Addendum to Eleftheriou and Michelacakis (2016)*

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Abstract

Following the publication of Eleftheriou and Michelacakis (2016a), it was brought to our attention that the problem identified and corrected in Eleftheriou and Michelacakis (2016a) affects more papers than just the Beladi et al. (2008). Two such instances of published papers that we know of are the Beladi et al. (2010a) and Beladi et al. (2010b). The aim of this short addendum is to warn the reader against the validity of the results in these two papers and perhaps others using the same basic duopoly model as in Beladi et al. (2008). We look into the origins of the fallacy and make an announcement of corrected versions of some of the affected conclusions referring elsewhere for precise details.

**JEL classification:** L13, L42, D43, R32, F10, F12

**Keywords:** Price-discrimination; Spatial-competition; Firm-location; Cross-border merger

In 2010, Beladi, Chakrabarti and Marjit attempted to generalize results previously obtained by Braid (2008) to the case of a vertically structured industry. They published two papers, one in February 2010 (Beladi et al., 2010a) and a second one in November 2010 (Beladi et al., 2010b). Unfortunately, both papers are plagued with the same technical and conceptual flaws identified in their previous paper Beladi et al. (2008) by Eleftheriou and Michelacakis and corrected in Eleftheriou and Michelacakis (2016a).

Beladi et al. (2010a) examine the effect of a vertical merger on the equilibrium locations of two downstream firms, which are engaged in sequential spatial competition and sell different varieties of a product. Additionally, Beladi et al. (2010b) examine the effect of a

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horizontal, cross-border merger between upstream firms on the equilibrium locations of two
downstream firms selling different varieties of a product, exercising spatial price discrimi-
nation and making simultaneous location calls.

Maintaining the exact same notation as in Beladi et al. (2010a, 2010b), we present, for
the sake of brevity, a fused model of vertically structured industry. The market is represented
by the closed interval \([0, 1]\). Downstream, we have two competing firms \(R_1 \equiv R\) (home)
and \(R_2 \equiv R^*\) (foreign) in Beladi et al. (2010a) (in Beladi et al. (2010b)) located at \(x\) and \(y\),
respectively, with \(x < y\) in \([0, 1]\). Upstream, there is a single supplier of an intermediate good
in Beladi et al. (2010a) while in Beladi et al. (2010b) there are two suppliers \(M\) (home) and
\(M^*\) (foreign). The two downstream firms transform the intermediate good provided by the
supplier(s) into differentiated final goods sold to consumers uniformly distributed in \([0, 1]\).
Three varieties of a differentiated product are offered to consumers: \(U\) and \(W\) from firm \(R_1\)
and \(V\) and \(W\) from firm \(R_2\); the fraction of consumers buying only good \(U\) or only good \(V\)
is \(c\) while the common good \(W\) is bought by a fraction \(b\) of consumers.

Absent a merger the supplier(s) charges(charge) different fixed fees \(F_1 \equiv F\), \(F_2 \equiv F^*\)
and different wholesale prices, \(w_1 \equiv w\), \(w_2 \equiv w^*\) for the intermediate good they provide the
downstream competitors \(R_1\) and \(R_2\), respectively. The source of all problems in Beladi et al.
(2010a, 2010b) as well as in Beladi et al. (2008) is that the authors fail to properly account
for the effects of the wholesale prices. All three papers are affected by the authors failure to
realize that the profits of \(R_1\) realizing a sale of the common good \(W\) to a customer located
at place \(z\), is equal to \(w_2 + t(y - z) - t|z - x| - w_1\). The corresponding profits for \(R_2\) can be
deduced analogously. Further, the misconception regarding the delivered costs leads to the
incorrect derivation of the threshold determining the segment of the market captured by each
firm with regards to the common good \(W\). This threshold is the location of the indifferent
consumer, \(s\), which can be found by setting \(t(y - s) + w_2 = t(s - x) + w_1 \Rightarrow s = \frac{x + y}{2} + \frac{w_2 - w_1}{2t}\).
Following this analysis, it becomes clear that equations (1), (2) and (3) in Beladi et al.
(2010a) are all incorrect and should be replaced by
\[ \Pi_2(x, y) = \left( c(k - w_2) - \frac{ct}{2} \left[ y^2 + (1 - y)^2 \right] \right) \]
\[ + \left( \int_y^{y+y} b[t(2z - x - y) + w_1 - w_2]dz \right) - F_2 \]  
(1)

\[ y(x) = \frac{4t(b + c) + 2tix + 2b(w_2 - w_1)}{2t(4c + 3b)} \]  
(2)

and

\[ \Pi_1(x, y(x)) = \left( c(k - w_1) - \frac{ct}{2} \left[ x^2 + (1 - x)^2 \right] \right) \]
\[ + \left( \int_x^x b[t(y(x) - x) + w_2 - w_1]dz \right) - F_1 \]  
(3)

respectively.

The same holds true for equations (1) and (2) in Beladi et al. (2010b), which should be replaced with equations (3) (where \( y(x) \) is replaced with \( y \) and (1), as well as practically for all equations involving two different wholesale prices, \( w_1 \neq w_2 \). A further clarifying remark regarding the nonegativity of profits of the two firms ought to be made. We observe that the profits of \( R_1 \) are positive on its market share, \([0, s]\), because the quantity \( t(y - z) - t |z - x| + w_2 - w_1 \) is always positive on the interval \([0, x]\) and is also positive on the interval \([x, s]\) by the very definition of \( s \). The case of the profits of \( R_2 \) is treated similarly.

It is unfortunate that the above mistakes affect all propositions in Beladi et al. (2010a) and nearly all propositions in Beladi et al. (2010b). Specifically,

- if the downstream leader merges upstream, both downstream firms move away from the socially optimal location to the direction of the un-integrated firm by amounts different than the ones indicated in Proposition 3 of Beladi et al. (2010a), while
- if the downstream follower merges upstream, both downstream firms move away from
the socially optimal location towards a direction depending on the wholesale price the
follower is charging the un-integrated leader through the upstream monopolist. This
result is the correct version of Proposition 5 in Beladi et al. (2010a).

Similarly, opposite to what Beladi, Chakrabarti and Marjit obtain in Beladi et al. (2010b),
following a cross-border, horizontal upstream merger, the downstream Nash equilibrium lo-
cations remain socially optimal.

The insistence of an editor for brevity has prevented us from giving complete explanations
for the above results. We therefore refer the interesting reader for details concerning the
mistakes in Beladi et al. (2010a) to Eleftheriou and Michelacakis (2016b) and for mistakes
in Beladi et al. (2010b) to Eleftheriou et al. (2016).

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References


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