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Discrete Elastic Residential Load Response under Variable Pricing Schemes

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Abstract—The introduction of variable pricing schemes has potential impacts for low voltage (LV) distribution networks with regards to load diversity and peak demand. Households are being equipped communication systems that give consumers a greater visibility of electricity prices facilitating greater market engagement and demand response. This paper presents a bottom-up load model coupled with a novel methodology to capture the discrete, bounded and uncertain consumer response to variable prices. The model uses Monte Carlo simulation techniques and price elasticity matrices to affect the probability of consumption, taking into account detailed consumer characteristics and appliance operation. The model is used to run a high resolution simulation of residential load response and to quantify behavioral changes using standard load metrics.

Index Terms-- Demand Response, Distribution Networks, Load Modelling, Smart Grids, Residential Load Sector, Price-Response

I. INTRODUCTION

Crucial to the implementation and development of future smart networks is the need to assess how changes in consumer participation in the electricity market impact residential demand and the distribution network. Traditional distribution networks have been built around long established values for standard load metrics developed over decades of operational experience. The introduction of variable pricing schemes such as time of use pricing (TOU), day ahead pricing (DAP), critical peak pricing (CPP) and real-time pricing (RTP) have potential impacts on assumed values for after diversity maximum demand (ADMD) and diversified peak demand. Traditional analysis of medium and high voltage networks use top-down models to simulate and forecast aggregate demand. These are not suitable for assessing how demand might react to new pricing schemes, as they do not capture the high resolution and discrete nature of low voltage (LV) loads.

Increased provision of smart meters and smart home technologies allows for the large potential for demand response of the residential sector, which accounts for approximately 20% of the total annual electrical energy demand in Ireland, to be realized [1], [2]. End-use, or bottom-up approaches, provides the capability to capture consumer behavior and appliance operation at the LV level. Bottom-up models require demographic, socioeconomic, lifestyle and appliance operation data [3]–[5]. Using these data, time-varying probabilities for consumer occupancy and activity can be extrapolated for which appliance operation is dependent. Recent models have taken Time Use Survey (TUS) data to model consumer behavior and, subsequently, appliance operation [6], [7]. A Markov Chain Monte Carlo (MCMC) approach to developing occupancy profiles is developed in [8] based on TUS data. Occupancy and activity profiles are then used as determinant in the switch-on events of consumer operated appliances. Lighting models that include dependence on irradiance levels have also been established allowing lighting seasonal variations to be modelled [9], [10].

Previous research has been conducted into the elastic response of demand using price elasticity matrices (PEM) and the assumption that demand is a continuous function and available to respond across all hours [11]. Other research into developing residential demand response models uses time-varying values for self and cross elasticity of demand to determine the potential distribution network impacts at an hourly resolution and assuming continuous demand [12]. In [13] a bottom-up simulation of household electricity demand is presented under variable pricing, without simulating elastic consumer response, but automating the operation of appliances that require minimum or no consumer interaction such as wet and cold appliances.

In this paper, a novel methodology is developed to simulate high resolution discrete elastic residential demand response to variable pricing schemes by combining the research streams of bottom-up residential load model and elastic demand response using PEMs. A case study of the effects of TOU and DAP pricing schemes on residential demand is also presented. Section II presents the methodology of the synthesis of residential energy demand. In Section III

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the methodology of using PEMs to affect switch-on probabilities is presented, including methods to incorporate occupancy, demand duration and shifting window of operation. Finally, Section IV presents the case study, results are shown in Section V and Section VI gives the conclusions of this paper.

II. RESIDENTIAL LOAD MODEL

A. User Occupancy

User occupancy profiles can be determined by using National TUS data, with this paper using Irish TUS data [14]. From TUS data, occupancy transition probability matrices can be established; the model used here is for up to four residential occupants, for both weekdays and weekends, with the matrices available from previous research [15]. The occupancy profiles for each residence is modelled at minute resolution using a MCMC approach; the methodology for which is outlined in [8]. Occupants can either be active or non-active, with active defining the state in which an occupant is at the house and not asleep, and non-active defining both out of house and/or asleep. To obtain accurate dwelling compositions, household size is allocated based on statistics on household size; obtained here from [1]. Dwelling size can be linked to appliance ownership by correlating household size and energy consumption through weekly electricity expenditure [16].

B. Activity Modelling

From the TUS data, activity profiles can be created for a set of basic occupant behaviors, in this paper 6 different activities, for up to four occupants and for both weekday and weekend, were available from [15] using the methodology from [6]. These activity profiles represent a basic set of occupant activities which can then be related to appliance use. The profiles capture the temporal probabilities of occupants participating in each activity, for example, cooking activity probabilities peak around typical meal-times, television activities shortly after. Each activity is associated with a set of appliances, with some appliance-usage being solely dependent on an active occupant being present, see Table I. This allows the capture of temporal preferences of occupants, which has impacts for demand response, in particular highlighting that consumers can have very low preference for some non-peak hours reducing the potential for demand shifting.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Appliances</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV</td>
<td>DVD, Games Console, TV1, TV2, Set-top Box</td>
</tr>
<tr>
<td>Cooking</td>
<td>Dish Washer, Hob, Microwave, Oven, Small Cooking Appliances</td>
</tr>
<tr>
<td>House Cleaning</td>
<td>Vacuum Cleaner</td>
</tr>
<tr>
<td>Ironing</td>
<td>Iron</td>
</tr>
<tr>
<td>Laundry</td>
<td>Tumble Dryer, Washer Dryer, Washing Machine</td>
</tr>
<tr>
<td>Wash/Dress</td>
<td>Electric Shower, Hair Dryer, Immersion Heater</td>
</tr>
<tr>
<td>Active</td>
<td>Answering Machine, CD Player, Hi-Fi System, Kettle, Laptop, Lighting, PC, Phone, Printer, Misc, Electronic Goods</td>
</tr>
<tr>
<td>Not Active Dependent</td>
<td>Refrigerator, Freezer, Fridge-Freezers, Storage Heaters</td>
</tr>
</tbody>
</table>

The sharing of appliances is captured both through using occupancy profiles and activity profiles that relate to the number of occupants present. This results in a non-linear increase in probability with an increase in occupants.

C. Appliance Use

The initialization of the demand model assigns ownership for the most popular consumer appliances based on available statistics, along with data on appliance characteristics [16]–[20]. Each appliance has data on cycle power, duration, standby power and cycles per year, amongst other parameters. Power ratings for each appliance are given by assuming a normal distribution around the mean power assigned to each appliance with a set standard deviation.

1) Occupant & Activity Dependent Appliances

The switch-on probability, \( P_a \), of any device for any time step, \( t \), is dependent on a number of factors. These are the binary variable of consumer occupancy, \( O(t) \), dependent on the presence of an active occupant, the calibration scalar, \( C_a \), for each appliance, \( a \), which calibrates the number of switch-on events based on appliance data, and the activity probability itself, \( A(t,n) \), dependent on both the number of occupants, \( n \), and time. The probability of switch-on events for appliances that have an associated activity are determined by (1), and switch-on events of appliances solely dependent on occupancy are determined by (2).

\[
P_a(t) = (O(t) \times C_a \times A(t,n)) \quad (1)
\]

\[
P_a(t) = (O(t) \times C_a) \quad (2)
\]

2) Lighting

At initialization of the model each household is allocated a set of one of 500 possible lighting configurations based on statistics on bulb type penetration, installed wattage from UK data [21]. The lighting model uses the methodology established in [9]. A switch-on event in the model is dependent on five factors; an irradiance threshold, relative use weighting of the bulb \( W_b \), the occupancy binary variable \( O(t) \), the effective occupancy \( Ef(t,n) \) and a lighting calibration scalar \( CL \). The irradiance threshold is used to compare outside irradiance levels to determine the binary variable \( Irr(t) \), with the irradiance level given by a normal distribution with a standard deviation to provide for different effective irradiance levels in each house. Each bulb, \( b \), is given a relative use weighting, \( W_b \), representing that the location of each bulb in a house determines its frequency of use. The effective occupancy is used to represent a non-linear increase in probability with increased occupancy. The model is adjusted such that 20% of switch-on events occur independent of the binary variable \( Irr(t) \) to better fit statistical data [18].

\[
P_b(t) = (O(t) \times CL \times Irr(t) \times W_b \times Ef(t,n)) \quad (3)
\]

D. Load Profiles

The developed model was implemented in MATLAB [22], and was tested against Irish residential data. Table II shows the results for the average yearly household energy consumption for simulation of 10,000 houses and compared with Irish data [23]. As the model does not try to capture the
full operation of space and water heating appliances, for which the penetration is low but energy consumption is high, there is a low representation of these categories and associated heat pumps and fans.

Table II Energy Contribution Comparison

<table>
<thead>
<tr>
<th>Category:</th>
<th>Data [23]</th>
<th>Model</th>
<th>% Captured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold</td>
<td>490 kWh</td>
<td>514.5 kWh</td>
<td>105 %</td>
</tr>
<tr>
<td>Hot Water</td>
<td>1225 kWh</td>
<td>333.2 kWh</td>
<td>27.2 %</td>
</tr>
<tr>
<td>Space Heating</td>
<td>343 kWh</td>
<td>245 kWh</td>
<td>71.42 %</td>
</tr>
<tr>
<td>Wet</td>
<td>539 kWh</td>
<td>588 kWh</td>
<td>109 %</td>
</tr>
<tr>
<td>Cooking</td>
<td>392 kWh</td>
<td>441 kWh</td>
<td>112 %</td>
</tr>
<tr>
<td>ICT &amp; CE</td>
<td>931 kWh</td>
<td>882 kWh</td>
<td>94.7 %</td>
</tr>
<tr>
<td>Lighting</td>
<td>784 kWh</td>
<td>832.2 kWh</td>
<td>105 %</td>
</tr>
<tr>
<td>Pumps &amp; Fans</td>
<td>196 kWh</td>
<td>0 kWh</td>
<td>0 %</td>
</tr>
<tr>
<td>Total</td>
<td>4900 kWh</td>
<td>3826.9 kWh</td>
<td>78.1 %</td>
</tr>
</tbody>
</table>

Fig.1 presents the comparison of the average Winter load profile for 10,000 houses with that of Irish residential data [24]. The Winter average is shown as it represents annual peak load, and is the time of year simulated in Section V. The morning and evening rises are late, most probably due to the use of timers on electric water and space heating. The data and the model have a Pearson correlation coefficient of 0.94 and the model gives a root mean square error of 12.5%.

A. Price Elasticity Matrix

In order to be able to quantify how consumers react to price changes, a measure of consumer response to prices must be used, that is price elasticity of demand. Elasticity is a linearization of the demand curve around a given point, relating a change in price to a change in demand through the relative slope; its elasticity. This shows that for a given change in price, $\Delta p$, the change in demand, $\Delta q$, can be calculated given its elasticity, $\varepsilon$, with respect to an equilibrium point $(q_0, p_0)$ (4).

$$\varepsilon = \frac{q_0}{\Delta q} \frac{\Delta p}{p_0}$$  \hspace{1cm} (4)

In the model, elasticity is used to affect the probability of a switch-on event, $p$, to capture the discrete non-continuous nature of residential demand. It is observed that average demand levels over a large number of Monte Carlo simulations exhibit the exact same characteristics shown in the classical continuous approach, as the classical approach is but an approximation of discrete changes in demand. Elasticity is thus defined as relating a change in price to a change in the probability of a switch-on event (5). Hence, variable pricing will only increase or decrease the probability of consumption, meaning that not all consumers will react rationally to pricing signals capturing the lack of knowledge of complete consumer utility.

$$\varepsilon = \frac{p}{\Delta p} \frac{\Delta q}{q_0}$$  \hspace{1cm} (5)

Electricity has the property of having both self-elasticity, $\varepsilon_{ii}$, and cross-elasticity, $\varepsilon_{ij}$, coefficients as different time intervals, $(i,j)$, can serve as substitutions for the current interval, (6) and (7) respectively. A step change in electricity price results in both intra and inter-temporal changes in electricity demand, illustrating that demand response can both be a reaction to current, upcoming and past prices.

$$\Delta P_i = \varepsilon_{ii} \frac{\Delta p_j}{p_0} P_j$$  \hspace{1cm} (6)

$$\Delta P_i = \varepsilon_{ij} \frac{\Delta p_j}{p_0} P_j$$  \hspace{1cm} (7)

Using (6) and (7) a PEM can be created which relates both cross and self-elasticity to demand changes in any given period (8). Elasticity matrices have been used in previous research in electricity markets, but always to affect demand assuming it was a continuous function [11].

$$\left( \frac{\Delta p_i}{\Delta P_i} \right) = \left( \begin{array}{cc} \varepsilon_{ii} & \varepsilon_{ij} \\ \varepsilon_{pi} & \varepsilon_{pj} \end{array} \right) \left( \begin{array}{c} \frac{\Delta p_j}{p_0} \\ \frac{\Delta p_j}{p_0} \end{array} \right)$$  \hspace{1cm} (8)
PEMs allow the effects of prices both associated and external to the current time period to be related to a change in demand (9). In this paper the window of time for which a consumer is willing to shift their demand both forward and back in time is denoted \( W \). For a flexible consumer, \( W \) may incorporate a large number of time periods and for an inflexible consumer, \( W \) would be over a smaller range. Also, \( W \) could also be set to include only future hours or past hours, reflecting the cases of anticipating and postponing consumers respectively.

\[
\Delta P = \sum_{j=W} D \varepsilon_j \frac{p_j}{p_0}
\]  
(9)

The elasticity coefficients can be used to represent two cases, one where the total change in demand is zero, and the second where it is non-zero. These cases are represented by having a lossless (10), and lossy (11) elasticity coefficients respectively.

\[
\sum_{j=W} \varepsilon_j = 0
\]  
(10)

\[
\sum_{j=W} \varepsilon_j < 0
\]  
(11)

B. Effect of Discrete Duration Appliances

Consumers make discrete decisions on the operation of electrical appliances in single time periods taking into account the duration of operation. For example, a consumer is unlikely to switch on a dishwasher at a time of low prices if they will very shortly be subjected to high prices, most rational consumers would initialize operation of the dishwasher a sufficient amount of time before peak prices. As the likelihood of switching on a device of discrete time duration, \( D \), is represented by a single switch-on probability, \( P \), consumers would react to the prices over that discrete duration in determining how to react to variable prices. So the change in probability of any single instant, \( i \), is a function of the price differentials experienced over the entire window of operation (12).

\[
\Delta P = \sum_{j=W} P_i \varepsilon_{ij} \left( \sum_{d=1}^{D-1} \frac{p_d}{D} \right) - p_0
\]  
(12)

C. Effect of Consumer Occupancy

Consumers are likely to react differently to price signals given their occupancy profile. Occupancy can affect the shifting window and can eliminate increases in probability should the occupant not be present. It also eliminates cross-elasticity coefficients for periods the consumer is not present for. To account for these effects the cross-elasticity coefficients are weighted by the factor \( a_i \) should the occupant not be present for the entire window of operation, and all coefficients are multiplied by the binary occupancy variable \( O_j \) (13, 14).

\[
a_i = \frac{2W}{\sum_{j=W} O_j} \quad \text{for } i \neq j, \quad a_i = 1 \quad \text{for } i = j
\]  
(13)

\[
\Delta P_\text{d} = \sum_{j=W} O_j a_i p_i \varepsilon_{ij} \left( \sum_{d=1}^{D-1} \frac{p_d}{D} \right) - p_0
\]  
(14)

D. Overall Implementation

Fig. 2 shows the developed methodology giving the change in normalized probability of a switch-on event for a TOU price signal and reference price given in Section IV. The change is for a self-elasticity coefficient of 0.3 and a cross-elasticity coefficient of 0.001667 based on a lossless simulation for a window of 90 minutes.

![Figure 2. Developed methodology for residential elastic response, change in normalized probability due to TOU price signal.](image)

IV. CASE STUDY

Four different pricing schemes are examined in this paper; flat rate tariffs, two TOU pricing schemes and DAP. Both the historical flat rate tariff and TOU pricing signal, herein named TOU1, are taken from a study by the Irish Commission for
Energy Regulation (CER) using the tariff which had the biggest peak and day price differential of those trailed, see Table III [20]. A second TOU signal, herein named TOU2, is also tested, with the peak period shifted to 18:00-20:00 to account for the temporal differences in the peak between the model and available data. The DAP price used in this paper is the ex-ante (EA) system marginal price (SMP), that is released by the Single Electricity Market Operator (SEM-O) 24 hours in advance, the prices used are for a Winter week and are of half hourly resolution [25]. A day and night distribution use of system (DuOS) charge is added to the SMP to obtain a more realistic price that would be experience by the consumer [26].

For the simulations in this paper a self-elasticity coefficient of -0.3 is used based on standard values in the literature [27]. A cross-elasticity coefficient of 0.001667 is chosen based on a shifting window of 90 minutes, $W$, to ensure a lossless simulation, the reference price and flat rate tariff was taken to 14.1 cents/kWh [20]. Irish global irradiance data of 15 minute resolution is used for the corresponding Winter week for the lighting model. All appliances are assumed to give an elastic response apart from lighting, cold appliances and storage heaters. Elastic lighting response is presumed to mainly come from energy efficiency measures and so is not modelled in this paper.

The simulation is for a Winter week for 1,000 Irish residential households whose energy consumption and residency occupant size is representative of the total population. The demand under each pricing scheme can be directly compared as the model uses the same testing random numbers under each simulation. The model simulates occupancy, appliance operation and consumer response at minute resolution. The PEMs are implemented as sparse matrices; as only the diagonal elements of the matrices within the window size of operation are non-zero.

V. RESULTS

The methodology developed allows the capturing of complex demand response functions of the consumer, catering for the effect of occupancy, activity probabilities and the discrete nature of both the magnitude and duration of residential demand. The results show low potential to shift into hours with low occupancy or low activity probabilities. The stochastic nature of the consumer is also captured, especially at lower demand levels, which has particular relevance for the distribution network. Overall change in demand from the flat tariff case to variable pricing schemes is less than 1%, confirming a lossless simulation.

A. Change in Demand

Consumers are shown to react to peak prices up to two hours in advance, with the magnitude of response increasing up until close to the transition, reflecting the reaction to the range and distribution of the price differentials experienced by different appliances. Fig. 3 shows the change in demand around one of the weekday peaks, it can be seen that both the TOU signals and DAP cause the creation of new peaks.

The pricing schemes introduced result in the creation of rebound peaks, whose magnitude is heavily dependent on consumer occupancy and activity probabilities, and whose duration is dependent on the variation of price differentials for the range of appliances. The peaks of different activity probabilities heavily influence the magnitude of the rebound peaks, reflecting temporal preferences of consumers.

<table>
<thead>
<tr>
<th></th>
<th>Time (4 h):</th>
<th>23-08</th>
<th>08-17, 19-23</th>
<th>17-19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekday Prices (c/kWh)</td>
<td>9</td>
<td>12.5</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Weekend Prices (c/kWh)</td>
<td>9</td>
<td>12.5</td>
<td>12.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Time (hrs)</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>Price (cents/kWh)</td>
<td>40</td>
<td>35</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>TOU1</td>
<td>Price (cents/kWh)</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>TOU2</td>
<td>Price (cents/kWh)</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DAP</td>
<td>Price (cents/kWh)</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3. Impact of pricing schemes on a weekday peak; change in demand.

Fig. 4 shows the top five peaks for each of the four pricing schemes; all five peaks occur during weekdays and are spread over a relatively large time frame. This is significant, as although at the transmission level peak demand occurs approximately at the same time each day, the same is not true at the distribution level. The result is that by targeting
transmission level peaks, price signals may introduce a rebound peak on top of an existing distribution level peak, which may have impacts for the network. Fig. 4 also shows that, in general, the pricing schemes introduced cause the creation of new peaks, rather than pure peak reduction.

![Figure 4. Comparison of top five peaks for each pricing scheme.](image)

### B. Load Metrics

The different pricing schemes do have a marginal effect on distribution load metrics, see Table IV. Maximum diversified demand is seen to increase in all pricing schemes, meaning the pricing schemes gave no reduction in peak demand.

<table>
<thead>
<tr>
<th>TABLE IV DISTRIBUTION LOAD METRICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Signal:</td>
</tr>
<tr>
<td>Maximum diversified demand</td>
</tr>
<tr>
<td>Maximum non-coincident demand</td>
</tr>
<tr>
<td>After diversity maximum demand</td>
</tr>
<tr>
<td>Diversity factor</td>
</tr>
<tr>
<td>Load Factor</td>
</tr>
<tr>
<td>Demand Factor</td>
</tr>
</tbody>
</table>

Standard network ADMD values in Ireland are specified at 2.5 kVA, which operating at a power factor of 0.95 would mean an ADMD of 2.375 kW [28]. The values presented in this paper are slightly less than half of that, most probably a reflection of the low representation of space and water heating in the model, which would increase the evening base load.

### VI. Conclusion

This paper has presented a novel methodology for simulating the discrete elastic load response from the residential sector. The methodology takes into account probabilities of consumption, consumer occupancy, and the discrete duration of appliances using PEMs and Monte Carlo simulation techniques. The residential response to variable pricing schemes was modelled, with none of the presented schemes reducing peak demand due to effects of rebound peaks.

### ACKNOWLEDGEMENT

The authors wish to thank Mr. O. Neu for providing the activity profiles and occupancy transition probability matrices used in this paper.

### REFERENCES