Footprint caused by a vehicle configuration on the dynamic amplification of the bridge response

Omar Mohammed\textsuperscript{1} and Arturo González\textsuperscript{1,2}

\textsuperscript{1}School of Civil, Structural and Environmental Engineering; Newstead; University College Dublin; Belfield, Dublin 4; Ireland

Abstract: The passage of a vehicle over a bridge leaves a unique footprint in the form of measured strains (or displacements) across the structure. This paper proposes a new level I damage detection method for short-span bridges using footprints of Dynamic Amplification Factor (DAF) versus vehicle speed. The total response of a bridge to a moving load is time-varying, and it can be assumed to be made of two components: ‘static’ and ‘dynamic’. Here, DAF is defined as the ratio of the maximum total response to the maximum ‘static’ component. For a given bridge, DAF patterns will vary with vehicle configuration. However, for a vehicle configuration (or a number of them), the mean DAF pattern measured on the bridge will remain unaltered unless the conditions of the bridge changed. The latter is the subject of investigation in this paper. In order to test the feasibility of using these patterns for monitoring purposes, damage is simulated within a bridge model as stiffness losses of 10\% and 30\% at mid-span. Changes in stiffness are identified by differences between DAF patterns corresponding to the healthy and damaged bridges. Results show to be more sensitive to damage than a traditional level I damage detection method based on variation of natural frequencies.

1. Introduction

Bridge damage detection techniques have gained increasing consideration from the engineering communities to prevent sudden structural failure leading to catastrophic, economic, and human life losses. Local damage detection techniques, such as ultrasonic methods and X-ray methods, entail that there is damage and that the vicinity of damage is known a priori and readily accessible for testing. Therefore, vibration-based damage detection methods are typically used first to establish if there has been damage (level I technique), the location (level II), its quantification (level III) and finally the remaining life of the structure (level IV) [1]. The essential idea behind vibration-based damage identification is that damage induces changes in the physical properties (mass, damping, and stiffness) that will be noticeable in modal properties (i.e., natural frequencies, damping and mode shapes). Clearly, a drop in stiffness in a narrow localised portion of the bridge will go unnoticed to the ‘static’ component of a strain measurement (i.e., in a static loading test), unless the transducer would be located at the damaged location [2]. However, the total strain due to a moving load will be affected by a localised loss in stiffness, as a result of the changes in the ‘dynamic’ component which is related to the modal properties of the bridge.

As a vehicle traverses a bridge, strain at a given section varies with the vehicle position. If the same vehicle crossed the bridge along the same transverse path with the same speed a second time, the

\textsuperscript{2}Corresponding author: Phone +353-7163219, Fax: +353-1-7163297, email address: aturo.gonzalez@ucd.ie
bridge strain would not vary except for a possible alteration in the initial conditions. If the vehicle ‘static’ configuration (vehicle ‘static’ configuration refers here to number of axles, their weights and spacing) or speed changes, then the response of the bridge will also change. Similarly, the measured strain due to a given vehicle ‘static’ configuration will change with an alteration of the structural bridge parameters (boundary conditions, stiffness distribution). Although ‘dynamic’ mechanical properties of the vehicle (i.e., stiffness and damping of tires and suspension, rotationary moments of inertia associated to the sprung masses, etc…) will also affect the bridge response, these are not considered here, as it is impracticable to assume that these properties could be identified on the field. However, ‘static’ mechanical properties and speed of a vehicle can be readily captured via a Weigh-In-Motion pavement-based system near the bridge or a Bridge Weigh-In-Motion installed on the bridge. Therefore, it is assumed that the impact of a vehicle fleet with a given ‘static’ configuration on a bridge can be characterized by a mean response, except for deviations due to vehicular ‘dynamic’ parameters. The road profile will also have a significant impact on the vehicle forces. While rough profiles will typically lead to a high variability in the bridge response, smooth profiles will produce more consistent (less variable) bridge responses. In fact, the response of a bridge to a sprung vehicle model running on a smooth (flat) profile is practically identical to that of a simpler model made of constant forces equal to the static axle weights of the sprung model. Therefore, this paper analyses the Dynamic Amplification Factor (DAF) patterns resulting from different vehicle ‘static’ configurations and how the “vehicle configuration – DAF” footprint on the bridge response can be used to monitor not only the passing traffic, but also the structural health of the bridge.

The impact of vehicle speed and ‘static’ configuration, namely number of axles, axle weights and their spacings, on the bridge response is now well understood in terms of their incremental static load effect as shown by the literature. For example, vehicle speed is considered a factor of supreme importance affecting bridge dynamics [3-7]. General speaking, the bridge dynamic response tends to: (i) increase with an increase in vehicle speed; (2) decrease as the vehicle weight increases [8-11] although [12] and [13] find that both static and dynamic components increase as the mass of the vehicle increases. The influence of the number of axles on the bridge response to a moving vehicle are specified by [14] (OMT 1991) and [15] (CSA 2006), however, their conclusions are related to particular scenarios and need to be interpreted with caution.

In order to test the use of changes in DAF patterns as a monitoring tool, a bridge is modelled as simply supported finite element discretized beam and vehicles are modelled as series of moving constant loads. This relatively simple analysis ignores interaction between vehicle and bridge, but it is used to capture and understand the underlying pattern (i.e., a contour plot of measured load effect versus time and section location) associated to different ‘static’ vehicle configurations that could be hindered by more complex models. This ‘healthy’ pattern is characterized by matching effects between the natural period of the bridge, the vehicle speed and configuration. Then, DAF is employed to detect changes in structural stiffness. In this process, the identification of the correct vehicle ‘static’ configuration, i.e., via a Weigh-In-Motion installed on the site, is key to the successful application of this method.

2. Simulation models

2.1 Bridge model

The bridge model consists of 150 discretised 1D beam elements (each element being 0.1 m long with two degrees of freedom at each end node). The beam section is assumed to have an inertia of 0.5273 m^4 and a mass per unit length of 28125 m^{-1} kg. The beam material has a modulus of elasticity of 35 GPa. The bridge in its healthy form is assumed to have constant properties throughout the bridge length. The values adopted for these properties are typical of a simply supported short-span solid slab deck, and they lead to main natural frequencies of 5.65 and 22.62 Hz. Damping ratio is assumed to be 0.03 for all modes.
2.2 Vehicle model

A model consisting of a series of constant point loads is used to represent a single vehicle crossing a simply supported bridge at uniform speed. This simulation model is obviously different from the reality in many aspects, as it does not take into account vehicle dynamics, road roughness and the interaction between vehicle and bridge. However, it does provide some insight into the underlying patterns in more complex simulation models. Three types of vehicle configurations based on the Eurocode 1 part 2 [16] are investigated (table 1).

<table>
<thead>
<tr>
<th>Axle spacing (m)</th>
<th>Frequent axle loads (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>90</td>
</tr>
<tr>
<td>190</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>80</td>
</tr>
<tr>
<td>1.3</td>
<td>140</td>
</tr>
<tr>
<td>140</td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>90</td>
</tr>
<tr>
<td>180</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>110</td>
</tr>
<tr>
<td>110</td>
<td></td>
</tr>
</tbody>
</table>

2.3 Calculation of the Bridge Response

The response of the discretized beam model to a series of moving time varying forces is given by the system of equations in Equation (1).

\[ M\ddot{y} + C\dot{y} + Ky = Nf \]  

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices respectively of the beam model and \( \ddot{y}, \dot{y} \) and \( y \) are vectors of nodal bridge acceleration, velocity and displacement respectively. \( f \) is the time varying external forces vector. \( N \) is a \((n \times nf)\) matrix for distributing \( nf \) forces to the \( n \) nodal degrees of freedom of the finite element beam. Rayleigh damping is used here to model the damping and is given by:

\[ C = \alpha M + \beta K \]

where \( \alpha \) and \( \beta \) are constants. These constants are obtained from \( \alpha = 2\zeta \omega_1 \omega_2 / (\omega_1 + \omega_2) \) and \( \beta = 2\zeta / (\omega_1 + \omega_2) \), where \( \omega_1 \) and \( \omega_2 \) are the first two natural frequencies of the bridge in rad/s. The damping ratio \( \zeta \) is assumed to be 0.03, the same for all modes [17].

Equation (1) is implemented in MATLAB and solved using the Wilson-Theta integration scheme [18] with a time interval of 0.002 s. The optimal value of the parameter \( \theta = 1.420815 \) is used for unconditional stability in the integration scheme. Initial conditions are assumed to be zero for all displacements, velocities and accelerations.
3. Definition and Testing of the Proposed Level 1 Damage Detection Algorithm
DAF has been used to assess the dynamic bridge response due to the crossing of vehicles by many researchers [19-22]. In this paper, DAF will be applied to damage detection. For this purpose, nine sensors are assumed to be uniformly spaced along the beam described in Section 2 and DAF is calculated for each sensor location using simulations of the response to a passing vehicle. The first and last sensors are located at 1.5 m and 14.5 m respectively from the first support and sensors in-between are spaced by 1.5 m each. Here, DAF refers to the ratio of maximum total strain (‘static’ + ‘dynamic’) to maximum ‘static’ strain for a given sensor location. Strain is calculated at the bottom of these nine locations based on the assumption that the neutral axis is located at 0.375 m of the measurement point.

The damage scenario is represented as a localised reduction in stiffness at 10 elements at and around the mid-span section (covering a full length of 1 m). Two different damage levels are simulated, i.e., stiffness reductions of 10% and 30%. The first natural frequencies of the bridge for each of the three states and the frequency ratios of the damaged states to the healthy one are shown in Table 2 for reference purposes. It can be noticed that the difference in first natural frequency amongst the different states is relatively small.

<table>
<thead>
<tr>
<th>Bridge type</th>
<th>First natural frequency (Hz)</th>
<th>Frequency ratio of damaged to healthy</th>
</tr>
</thead>
<tbody>
<tr>
<td>healthy</td>
<td>5.65</td>
<td>1</td>
</tr>
<tr>
<td>10 % stiffness reduction at mid-span</td>
<td>5.61</td>
<td>0.99</td>
</tr>
<tr>
<td>30 % stiffness reduction at mid-span</td>
<td>5.48</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 2. Main frequencies of the three bridge states under investigation

The damage detection algorithm relies on a prior knowledge of the DAF pattern for the ‘healthy’ structure (or structure at a time $T$), and by comparison to this DAF at time $T$, it establishes if the structural conditions at time “$T + \Delta T$” have changed. Hence, the first step of the algorithm is the calculation of DAF for the ‘healthy’ bridge due to the three vehicle configurations under investigation.

3.1 DAF pattern at a point in time $T$
For the calculations in this section, the bridge at time $T$ is assumed to be in a ‘healthy’ state (i.e., as defined in Section 2.1). Figure 1 shows a 3D plot of DAF versus velocity (km/h) due to the crossing of the 2-axle truck defined in table 1. Velocity increments of 1.08 km/h are employed in the generation of this graph.

Figure 1. DAF at 9 bridge locations due to a 2-axle truck versus velocity in a healthy bridge

Then, the plot is divided into slices to work out a damage index. For example, figures 2(a) and (b) show slices at two consecutive velocities, 42.12 km/h and 43.2 km/h. The area under each slice is calculated using the trapezoidal rule. These areas are used to calculate the volume of each strip between two slices using Equation (3).
\[ Vol_{\text{strip}}(i) = \frac{A_{\text{slice}(j)} + A_{\text{slice}(j+1)}}{2} \times 1.08 \] (3)

where, \( i \) and \( j \) are the number of the strips and slices respectively. The total volume of the plot is calculated by the sum of the volumes of all individual strips between consecutive slices.

Figure 2. DAF at nine locations of the healthy bridge due to the passage of a 2-axle truck:
(a) velocity of 42.12 km/h, (b) velocity of 43.2 km/h

3.2 DAF pattern at a point in time \( T + \Delta T \)

Figure 3 shows the 3D plot of DAF versus velocity (km/h) for a 2-axle vehicle passing over bridge with a 30% stiffness loss localised at mid-span. The total volume of this plot is calculated using the same method defined in Section 3.1. Figures 4(a) and (b) show slices at two subsequent velocities 42.12 km/h and 43.2 km/h.

Figure 3. DAF versus velocity due to a 2-axle truck crossing a bridge with a 30% stiffness loss at mid-span

Figure 4. DAF at nine locations versus two subsequent velocities of a 2-axle truck crossing damaged bridge with 30 % stiffness loss at mid-span: (a) velocity of 42.12 km/h, (b) velocity of 43.2 km/h
3.3 Comparison of DAF patterns at different points in time

DAF corresponding to two points in time are subtracted for depicting if significant change had occurred. To quantify the damage, the resultant DAF ("damaged-healthy") is shown in the contour plots of figures 5-7 for the two percentages of stiffness loss and the three vehicle configurations.

Figure 5. Resultant DAF using a 2-axle truck: (a) 30% stiffness loss, (b) 10% stiffness loss

Figure 6. Resultant DAF using a 3-axle truck: (a) 30% stiffness loss, (b) 10% stiffness loss

Figure 7. Resultant DAF using a 5-axle truck: (a) 30% stiffness loss, (b) 10% stiffness loss
As expected, changes in DAF for 10% stiffness loss at mid-span are not as clear as for 30% stiffness loss. Table 3 shows the values of the DAF damage indexes. The resultant DAF damage indexes are the subtraction between the two volumes of the damaged and ‘healthy’ bridges for each of the vehicle configurations (the volumes have been calculated using Equation (3)). From table 3, it can be noticed that, the bridge under the impact of a 2-axle truck is more sensitive to the mid-span damage than the other two vehicles (3-axle and 5-axle trucks). The results show that the DAF damage index is comparatively more sensitive to changes in stiffness than those changes in frequencies shown in table 2.

<table>
<thead>
<tr>
<th>Truck Configuration</th>
<th>Percentage of stiffness loss</th>
<th>10%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-axle truck</td>
<td>0.663</td>
<td>2.621</td>
<td></td>
</tr>
<tr>
<td>3-axle truck</td>
<td>0.615</td>
<td>2.320</td>
<td></td>
</tr>
<tr>
<td>5-axle truck</td>
<td>0.265</td>
<td>1.173</td>
<td></td>
</tr>
</tbody>
</table>

The authors acknowledge that a beam can be a crude one-dimensional representation of a bridge deck. However, the aim of this paper is carrying out a preliminary test of the potential of a new damage detection method before characterizing its sensitivity and accuracy levels in more realistic situations. While low DAFs take place as a result of destructive interference between the ‘static’ and ‘dynamic’ components of the response, high DAFs are due to simultaneous occurrence of peaks in the both components. Consequently, DAF depends on the synchronization between the two components (‘static’ and ‘dynamic’); which may vary due to changes in the structure, including damage. Therefore, the response of a beam to moving loads and bridge inertial forces, which contains ‘static’ and ‘dynamic’ features that can be found in more complex models and in reality, has been deemed to be sufficient to meet this aim. As mentioned in Section 1, a weigh-in-motion installation is needed at the site to measure vehicle speed, static weights, configurations and multiple vehicle presence, and undoubtedly, there will be inaccuracies in these measurements as well as in those of the bridge response due to these vehicles. Additionally, it is true that vehicle dynamics will introduce deviations with respect to a reference mean pattern (i.e., symbolized by a series of constant moving loads in this paper).

In spite of the large number of uncertainties associated to mathematical models and field data, González et al [20] demonstrate that a vehicle model represented by a series of moving constant loads can resemble DAF patterns found in more complex Vehicle-Bridge Interaction models or in measurements. More specifically, they measure the response of a two-lane 24.8 m long simply supported bridge to a population of more than 16,000 two-axle trucks to gather a site-specific mean DAF pattern. Information on static weights, spacings and speed of each truck is gathered via a weigh-in-motion system installed on the site. They also carry out theoretical simulations using an orthotropic plate finite element model that vibrates at the same frequencies than the true bridge. In their simulations, they use different vehicle models: (a) based on a series of constant moving loads with weights and spacings that match those measured by the weigh-in-motion system; (b) based on body masses supported by suspension and tire spring-damper systems. They conclude that, it is possible to find consistent DAF patterns for a given truck population, although larger variability in DAF (common when using complex models with a rough profile or in a real-life situation) demands a larger sample size for an accurate pattern characterization. If a mean DAF pattern can be reproduced for a bridge crossed by a truck population identified via a weigh-in-motion system (as in [20]), clearly there is scope for the application of a damage indicator based on relative differences between DAF patterns in two points in time.

4. Conclusions
This paper has introduced a new level I damage detection method based on changes in DAF at different points in time. The method assumes that the vehicle speed, number of axles and their spacing
and weight distribution can be obtained on-site, i.e., via a weigh-in-motion system. It has been theoretically tested with the simulated response of a 15 m simply supported bridge to a series of moving constant loads resembling three types of vehicle configurations (2-axle, 3-axle and 5-axle trucks). The responses have been obtained at nine equally spaced locations that allowed gathering a full picture of the DAF patterns throughout the length. Damage has been modelled as a localised reduction in stiffness of 10% and 30% at bridge mid-span. The DAF damage index that has resulted from subtracting the DAF of a ‘healthy’ bridge from that of damaged ones have shown to be potentially able to detect damage. The resultant 3D plots have been shown to be more sensitive to the crossing of the 2-axle truck than the other vehicle configurations. Despite the natural frequency of the damaged bridge having changed very slightly for the 10% stiffness loss, the new method has still been able to successfully detect changes in the structural response. However, a number of issues still need to be addressed in the future which include: influence of variations in vehicle dynamics and road profile on the pattern, inaccuracies in obtaining the static response via filtering, etc...

5. References:
[15] Canadian Standards Association (CSA) 2006 Canadian highway bridge design code, Mississauga, ON, Canada.
Acknowledgement
The authors are grateful for the financial support received from the Iraqi Ministry of Higher Education and Scientific Research (MoHESR) towards this investigation.