Assortativity Anomalies in a Large Test System

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Abstract—This letter compares the degree assortativity of the various test power systems that compose the NESTA archive. One outlier is detected, whose network structure is implausible. A new visualization of this network supports this conclusion.

Index Terms—Data visualization, network structure.

I. INTRODUCTION

The NESTA test case archive [1], version 0.5.0, contains thirty-five publically available test power systems, including some large power systems developed as part of the PEGASE project [2]. One of these systems, the 9241 bus network, is depicted in Fig. 1, using recently developed visualization techniques [3]. These techniques, briefly recounted below, plot buses in an electrically meaningful way. The depiction in Fig. 1 is immediately striking: why is there such a concentration of hub nodes, shown in red, visible in the center of the diagram?

II. VISUALIZATION

A. Power Transfer Distance

Firstly, a distance measure, $PT_{ij}$, is defined to measure the electrical distance between every bus pair, $i$ and $j$, in the power system. This particular measure uses the DC power flow assumptions to record the total shift in active power flows, $P^t$, across all branches, $B$, in the system, for a unit active power injection at $i$ and corresponding withdrawal at $j$:

$$PT_{ij} = \sum_{B} E_{B}^{ij}$$

(1)

B. Multi-dimensional Scaling

Multidimensional scaling iteratively positions each bus in our chosen two dimensions ($N = 2$), so that the resulting fitted distances between node pairs, $d_{ij}$, are maximally consistent with the desired input distances, $d'_{ij}$ ($= PT_{ij}$) This work uses the Sammon stress function [4], which defines the error function, $E$, to be minimized as follows:

$$E = \frac{1}{N} \sum_{i<j}^{N} \frac{(d_{ij} - d'_{ij})^2}{d_{ij}^2}$$

(2)

The inter-bus power transfer distances for the PEGASE 9241 system embed well in two-dimensions, giving a legible layout, with a suitably low stress value $E = 0.0267$ achieved.

III. STRUCTURAL METRICS

Two large-scale structural features are immediately evident in Fig. 1: the various loosely-interconnected regions around the periphery, presumably corresponding to distinct national grids, and the dense system core in the center. In this figure, nodes which have a degree greater than 20 are marked in red: work such as [5] has shown that such hub nodes are rare in power systems. Why are so many of these hubs connected together in the heart of this system? Why doesn’t each national grid possess its own hub nodes, as may be anticipated?

The likelihood of finding such interconnected hubs can be initially quantified using network degree assortativity [6]. This metric records the Pearson correlation of degree between mutually connected nodes, and is a meaningful summary classifier of network structure. Positive assortativity means that nodes tend to connect to other nodes that are similar in degree, whereas negative assortativity implies the reverse.

Fig. 2 shows node degree metrics for the 30 non-trivial systems in the NESTA archive (those with less than ten buses have been excluded) Nearly all of these systems have a maximum degree less than 15, and a degree assortativity within ±0.5. By contrast, the outlying PEGASE 9241 has a maximum nodal degree of 41, and a very high positive assortativity of 0.7752.

Even in networks with low aggregate assortativity, it could still be possible that the extant hub nodes connect disproportionately amongst themselves. Such a hub-of-hubs can be termed a ‘rich-club’, and the strength of same can be gauged using the rich-club coefficient [7]:

$$\phi(k) = \frac{2E_{>k}}{N_{>k} \binom{N_{>k}}{2}}$$

(3)

Where $E_{>k}$ is the number of edges between nodes of degree greater than or equal to $k$, and $N_{>k}$ gives the number of nodes meeting this criteria.

Fig. 3 shows the spectrum of this coefficient on the PEGASE 9241 network. It is clear that the set of nodes with $k > 30$ form a fully connected sub-network, which is an unexpected structure for a continent-spanning network of this type.

IV. CONCLUSION

Based on its strongly positive assortativity, unusual rich-club spectrum, and by visual inspection, this author concludes that the PEGASE 9241 system is structurally anomalous, and is not truly representative of a real power system. This recommends against its use as a validation platform for power system algorithms, which is its raison d'être.
V. REFERENCES