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Can We Infer External Effects from a Study of the Irish Indirect Tax System?

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Abstract: This paper estimates implied external effects for the Irish indirect tax system for the year 1987. The study uses the inverse optimum technique of Christiansen and Jansen (1978) which estimates implied external effects, given the assumption that the economy is at an optimum with regard to the indirect tax system. External effects are estimated for three goods: tobacco, alcohol and petrol and in all cases the estimated external effects are of the expected sign. The paper also estimates the implied degree of inequality aversion in the Irish indirect tax system and finds that the government’s social welfare function as implied by the indirect tax system is virtually utilitarian.

I INTRODUCTION

The justification for the existence of corrective or Pigovian taxes or subsidies to take account of external diseconomies or economies of consumption is well known. Owing to the divergence between private and social utility of consuming various goods, it may be optimal for taxes or subsidies to be imposed. Various attempts have been made to estimate what these external effects are and how they may influence the rate of taxation on different goods. Examples of such work in the Irish context are the papers by Walsh


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1. We will examine the case of corrective taxes although, of course, our arguments could also be applied to corrective subsidies.
(1987) and O'Hagan (1983) and there are numerous examples from the international literature.

This paper looks at the question of corrective taxation in Ireland from the approach of the inverse optimum and marginal tax reform literature of Christiansen and Jansen (1978) and Ahmad and Stern (1984). Thus, consistent with the tradition in this literature, rather than attempting to estimate what the optimal tax rates should be, we attempt to estimate what are the external effects implicit in the existing Irish indirect tax system. In doing so we also infer the degree of inequality aversion in the Irish indirect tax system.

II INCORPORATING SOCIAL COSTS INTO MARGINAL TAX REFORM MODELS.

Recent work carried out by the author (Madden 1989, 1992a) examines the Irish indirect tax system from the point of view of the marginal social cost (MSC) of taxation associated with raising the tax rates on different goods. A feature of the results was the relatively wide dispersion of values of the MSC for the different goods, suggesting that there was room for marginal reforms that would raise welfare while leaving revenue unchanged. A further feature of the results was the consistently high MSC of increasing the tax on tobacco. As the model used in that work did not take account of the possibility of externalities associated with the consumption of various goods, it was speculated that the very high tax on tobacco might reflect a corrective or Pigovian tax, arising from the divergence between the private and public marginal evaluation attached to the consumption of tobacco.

This paper attempts to incorporate such externality effects into the model used in previous work on tax reform. It does so using a technique known as the "inverse optimum" which infers the external effects implicit in the indirect tax system. Furthermore, the degree to which the government is concerned with distributional issues may also influence indirect taxes, and so the implied inequality aversion of the government is also estimated.

The model used is essentially that of Madden (1989, 1992a) except that we incorporate external effects. We have a Bergson-Samuelson social welfare function:

\[ V(q) = W(v^1(q), v^2(q)\ldots, v^h(q)). \]  
(1)

where \( v^h \) is the indirect utility function of household \( h \), giving the maximum utility attainable at prices \( q \). We assume incomes are fixed.

The aggregate demand vector is given by
\[ X(q) = \sum_h x^h(q) \]  

and government revenue is given by

\[ R = t \cdot X = \sum_i t_i X_i \]  

where \( t \) is a vector of specific taxes.

The tax reform model we use involves the calculation of the MSC of increasing the tax on different goods. Formally the expression for MSC, more commonly referred to as \( \lambda_i \), is:

\[ \lambda_i = -\frac{\frac{\partial V}{\partial t_i}}{\frac{\partial R}{\partial t_i}} \]  

Intuitively it is obvious that at a welfare optimum these \( \lambda_i \) should be equal, since otherwise it would be possible, by lowering the tax on a good with a high \( \lambda_i \) and raising the tax on a good with a low \( \lambda_i \), to increase welfare for constant revenue.

\( \lambda_i \) can be re-expressed in a way that is readily calculable (see Ahmad and Stern 1984 for the derivation).

\[ \lambda_i = \frac{\sum_h \beta_i^h q_i x_i^h}{q_i X_i + \sum_k \varepsilon_{ki} \tau_k q_k X_k} \]  

where \( \beta_i^h \) is the welfare weight of household \( h \) (introduced to take account of distributional considerations), \( q_i x_i^h \) is the value of expenditure on good \( i \) by household \( h \), \( q_i X_i \) is the total expenditure on good \( i \), \( \varepsilon_{ki} \) is the uncompensated cross-elasticity of demand for good \( k \) with respect to a change in the price of good \( i \) and \( \tau_k \) is the \textit{ad valorem} tax on good \( k \), expressed as a fraction of the consumer price.

Formally it is quite straightforward to incorporate external diseconomies, or social costs into this model. In the expression above the social welfare function (Equation 1) is of the form \( V = W(v^1, ..., v^h, v^H) \). This function can be modified to take account of the social costs associated with the consumption of certain goods by including total expenditure of the goods in question as separate arguments in the social welfare function. Thus, our social welfare function becomes \( W(v^1, ..., v^h, v^H, X_1, ..., X_N) \) where the external effect is introduced via the inclusion of the aggregate consumption of the good into the social welfare function. The sign of \( \frac{\partial V}{\partial X_i} \) depends upon whether the good in question is a social "good" or "bad". This inclusion of this term alters the expression for the numerator of \( \lambda_i \) as follows:
\[
\frac{\delta W}{\delta t} = -\Sigma_h \beta^h x^h_i + \Sigma_k \mu^*_k \left( \frac{\delta X_k}{\delta q_i} \right)
\]  
(6)

where \( \mu^*_k = \frac{\delta W}{\delta X_k} \). Thus the expression for \( \lambda_i \) becomes:

\[
\lambda^*_i = \frac{\Sigma_h \beta^h q_i x^h_i - \Sigma_k \mu_k \epsilon_{ki} q_k X_k}{q_i X_i + \Sigma_k \epsilon_{ki} \tau_k q_k X_k}
\]  
(7)

where \( \mu_k = \frac{\mu^*_k}{q_k} \).

Alternatively we can express it as follows

\[
\lambda^*_i = \lambda_i - \frac{\Sigma_k \mu_k \epsilon_{ki} q_k X_k}{q_i X_i + \Sigma_k \epsilon_{ki} \tau_k q_k X_k}
\]  
(8)

Thus, a non-zero value for \( \mu_k \) affects the MSC of an indirect tax on good \( k \) itself, and in general also affects that on every good, through the workings of cross-price effects.

It is worthwhile working through a couple of examples to see how the \( \mu_k \) term affects the MSC. Since it does not enter the expression for the denominator, we need only concentrate on the expression for the numerator. To further simplify matters we will neglect distributional issues, and assume \( \beta^h = 1 \) for all households (i.e. all households have equal welfare weights). Suppose we take the example of a good such as tobacco, where we assume that \( \mu_k < 0 \). The numerator of the expression for \( \lambda_i \) then becomes: \( q_i X_i - \mu_{tob} \epsilon_{tob,i} q_{tob} X_{tob} \). If the two goods are substitutes \( \epsilon_{tob,i} > 0 \), and thus the second term in the expression above is also positive, thus tending to increase the marginal social cost of the tax on good \( i \). The intuition behind this is that if the goods are substitutes, then increasing the tax on good \( i \) will cause a substitution towards consumption of tobacco, which will have negative external effects. An analogous explanation can be presented for the case where the goods are complements.

The two crucial questions to be addressed are: How do we obtain estimates of the \( \mu_k \) and for which goods do we attempt to find these estimates? In principle we could attempt to find estimates of the \( \mu_k \) for all goods. However, as will be explained below, this is not desirable using the approach adopted in this paper. There is, to some extent, a trade-off between the number of goods for which we will attempt to find estimates of \( \mu_k \) and the reliability of those estimates. Thus, we arbitrarily choose which goods we feel may reasonably be expected to have \( \mu_k \) different from zero. The relatively broad aggregates of goods we are using constrained us to choosing three goods which we feel, a
priori, might have $\mu_k < 0$. They were tobacco, alcohol and petrol. Given the relatively broad aggregate of goods used in this study, we did not feel there was any good for which $\mu_k > 0$.

The other question to be answered is where to obtain estimates of the $\mu_k$? We could simply impose exogenous values, as is usually done with the $\beta^h$, perhaps deriving them from health studies, in the case of alcohol and tobacco, or from environmental studies in the case of petrol. (For a recent example of such an approach in the case of alcohol, see the paper by Holm and Suoniemi 1992). An alternative route is to attempt to derive them from what has become known as the "inverse optimum" approach. This approach was first adopted by Christiansen and Jansen (1978) for the case of Norway, who also, in the same paper, calculated MSCs for the Norwegian indirect tax system. (Other examples of this approach are Decoster and Schokkaert 1989 and Craggs 1990 for the Belgian and Canadian tax systems respectively.) Christiansen and Jansen's paper attempted to find the social preferences implicit in the Norwegian indirect tax system, viz. the implicit value of $\epsilon$ (the inequality aversion parameter, which enters the expression for $\lambda_i$ via the $\beta^h$ term), the implicit equivalence scales and the implicit social costs associated with certain goods. In this paper we will apply this methodology to the Irish indirect tax system, although we will not address the issue of the implicit equivalence scales.

Adapting their solution procedure to our model, we maximise a social welfare function with respect to specific taxes, $t_1, \ldots, t_n$ subject to the fiscal requirement $\Sigma t_j X_j = R$. We form the Lagrangian:

$$ L = W(v^1, \ldots, v^H, X_1, \ldots, X_n) - \lambda(\Sigma t_j X_j - R). $$

(9)

$\lambda$ is the Lagrange multiplier associated with the fiscal constraint. We derive the first order conditions for the social optimisation problem as:

$$ -\Sigma_i \beta^h x^h_i + \Sigma_k \mu_k^* \delta X_k / \delta q_i = \lambda(X_i + \Sigma_k t_k \delta X_k / \delta q_i) $$

(10)

where the notation is as before. Note the similarity between this condition and the expression for $\lambda_i$ in (7), with the only differences being that we have not expressed it in terms of elasticities and ad valorem taxes and we are assuming a common value of $\lambda_i$ for all goods. What we are essentially saying here is that assuming the existing indirect tax system is optimal, in the sense

2. In this study we use the same breakdown of expenditure as in Madden (1992a). There are ten goods: food, alcohol, tobacco, clothing and footwear, fuel and power, petrol, transport and equipment, durables, other goods and services. We estimate values of $\mu_k$ for 1987.
that all the $\lambda_i$ are equal, what are the implied values of the $\beta^h$, the $\mu^*_k$ and $\lambda$? We can thus solve for these parameters on the assumption that the existing tax system is optimal.

Before going any further, it is necessary to provide a more explicit functional form for the $\beta^h$. We use the utility of income function first introduced by Atkinson (1970). Atkinson generated welfare weights from the following function:

$$U^h(I) = \frac{kI^{1-e}}{1-e}, \quad e \neq 1, \quad e > 0$$

$$= k \log(I), \quad e = 1$$

where $I^h$ is total expenditure per equivalent adult of the $h^{th}$ household. $\beta^h = U^*(I^h)$ and we can normalise $\beta^h$ through choice of $k$ so that $\beta^h$ for the poorest household is unity. Then we have $\beta^h = (I^1/I^h)^e$. Thus $e$ can be viewed as an inequality aversion parameter, since $e = 0$ implies equal weights for all households, while $e > 0$ implies $\beta^h < 1$ for $h > 1$, so that increments of expenditure to the poor are seen to have a higher marginal social value than those to the rich.

Using this definition of $\beta^h$, and following the usual manipulation to express the above first-order condition in elasticity terms, we have the following expression:

$$-\Sigma_{h} (I^1 / I^h)^e q_i x_i^h + \Sigma_k \mu_k \varepsilon_{ki} q_k X_k = \lambda (q_i X_i + \Sigma_k \varepsilon_{ki} T_k q_k X_k)$$

where $\mu_k = \mu^*_k / q_k$.

In the above expression we can obtain values for $q_i x_i^h$ and $q_i X_i$ from the Household Budget Survey, $T_k$ from Revenue Commissioners' Reports and Budget booklets, while the $\varepsilon_{ki}$ used are taken from Madden (1992a). The unknowns in the above expression are $e$, $\lambda$, and the $\mu_k$. We can estimate them using a non-linear estimation procedure as we have ten observations (one for each good) on this equation. However, before involving ourselves with non-linear estimation, we can adopt a simpler route, by simply imposing a value of $e$, which effectively linearises the above expression.

We will initially examine the case where $e = 0$. Thus, we are looking at the

3. The particular elasticities chosen were obtained from an unrestricted Almost Ideal Demand System estimated in levels and evaluated for 1987. For details see Madden (1992a, 1992b). Obviously the values of $e$, $\lambda$ and $\mu_k$ estimated will be sensitive to these elasticities. While the issue of sensitivity will not be addressed here, see Madden (1992c) for a discussion of the sensitivity of $\lambda$, in general to the estimated elasticities.
case where we are assuming that the government's preferences are of the extreme utilitarian variety i.e. it is not concerned with distributional considerations. The above expression can then be estimated by OLS. (A further reason for carrying out linear estimation first is that it gives a reasonable set of starting values for the non-linear estimation.)

An alternative equation which we could attempt to estimate is Equation (8), since we have estimates of the $\lambda_i$ from Madden (1992a). Thus our estimate of $\lambda^*$ would simply be the estimate of the constant from this regression. Note however, that although the $\mu_k$ in both expressions are the same, we would not expect the estimates to be the same since Equation (12) is OLS without an intercept while Equation (8) is WLS with an intercept. In fact, the estimates of $\mu_k$ turn out to be quite similar in magnitude. We will concentrate on the estimates from Equation (12) since the non-linear version that we estimate is in this form also.

We now explain why there is a trade-off between the number of $\mu_k$ we try to estimate and the reliability of these estimates. Firstly, the more $\mu_k$ are included on the RHS of (11), the fewer degrees of freedom we have and the more highly determined the equation becomes until eventually we would be solving a system of equations rather than estimating a relationship. The inclusion of a $\mu_k$ for all goods would mean that we are essentially saying that the tax system is as it is because the government has decreed that there are special effects attaching to all goods, and therefore the current system must be optimal. In only choosing three goods, what we are saying is that the current system of indirect taxation is optimal (barring estimation errors) when external effects for these goods are included.

Table 1 in the Appendix gives the results of this estimation. The coefficients are negative, as expected, with a common value of $\lambda$ of 1.06. The standard errors for the $\mu_k$ are comparatively large but in line with those obtained for the Norwegian and Canadian cases. Thus, these are the external effects implicit in the Irish indirect tax system of 1987, given zero inequality aversion. We normalise these values by dividing by $\lambda$. There are two reasons for doing this. Firstly, we will be estimating $\mu_k$ for the case where $e>0$, which will essentially involve a normalisation of the welfare function and so to facilitate comparison of the $\mu_k$ across the cases of different levels of inequality aversion we need to normalise. Secondly, the choice of $\lambda$ as normalising variable seems reasonable as it gives the externality effects in terms of the gross welfare loss to taxpayers of raising one extra unit of revenue. Thus at the margin, the consumption of one unit extra of tobacco gives the same social welfare loss as would the raising of 0.83 units of revenue.

Further understanding of these figures can be obtained by seeing how they affect $\delta V/\delta t_i$ as shown in Table 2 in the Appendix. This expression is
\[ \Sigma_h \beta^h q_i x_i^h - \Sigma_k \mu_k \epsilon_{ki} q_k X_k, \text{ with } \beta^h = 1 \text{ in this case. Thus taking the case of food, } q_i X_i = 49.977, \text{ while } \Sigma_k \mu_k \epsilon_{ki} q_k X_k = 12.04. \] The numerator thus becomes 37.94, indicating that the MSC of food is reduced, owing to its cross-price effects with the goods which have external effects i.e. on balance food is complementatory with goods with negative social effects. Of course, the effect could work the other way. If \( \Sigma_k \mu_k \epsilon_{ki} q_k X_k < 0 \), then the MSC of a good will be increased by the inclusion of external effects. This happens for alcohol, fuel and power, services and is marginally the case for clothing and footwear and transport and equipment.

Having obtained estimates for the \( \mu_k \) we can now recalculate the MSC of each good including the \( \mu_k \) in the expression for \( \delta V/\delta t \) (what we termed \( \lambda^*_i \) above; these are presented in Table 2). However, a word of caution should be entered here. There is a certain sleight of hand in calculating these \( \lambda^*_i \). We estimated the \( \mu_k \) on the basis that \( \lambda = \lambda \) for all \( i \), and then used these \( \mu_k \) in the calculation of the \( \lambda^*_i \). In many ways the different \( \lambda^*_i \) may merely reflect the poor fit in the calculation of the \( \mu_k \) in the inverse optimum problem. Thus, in some ways it is inconsistent to “solve” an inverse optimum problem and then calculate welfare improving directions of tax reform. In defence of this procedure it must be pointed out that estimates of the \( \mu_k \) must be obtained from somewhere and their calculation from the inverse optimum problem is a potentially interesting option.

Before presenting these \( \lambda^*_i \) we must point out a further possible paradox. As we have seen, if \( \Sigma_k \mu_k \epsilon_{ki} q_k X_k > 0 \), then the MSC of a good is reduced. It is possible that \( \Sigma_k \mu_k \epsilon_{ki} q_k X_k \) be so large as to make the numerator negative i.e. \( \Sigma_h \beta^h q_i x_i^h - \Sigma_k \mu_k \epsilon_{ki} q_k X_k < 0 \). Thus, welfare would be increased by raising the tax on this good owing to the reduced consumption of social “bads” it would induce. This actually occurs for two goods in the Irish case, tobacco and durables. Coincidentally, these are the two goods for which \( \delta R/\delta t \) was found to be less than zero in Madden (1992a) i.e. those goods whose tax was so high it was beyond its revenue maximising level. Thus, when we recalculate \( \lambda^*_i \) including external effects, calling them \( \lambda^* \), we find that the MSC returns to being positive, but for the “wrong” reason. Instead of \( -\delta V/\delta t > 0 \) and \( \delta R/\delta t > 0 \), we have \( -\delta V/\delta t < 0 \) and \( \delta R/\delta t < 0 \! \).

The inclusion of the \( \mu_k \) also alters the ranking of goods by MSC as well as narrowing their spread. While the fact that the rankings are very sensitive to the inclusion of social costs may seem somewhat alarming, it is worth noting that the same phenomenon occurs in the Norwegian and Belgian cases. The inclusion of social costs has differing effects on the MSCs of the three externality-creating goods. As indicated above, the case of tobacco is difficult to interpret. However it seems reasonable to suggest that if the tax on tobacco were at a rate such that \( \delta R/\delta t > 0 \), then the inclusion of social costs would
lower the MSC of raising its tax. The same is true of petrol, but as was
tioned above, the inclusion of external effects actually increases the MSC
of raising the tax on alcohol. This may appear strange, given that we have
identified a negative external effect for alcohol, but it is explained by the
pattern of substitutability and complementarity with the other goods with
external effects. In particular, our elasticity estimates suggest that alcohol is
highly complementary with petrol and it is this which contributes most to the
paradoxical result.4

Of course, ideally we would like to estimate the value of e rather than
impose it. It is possible that what we estimate as an external effect may, in
fact, reflect distributional considerations on behalf of the government. For
example, an estimated negative externality for a good such as alcohol may
reflect the fact that it is disproportionately consumed by higher income
households and so the social cost in fact reflects distributional concerns.
Before attempting to estimate e, our inequality aversion parameter, it is
worth looking at what Feldstein terms the "distributional characteristic" of
each good for different levels of e (see Feldstein 1972 and Atkinson and
Stiglitz 1980). This essentially shows what the relative MSCs of goods are,
when we are solely taking account of distributional considerations i.e. the
values of \( \lambda_i \) when \( \epsilon_i = 0, \forall i,j \). Table 3 shows these characteristics. When \( e = 0 \)
they are all equal to unity. However, as \( e \) increases, goods that are necessities
get relatively higher and luxuries a relatively lower value for the distribu-
tional characteristic. As might be expected, food has a relatively high
characteristic and so too has tobacco. Petrol and alcohol have relatively low
characteristics. Thus, we might expect that the introduction of distributional
considerations might reduce the estimated externality attached to tobacco,
while leaving those for petrol and alcohol relatively unchanged.

Recall expression (12) for the first-order condition when including the
full expression for \( \beta^h \). As we can see, this is a non-linear expression with
unknowns, e, \( \lambda \), \( \mu_{\text{obs}} \), \( \mu_{\text{pet}} \) and \( \mu_{\text{alc}} \). To facilitate non-linear estimation we re-
write the expression as follows:

\[
q_i X_i = \left[ \left( -\Sigma_h \left( I^1 / I^h \right) e_i X_i^h i + \Sigma_k \mu_k e_{k_i} q_k X_k \right) \right] / \lambda + \Sigma_k e_{k_i} t_k q_k X_k \tag{13}
\]

4. It should be noted that a number of the cross-elasticities estimated for alcohol were quite
sensitive to the imposition of such restrictions as homogeneity and especially symmetry. See
Madden (1992b) for details.
We have ten observations of this equation and five parameters to estimate. The non-linear programme in SHAZAM was used for estimation. This is a quasi-Newton method and the reader is referred to the SHAZAM manual (White et al. 1990) for further details. One of the crucial elements in non-linear estimation is a suitable choice of starting values for the parameters being estimated. A number of different starting values for the parameters were experimented with, commencing with the values obtained in the linear regressions and also including a few "rogue" values. In all cases the estimates converged to the values given in Appendix Table 1 suggesting that this is a global rather than merely a local optimum. The typical number of iterations was 15-20.

First of all, note that the estimate for $e$ is not significantly different from zero, suggesting that the Irish indirect tax system is not progressive. Even if the coefficient were significant, it would still indicate that the government's preferences, as indicated by the indirect tax system were virtually utilitarian. Furthermore, when Equation (13) was re-estimated with the $\mu_k=0$, a negative estimate of $e$ was obtained, suggesting that the government's preferences, in the absence of external effects, are regressive. (More formally, a value of $e \leq 0$ implies that the social welfare function violates "s-concavity". Sen (1973) provides a discussion of s-concavity). The lack of inequality aversion is most probably caused by the high tax on tobacco, the good with the highest distributional characteristic. This finding is consistent with the results of Madden (1992a) where it was shown that the ranking of goods by MSC was relatively insensitive to changes in $e$, suggesting that the Irish indirect tax system is relatively inefficient at addressing distributional issues. Sah (1983) provides a theoretical explanation of why, in general, one might expect this to be the case.

The estimates for the $\mu_k$ are very similar to those in the linear case, which is not surprising, given that the estimate of $e$ is so close to the value of zero which was imposed in the linearised case. Thus, we do not present the values of $\lambda_i^*$ for these $\mu_k$ since they are virtually identical to those presented in Table 2.

To summarise the results of this paper, we have applied the inverse optimum technique to estimate possible external effects and also the degree of inequality aversion implicit in the Irish indirect tax system. Our results suggest that there is virtually no inequality aversion in the indirect tax system and also underline the importance of patterns of substitutability and complementarity in analysing external effects. This latter aspect was also highlighted in Madden (1992a) and it reinforces more than ever the importance of reliable elasticity estimates. The sensitivity of Irish consumer demand elasticities to such factors as stochastic specification is examined in
Madden (1992b) where a wide range of elasticities is presented. Not for the first time in applied work on the Irish economy, the importance of having reliable estimates of crucial parameters is stressed.

REFERENCES


