moreover, our model may easily account for features like weighted spatial
lags. When the states of nature in neighbouring countries are more strongly correlated than in countries which are more distant from each other, a country’s tax policy would react more to its close neighbours’ tax rates than to more distant ones. In terms of measurement, this can give rise to spatial lags that include only the policies of neighboring countries, as done by Altshuler and Goodspeed (2007). An alternative is the use of political similarity weights, as utilized in Davies and Klasen (2013), who use a measure of political affinity based on United Nations voting as a weighting scheme for their study of overseas development assistance donations. Taking this idea a step farther, one can imagine a setting in which a subset of nations, for instance the EU, closely observe each other, but non-members are ignored. In this case, ‘clubs’ occur with different learning pattern inside and outside the EU. This would then fit the pattern found by Redoano (2014) and Davies and Vogel (2009), who find that while non-EU members respond equally to the corporate taxes of both EU and non-EU countries, members respond less to the taxes of non-members than to those of members.

Thus, we may conclude that the tax competition model and the learning-without-fiscal-externalities model (see Application 1) are, in many aspects, observationally equivalent. However, the policy implications differ diametrically. The theory of tax competition suggests that, under some circumstances, tax harmonization may yield a Pareto improvement. From the viewpoint of the model outlined above, however, policy harmonization may decrease efficiency as it prevents countries from learning.

6 Conclusion

This paper offers a model of tax competition in which the tax base elasticity is an a priori unknown variable which can be learned over time. This makes the existing tax competition model more realistic and yields some novel insights and empirical predictions. First, learning is gradual. That is, countries do not jump to a new equilibrium once there is a change in the underlying environment. Instead, they gradually approach the new equilibrium. Second, if states of nature are similar across countries, tax policy may
interact even in the absence of cross-border externalities in terms of resource flows. Third, learning may have a negative value and may thus be inefficiently fast. Fourth, tax coordination may be harmful because it prevents countries from learning. Similarly, tax havens which withhold information on assets within their borders prevent countries from learning about the tax base elasticity. \(^{15}\)

Although our model uses the elasticity of firm decisions as the unknown variable, the results here are easily applicable to a broad variety of situations, including the productivity of research and development, the ease of profit shifting, or the costs associated with relocation. Therefore while we are not suggesting that traditional motives for the correlation of taxes across borders (such as tax competition) are not important, we hope that our model provides insights that can further the debate on international taxation.

References


\(^{15}\)See Johannesen and Zucman (2014) for recent work on information exchange and investment in tax havens.


UCD CENTRE FOR ECONOMIC RESEARCH – RECENT WORKING PAPERS

WP14/17 Sarah Parlane and Ying-Yi Tsai: 'Optimal Sourcing Orders under Supply Disruptions and the Strategic Use of Buffer Suppliers' October 2014
WP14/19 Johannes Becker, Ronald B Davies and Gitte Jakobs: 'The Economics of Advance Pricing Agreements' November 2014
WP14/20 David Madden: 'Bridging the Gaps: Inequalities in Childrens' Educational Outcomes in Ireland' November 2014
WP14/21 Ronald B Davies, Julien Martin, Mathieu Parenti and Farid Toubal: 'Knocking on Tax Haven's Door: Multinational Firms and Transfer Pricing' December 2014
WP15/03 Ronald B Davies, Rodolphe Desbordes and Anna Ray: 'Greenfield versus Merger & Acquisition FDI: Same Wine, Different Bottles?' February 2015
WP15/04 David Candon: 'Are Cancer Survivors who are Eligible for Social Security More Likely to Retire than Healthy Workers? Evidence from Difference-in-Differences' February 2015
WP15/05 Morgan Kelly and Cormac Ó Gráda: 'Adam Smith, Watch Prices, and the Industrial Revolution' March 2015
WP15/06 Kevin Denny: Has Subjective General Health Declined with the Economic Crisis? A Comparison across European Countries' March 2015
WP15/07 David Candon: 'The Effect of Cancer on the Employment of Older Males: Attenuating Selection Bias using a High Risk Sample' March 2015
WP15/10 Morgan Kelly and Cormac Ó Gráda: 'Why Ireland Starved after Three Decades: The Great Famine in Cross-Section Reconsidered' April 2015
WP15/11 Orla Doyle, Nick Fitzpatrick, Judy Lovett and Caroline Rawdon: 'Early intervention and child health: Evidence from a Dublin-based randomized controlled trial' April 2015
WP15/12 Ragnhild Balsvik and Stefanie A Haller: 'Ownership Change and its Implications for the Match between the Plant and its Workers' May 2015
WP15/13 Cormac Ó Gráda: 'Famine in Ireland, 1300-1900' May 2015
WP15/15 Sandra E Black, Paul J Devereux and Kjell G Salvanes: "Healthy(?) Wealthy, and Wise - Birth Order and Adult Health" July 2015
WP15/18 Vincent Hogan and Patrick Massey: 'Teams’ Responses to Changed Incentives: Evidence from Rugby’s Six Nations Championship' September 2015