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BRAIN DRAIN AND DISTANCE TO FRONTIER

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Brain Drain and Distance to Frontier*

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Abstract

In this paper we investigate the effects of emigration on growth in developing countries. We present a model in which productivity increases either through imitation or innovation, and both activities use the same types of human capital as inputs, albeit with different intensities. Heterogenous agents accumulate human capital responding to economic incentives, and might be able to emigrate. When no migration of skilled workers is allowed, backwards countries converge to the technological frontier. The possibility of migration, however, distorts the optimal accumulation of human capital and slows down, or even hinders, development. This effect is stronger the farther away a developing country is from the technological frontier. Thus, technologically backward countries are more likely to suffer from a negative brain drain effect. Among these countries, those which implement appropriate policies, subsidizing the accumulation of the most useful type of human capital, improve their growth performance. They converge faster, and possibly to a higher productivity level than countries where such policies are neglected.

JEL Classification: I28, F22, J24, O40.
Keywords: Education, Migration, Human Capital, Economic Growth.

1 Introduction

Classical theoretical studies on the Brain Drain hold that emigration of highly educated people is beneficial for destination countries and harmful for source ones (e.g. Borjas 1994, Borjas 1995). For immigration countries, the inflow of highly skilled individuals increases the pool of available human capital, and boosts economic growth in the long-run. A specular logic seems to imply that the outflow of ‘brains’ is damaging for the source countries.1

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1Several theoretical studies have pointed at the potential negative effect of the outflow of human capital on source countries, among others: Bhagwati and Hamada (1974), Kwok and Leland (1982), Galor and Tsiddon (1997), and Miyagiwa (1991).

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This theoretical prediction, however, is at odds with the experience of some sending countries that grew faster than their relatively more closed neighbours. Examples include Japan, South Korea, Taiwan and Singapore as opposed to Bangladesh, India and Indonesia, for example.\textsuperscript{2} A recent literature on the effects of the outflow of skilled workers has focused on the potential for a \textit{Beneficial Brain Drain (BBD)}, or a \textit{Brain Gain}.

The central proposition of studies such as Mountford (1997), Stark, Helmenstein, and Prskawetz (1997) and (1998), Vidal (1998), and Beine, Docquier, and Rapoport (2001) is that, if the possibility of emigration induces more skill-creation than skill-loss, source countries might actually increase their stock of human capital, as the possibilities of moving and working abroad increase. One of the simplest mechanisms behind results of this type is that the possibility of emigration might lead economic agents to invest more in their human capital. Yet, since not all of them emigrate in the end, also those who stay in the country of origin have a higher human capital than would otherwise have been the case. Under such circumstances, the simple ‘drain’ effects emphasized by earlier contributions are (possibly) more than compensated by these ‘gain’ effects.

Empirical investigations of the effects of skilled migration on source countries have provided mixed results. While most authors would agree that migration of skilled workers is positive for the destination country,\textsuperscript{3} there is no consensus as refers to the effects on the source economies. Recent empirical work by Beine, Docquier, and Rapoport (2004) has indeed shown that the net effect of the brain drain can be either positive or negative. Despite the significant and positive effect on human capital accumulation that they are able to identify, Beine, Docquier, and Rapoport show that the effects in terms of annual GDP growth are more mixed. Indeed, according to their estimates the BBD hypothesis is supported by the data only for a small number of countries. The authors conclude by noting that "the simple fact that, among sending countries there are winners and losers, points to the necessity of a better understanding of the circumstances and factors favouring the occurrence of a detrimental brain drain".\textsuperscript{4} In this paper we aim at contributing to the debate on the brain drain by focussing on the role played by the \textit{composition} of human capital in fostering productivity growth and, finally, economic development.

The BBD hypothesis implicitly assumes that the human capital that is accumulated with a view to emigration can prove useful once people remain in their country of origin. One might ask if this is a realistic assumption. Indeed, it runs counter to some empirical evidence showing that countries with similar levels but different compositions of education (which we use as a proxy for human capital accumulation) by type have very different performances in terms of convergence and growth. If all human capital would be useful, a higher level thereof would imply faster GDP growth, irrespective of its composition, all else equal.

Although not much addressed in the literature, the different roles played by different types of human capital at different stages of development has been recognized by a number

\textsuperscript{2}Japan and, to a greater extent, South Korea experienced high levels of skilled emigration in the past decades. South Korea, for example, still had a rate of brain drain of over 9% among highly skilled workers in 1990. In the same year Taiwan and Singapore exhibited even higher rates: 15.2% and 24.8%, respectively. By comparison India (3.9%), Bangladesh (2.1%), and Indonesia (3.9%) suffered a much smaller drain of human capital. This high rates of brain drain notwithstanding, Japan and the Asian Tigers where much more successful in terms of economic performance than the countries in the other group.

\textsuperscript{3}An excellent reference on these issues is Borjas (1990).

\textsuperscript{4}Beine, Docquier, and Rapoport (2004), p.35.
of authors. Both Durlauf and Johnson (1997) and Krueger and Lindahl (2001) provide evidence as to the heterogenous effects of education on growth across countries with different levels of development. Kalaitzidakis, Mamuneas, Savvides, and Stengos (2001), instead, discuss the existence of non-linearities in the education-growth relationship. Based on this, in what follows we claim that not all the human capital accumulated in view of possible emigration is *appropriate* for the technology available in the source country, and for its level of development. In particular, we postulate that the distance to the technological frontier is a key determinant for understanding the effects of human capital accumulation/composition on economic growth. While the accumulation of human capital seems to imply faster technological advancement and economic growth, we point at the different *types* of human capital that are most useful at different stages of development. This view reflects the idea that technological advances become available either through imitation or through innovation, and that each activity requires (a different combination) of different types of skills. It is reasonable to assume that imitation requires a more technically inclined work force, whereas the more complex activity of innovation requires more than technical skills alone. Indeed, the closer economies are to the frontier, the more complex their economic and institutional systems, the higher their need for a balanced work force comprising technical skills, creativity, humanistic competencies, legal and managerial expertise. Conversely, at earlier stages of economic development, when the main task is to copy and adapt available technologies, a more intense specialization in technical skills can prove helpful in catching up. Following Vandenbussche, Aghion, and Meghir (2004), we model two economies that can be parameterized by their distance from the technological frontier. Economic development is driven by productivity growth, and productivity improvements depend on the amount and the composition of the human capital available in the country, besides on the distance from the technological frontier. Once at the frontier of technology, productivity advances are only possible through innovation, whereas imitation occurs further away from the frontier. Following on our argument above, we assume that imitation is more intensive in technical skills than innovation. To investigate the distortionary effects of migration on the accumulation of human capital, we model human capital accumulation by agents as an endogenous decision. By letting the type of skills acquired be determined by the costs and benefits faced by heterogenous agents, we add one important dimension to our model. We are in fact able to investigate the interaction between labour market outcomes, migration possibilities and institutional arrangements, such as the existence of educational policies targeted at satisfying the needs of the local economy. Our results show that the possibility of migration distorts the incentives for agents to accumulate the type of human capital that is appropriate for the country of origin, given its level of development. We show that when migration becomes possible at early stages of economic development the growth rate of the source economy decreases. We discuss circumstances under which this process leads to development traps, i.e., situations where the process of convergence to the technological frontier stops prematurely. Furthermore, we show that educational policies, in the form of subsidies to particular types of skills, can counteract the negative effects of migration on growth. Assuming that in democratic societies migration cannot be (completely) prevented, our analysis delivers a clear policy recommendation: Countries that wish to maximize their convergence potential
should take this mechanism into account and increasingly subsidize appropriate skills, the further away they are from the technological frontier, and the easier the prospects of migration.

2 Education, migration and economic development

At the aggregate level, the relationship between brain drain and economic growth is far from univocal. To illustrate this point, we consider the growth rates of the GDP for 128 countries in 2000 and their rate of brain drain – measured as the percentage of tertiary educated residents who emigrate – ten years earlier. Figure 1 presents the scatter plot of the two variables and the regression line. The two variables show very little evidence of being correlated, in fact the correlation coefficient, $\rho$, equals 0.06.5

The regression equation is: $%\Delta GDP_{2000} = 2.35 + 0.91 \cdot %BrainDrain_{1990}$. The GDP growth rates are derived from the Penn World Table 6.1 from Heston, Summers, and Aten (2002), the rates of brain drain from Docquier, Lohest, and Marfouk (2005).

The lack of any significance of this aggregate relationship does not mean much, however, as it simply hides a whole range of situations where countries experienced different degrees of brain drain and various degrees of success in terms of economic growth. Among these, we find the experience of the East Asian economies to be one of the most interesting.

In the last fifty years countries like Japan, the Republic of Korea, Singapore, Hong Kong and Taiwan all exhibited astonishing growth rates. At the same time, they pursued a policy of open borders, i.e. a significant share of their highly skilled workers left over the years to work abroad. Compared to countries with similar rates of brain drain

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5 The regression equation is: $%\Delta GDP_{2000} = 2.35 + 0.91 \cdot %BrainDrain_{1990}$. The GDP growth rates are derived from the Penn World Table 6.1 from Heston, Summers, and Aten (2002), the rates of brain drain from Docquier, Lohest, and Marfouk (2005).
and initial levels of development, however, these East Asian economies performed much better and managed to catch-up with first-world standards of living (and technological knowledge) within a short time period.

There are many important lessons to be learned from the experience of these countries and, indeed, many pages have been filled with analyses of the East Asian “miracle.” Here we draw attention to one specific aspect of these economies that has not been fully appreciated by previous analyses: all these economies have exhibited a marked commitment of the government to promote the accumulation of particular types of skills. As World Bank (1993) puts it, “public funding of post-secondary education focused on technical skills [...] The result of these policies has been a broad, technically inclined human capital base well-suited to rapid economic development.”

Despite having shares of public expenditure on education in line with, and sometimes lower than those in other developing countries, the East Asian economies chose to support the accumulation of specific types of skills which were deemed most useful to economic development. As shown by the graph in Figure 2, there is no clear relationship between the accumulation of ‘technical’ skills and the level of economic development. The Figure reports the percentage of science and engineering students in the total, in 1980, for an indicative cross-section of developed and developing countries. One remarkable feature of these data is that both poor and rich countries exhibit either high or low shares of technical students (China vs. India or Finland and Sweden vs. New Zealand and Canada, for example), so that no clear pattern is visible. What is apparent,

![Figure 2: Share of science and engineering students on total tertiary education (1980).](image)

Source: Own calculations on United Nations Common Data-Base (UNCDB) data.

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6Two important references analyzing, and rethinking, the East Asian economies’ impressive performance are World Bank (1993) and (2001).

7Ibid., page 15. Emphasis added.

8In 1960, for example, the Republic of Korea spent 2.0% of its GDP on education, in the same year Brazil’s share was 1.9%, and the average for Sub-Saharan African countries was 2.4%. In 1989, Korea’s budget for education increased to 3.6%, Brazil’s reached 3.7% and for the same sub-set of African countries the share topped 4.1%. These figures are taken from World Bank (1993), table 5.3.

9Figure 2 is based on computations by the authors on UNCDB data.
instead, is that countries like the Republic of Korea, Hong Kong and Singapore are at the top of the distribution.

Can this high share of technically skilled workers explain, at least in part, the success of the East Asian economies? Another simple graph lends support to this claim. Figure 3 plots the growth rate of a number of developed and developing countries in 1990, against the share of science and engineering students on total tertiary education in 1980 and the corresponding regression line.\textsuperscript{10} The plot seems to imply that having a higher share of science and technology students is an advantage in terms of growth performance.

\textbf{Figure 3: GDP growth (1990) and the share of S&E students (1980).}
Source: Penn World Table 6.1 and UNCDB.

The other interesting aspect is how such a composition of human capital was obtained. Most East Asian economies, in fact, represent clear examples of the government’s intervention into the structure of the tertiary education. In Japan, for example, the system comprising the National Institute for Educational Policy Research (NIER), founded in 1949, and the Ministry of Education, Sports and Culture,\textsuperscript{11} has for a long time been perceived as the “Super-ministry” responsible for adjusting the structure of Japanese schooling to the needs of local industries. Similar institutional structures also exist in the countries of the group of the so-called “Asian Tigers”.

More recently, however, there has been an increasing effort to move away from the predominance of the government and towards the utilization of market mechanisms, especially in Japan where a deep reform of the educational system is currently under way. Analysts have argued that such moves reflect fundamental shifts in the mode and direction of social development. To quote a recent OECD report \textit{“the increased diversity...}
and complexity of the modern society and its needs, necessarily have made centralized decision and control obsolete [...] [M]arket mechanisms will be the only way to achieve diversified and multidimensional changes”.

This shift in paradigm is consistent with the main ideas of our proposed framework. Since the advancement of knowledge, which is at the basis of economic growth, can either occur through the creation of new technologies (ideas) or through the adoption of old technologies from abroad, and since the two activities require different compositions of human capital, the optimal structure of human capital depends crucially on the stage of development of any given economy. Thus, in the presence of distortions, different types of public policies can be necessary to favour the accumulation of different types of human capital at different stage of development.

We focus on the possibility of migration as one such potential distortion. By blurring the borders between economic systems at different levels of economic development, migration distorts the incentives for the optimal accumulation of human capital: agents in lagging countries prefer to acquire the type of human capital that would be more profitable in case of successful migration. Thus the distance to frontier in different countries could offer a useful key to understand the effects of the brain drain on economies at different stages of development.

Moreover, policy could provide a way to offset this harmful effect of brain drain on human capital composition, by regulating the structure of education, as we will argue in what follows.

3 The model

We describe an economy consisting of two countries, one large destination country and one small source one. We assume that the destination country (which we can think of as the group of the OECD countries for concreteness) is the technological leader, whereas the source country is technologically less developed.

The economies are populated by workers and firms. Workers accumulate skills, and supply skilled labour to firms. Skill accumulation is costly in that some time is necessary to acquire knowledge. We assume that workers differ in their abilities (their ‘talent’), so that certain types of skills are more difficult to accumulate (i.e. more time is required) for some agents than for others. For simplicity, we assume that all workers accumulate skills and that it is only possible to acquire two types of skills which we broadly label ‘technical’ and ‘general’. Consequently, in the model there are two types of workers: technically-skilled ($T$) and generally-skilled ($G$) ones.

Since the net rewards to the accumulation of different skills depends on the wage commanded by the specific skill and the cost it entails (in terms of foregone earnings), each worker decides on the type of skills she wants to acquire based on her specific type. Each firm engages in production of an intermediate, needed in the production of the

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12This quote is from the Japanese National Report of the OECD IMHE-HEFCE project on international comparative higher education financial management and governance, 2004.

13In what follows we use the terms destination country, technological leader, and leading country interchangeably. The same goes for source country, technological follower, and lagging country.
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final good, and invests in technology improvements. We assume that workers are only used in the latter activity. Hence, firms decide how many workers of each type to employ in the ‘research’ sector, given that technology can be improved either through R&D activities (innovation proper), or by adopting existing technologies from the world technological frontier (imitation).

In the next subsections we describe in greater details the accumulation decisions made by workers and the parallel innovation choices facing firms. We discuss the choices in a situation of autarchy, that is a situation in which no migration possibility exists. This discussion fully characterizes the destination country, given our assumption that it is large enough that smaller foreign markets are not relevant to its agents’ decisions. Notice, moreover, that throughout the paper we ignore the possibility that goods be traded; we do this to be able to clearly identify the effect of migration on workers’ accumulation decisions. Hence, the alternative to autarchy in our framework is simply a situation in which workers are allowed (with some positive probability) to move from the lagging to the leading country.

3.1 Investment in education

Each period new cohorts of workers of fixed size are born in each country, thus there is no population growth. We assume that the population size in the leading country, $L$, is larger than $L$, the population in the lagging country. We further assume that the share of entrepreneurs in the population is the same in both economies. This has two consequences: first, the number of firms is larger in the leading rather than in the lagging country; second, the number of workers per firm will be the same across countries. In this fashion, the relative size of the two economies plays no role in the model. Without loss of generality, as long as we only look at one country at the time, we can simplify the analysis by letting the population size equal 1.

Workers only live for one period: each period new agents are born, they decide about their education, they work for a wage, consume all their income and finally die.

Workers are risk neutral and differ only with respect to the cost they have to incur to accumulate different types of human capital. They are indexed by $j$ according to their talent and uniformly distributed over the interval $[0, 1]$, with the convention that $j = 1$ corresponds to the most talented individual. The talent of an agent determines her relative cost of acquiring general skills. Agent $j$ needs to spend a fraction $1 - j$ of her time to acquire these skills, while we assume that the time-cost of acquiring technical skills is independent of talent and equal to $1 - \xi$ for all workers, where $\xi \in (0, 1)$.

Agent $j$ will thus be able to offer $j$ units of general skills, or $\xi$ of technical skills. Our modelling choices don’t make general skills overly costly for any individual, however, for some of them technical skills are easier to acquire and they will therefore invest in that direction.

The composition of skills between technical and general ones will be determined by the relative costs of skills accumulation, and by the relative rewards to the particular kind of skills. Letting the salary for a $G$-skilled worker at time $t$ be $w_{Gt} = \omega_{Gt} A_{t-1}$ — where $\omega_{Gt}$ is the wage per effective unit of human capital provided at time $t$, and

\[\text{Effectively this only means that, using the difficulty of developing technical skills as a benchmark, general skills are relatively easier to acquire for some individuals, and more difficult for others.}\]
$A_{t-1}$ indicates the level of total factor productivity at time $t-1$ – and the salary for a $T$-skilled individual be $w_{Tt} = \omega_{Tt} A_{t-1}$, it is possible to identify the marginal worker, $j'$. Agent $j'$, the worker who is indifferent between acquiring technical skills (and earning $w_T$ per each unit she provides) and general ones (thereby earning $w_G$ per unit), must satisfy the following condition:

$$\omega_{Tt} \xi = \omega_{Gt} j'.$$

(1)

All agents indexed by $j \in [0, j']$, will accumulate technical skills, conversely, agents with $j \in (j', 1]$ will choose to become generalists.

Accordingly, the total supply of $G$-skilled labour equals,$^{15}$

$$G_t = \int_{j'}^1 j \, dj = \frac{1}{2} (1 - j'^2);$$

which can be easily solved for $j'$, yielding:

$$j' = \sqrt{1 - 2 G_t}.$$

(2)

Rearranging equation (1) and using the above expression to substitute for $j'$, we get the following expression for the supply of $G$-skilled labour:

$$\omega \equiv \frac{\omega_{Gt}}{\omega_{Tt}} = \frac{\xi}{\sqrt{1 - 2 G_t}}.$$

(3)

Finally, note that the constraint that $j \in [0, 1]$ implies that the supply of graduates with technical background depends on the supply of $G$-skills. Hence, the supply of $T$-skilled labour is given by:

$$T_t = \xi j' = \xi \sqrt{1 - 2 G_t}.$$

(4)

### 3.2 Production and technological progress

In the leading country there are $N\bar{f}$ firms, while in the lagging country there are only $N < \bar{N}$ of them. As discussed above, the number of firms in each country is proportional to the number of workers in each country, so that the size of the economy is immaterial.

Thus, for the sake of generality, we indicate the number of firms by $\nu$ in what follows.

Each firm produces one intermediate input for the production of final output, and engages in productivity-enhancing activities employing skilled workers.

Final output is produced competitively using a continuum of mass $\nu$ of intermediates, accordingly to the following production function:

$$Y_t = \int_0^\nu \int_0 A_t^{1-\alpha} x_{i,t} \, di,$$

(5)

where $\alpha \in (0, 1)$ and $x_{i,t}$ is the amount of intermediate good $i$ used to produce $Y$ at time $t$.

Each intermediate producer acts as a (local) monopolist and produces good $i$ using the final good with a one-to-one technology. It is then easy to show that, for a given

$^{15}$Notice that this specification implies $G \leq 1/2$. 
level of $A_t$, the equilibrium demand for input $i$ equals $x_{i,t} = \alpha \frac{2}{\alpha} A_t$. Hence, profit maximization on the part of intermediate goods’ producers implies that for each firm monopoly profits equal:

$$\pi_t = \zeta A_t,$$

(6)

where $\zeta \equiv \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}}$.

Moreover, using the expression for inputs’ demand derived above, it is straightforward that the level of final output is linear in the level of technology and, as a consequence, the growth rate of output will be the same as the growth rate of technology. Keeping this in mind, we now analyze the choice faced by firms in formulating their technological development plans.

Firms employ skilled workers to increase productivity. We assume that productivity can be improved by being directly involved in R&D activity or by adopting existing technologies from the world technological frontier:

$$A_t = A_{t-1} + A_{t-1}T_{nt}G_{nt}^{1-\phi} + (\bar{A}_{t-1} - A_{t-1})T_{mt}G_{mt}^{1-\sigma}.$$

(7)

Here $T_{nt}$ represents the amount of $T$-skills used in innovation at time $t$, while $T_{mt}$ refers to the amount in imitation. The same applies to $G_{nt}$ and $G_{mt}$. We assume that both types of skills are needed in both innovation and imitation, and that the two activities differ in that the productivity of $G$-skilled workers is higher in innovation than in imitation, i.e. we let $\sigma > \phi$.

Furthermore, the technological improvement function in (7) implies that imitation is more productive the further away a country is from the technological frontier, $\bar{A}$. This is intuitive since a larger technological gap means that more innovations can be usefully adopted from abroad. Innovation, instead, becomes more productive with the own technology level, $A$, formalizing the idea that a broader technological base is needed to push the frontier further.

4 Equilibrium under autarchy

To characterize equilibrium situations under autarchy, we need to discuss three possible types of equilibrium according to the regime of technological change that takes place. In what follows, we distinguish between equilibria that occur under innovation, equilibria that obtain under imitation, and mixed equilibria where both activities take place at the same time.

Let us start with the case where innovation is the only type of productivity-enhancing activity performed in equilibrium. In this case, new technologies develop according to:¹⁷

$$\bar{A}_t = \bar{A}_{t-1} + \bar{A}_{t-1}T_{t}^{\phi}G_t^{1-\phi}.$$

(8)


¹⁷From now on, we identify variables that refer to the innovation-only case by an upper bar. Variables without the upper bar refer the imitation-only equilibrium. When necessary, we will distinguish the mixed equilibrium variables with a tilde: $\tilde{}$. 
Profit-maximizing firms will choose the amount of each type of skilled labour to employ in innovation, in order to solve the following maximization problem:

$$\max_{G_t, T_t} \pi_t = \zeta \tilde{A}_t - \tilde{\omega}_{T_t} \tilde{A}_{t-1} \tilde{T}_t - \tilde{\omega}_{Gt} \tilde{A}_{t-1} \tilde{G}_t,$$  \hspace{1cm} (9)

subject to (8).

The first-order conditions for this problem are,

$$\tilde{\omega}_{Gt} \equiv \frac{\tilde{\omega}_{Gt}}{\tilde{A}_{t-1}} = (1 - \phi) \zeta \left( \frac{\tilde{T}_t}{\tilde{G}_t} \right)^{\phi}, \hspace{1cm} (10)$$

$$\tilde{\omega}_{Tt} \equiv \frac{\tilde{\omega}_{Tt}}{\tilde{A}_{t-1}} = \phi \zeta \left( \frac{\tilde{T}_t}{\tilde{G}_t} \right)^{\phi-1}. \hspace{1cm} (11)$$

The above equations, together with (3) constitute the equilibrium. Taking the ratio of (10) and (11) and substituting for $\tilde{T}$ from the expression in (4), one gets the following expression for the demand of $G$-skilled labour:

$$\tilde{\omega}_t \equiv \frac{\tilde{\omega}_{Gt}}{\tilde{\omega}_{Tt}} = \frac{1 - \phi}{\phi} \sqrt{\frac{1 - 2 \tilde{G}_t}{\tilde{G}_t}}, \hspace{1cm} (12)$$

Using this and the supply function in (3), we can illustrate the equilibrium graphically in the $(\omega, G_t)$ plane (see Figure 4, where $\omega \equiv \tilde{\omega}_{Gt}/\tilde{\omega}_{Tt}$). The demand for generalists is represented with the downward sloping curve $D$, whereas the supply is represented by the upward sloping curve $S$. The equilibrium obtains when both conditions are satisfied simultaneously, that is at a point like $(\tilde{\omega}^*, \tilde{G}^*)$. This point represents the equilibrium when a country is fully specializing in innovation, thus this is the equilibrium prevailing the destination country.

Analytically, it is straightforward to solve for the equilibrium level using (3) and (12), to get:

$$\tilde{G}^* = \frac{1}{\xi \Phi + 2}, \hspace{1cm} \text{and} \hspace{1cm} \tilde{\omega}^* = \xi \sqrt{\frac{\xi \Phi + 2}{\xi \Phi}},$$

where we have used $\Phi \equiv \frac{\phi}{1-\phi}$.

At the other extreme, we focus on the case in which a country only resorts to imitation to increase their technological level. Local firms, thus fully specialize in imitation. Except for this, they behave exactly like their counterparts in the previous case: amending the relevant production function, they choose $G_t$ and $T_t$ to maximize their profits,

$$\max_{G_t, T_t} \pi_t = \zeta [A_{t-1} + (\tilde{A}_{t-1} - A_{t-1}) \tilde{T}_t \tilde{G}_t^{1-\sigma}] - \tilde{\omega}_{T_t} A_{t-1} T_t - \tilde{\omega}_{Gt} A_{t-1} G_t. \hspace{1cm} (13)$$

Hence, their demand for $G$-skilled labour equals:

$$\tilde{\omega}_t = \left( \frac{1 - \sigma}{\sigma} \right) \frac{\sqrt{1 - 2 \tilde{G}_t}}{\tilde{G}_t}.$$  \hspace{1cm} (11)

Since $\sigma > \phi$, the demand curve for the case of imitation (the dashed line $D$ in Figure 4) lies below the demand curve for innovation. Intuitively, it is clear that, since skills of type
Figure 4: *The equilibrium without migration in the two countries.*

$G$ are more productive in innovation than in imitation, for any given relative wage firms specializing in innovation would demand relatively more $G$ skills than firms specializing in imitation. Just as before, the equality of demand and supply will determine the equilibrium levels of the relative wage and of the supply of $G$-skills:

$$G^* = \frac{1}{\xi \Sigma + 2}, \quad \omega^* = \xi \sqrt{\frac{\xi \Sigma + 2}{\xi \Sigma}},$$

where $\Sigma \equiv \frac{\alpha}{1-\alpha}$.

From $\sigma > \phi$, we conclude that when countries fully specialize, the country that does so in innovation will have a higher level of $G$.

To complete our analysis, we need to address what happens when firms don’t fully specialize in either activity. In this case, they will adopt the combination of the two activities which allow them to maximize their profits. In terms of Figure 4, equilibria of this type will correspond to points along the supply curve $S$, comprised between $E$ and $ar{E}$. The weight of each type of activity, innovation and imitation, will be determined by the relative productivity of each. The higher is the weight of imitation, the closer the mixed equilibrium will be to $E$, and *vice versa* for innovation.

As mentioned at the end of the previous section, the productivity of imitation is higher, the wider the technological gap. Innovation, on the other hand, is more productive the closer a country is to the technological frontier. Thus, at an intuitive level it seems reasonable that, as we move up along the technological ladder, we encounter countries progressively more active in innovation. In terms of Figure 4, this implies that the equilibrium would gradually shift from point $E$, where only imitation occurs, to point $\bar{E}$, where innovation is the only activity taking place.
Given the structure of (7), countries that have a low level of technology have larger incentives – represented by the term \((A_t - A_t)\) – to engage in imitation. These incentives, however, decrease with the reduction of the distance to frontier. Thus, it would seem that imitation occurs far away from the frontier; imitation and innovation coexist as the distance to frontier gets smaller; while only innovation takes place for low levels of the technological gap. Indeed, in the proposition that follows we show that the choice of the type of activity to undertake only depends on the distance-to-frontier parameter, that we define as \(a_t = A_t / A_t \geq 1\).

**Proposition 1.** Consider the economy described above. There exist two critical values of the distance to frontier – \(a_t\) and \(a_h\) – such that, when \(a_{t-1} < a_t\), only innovation occurs in the equilibrium; when \(a_{t-1} > a_h\), only imitation occurs in the equilibrium; and when \(a_{t-1} \in (a_t, a_h)\), both activities take place in the equilibrium.

**Proof.** See Section A.1 in the Appendix for the proof, and for the expressions of \(a_t\) and \(a_h\).

According to this proposition, there are values of the distance-to-frontier for which both innovation and imitation occur simultaneously: this is indeed the case when \(a_{t-1}\) lies between \(a_t\) and \(a_h\), as defined in Section A.1 of the Appendix. In this case, the equilibrium is characterized by a value of the wage that depends on \(a_{t-1}\), \(\bar{\omega}(a_{t-1})\), defined as

\[
\bar{\omega}(a_{t-1}) = \left( (a_{t-1} - 1) \frac{1 - \sigma}{1 - \phi} \left( \frac{\sigma}{1 - \sigma} \left( \frac{1 - \phi}{\phi} \right)^{\phi} \right)^{\frac{1}{\bar{\omega} - \sigma}}, \right.
\]

and such that \(\bar{\omega}(a_{t-1}) \in (\omega^*, \bar{\omega}*)\). The corresponding level of the total supply of skills, \(\bar{G}(a_{t-1})\) say, can be read on the labour supply curve, \(S\) in Figure 4.

Hence, our economies have an equilibrium at \((\omega^*, G^*)\) for all levels of \(a_{t-1} \in [a_h, +\infty)\), in which case full-specialization in imitation will obtain; the equilibrium switches to a non-specialization regime with both imitation and innovation happening at the same time, and \((\bar{\omega}(a_{t-1}), \bar{G}(a_{t-1}))\) for intermediate levels of \(a\) i.e. for \(a_{t-1} \in (a_t, a_h)\); finally, full specialization in innovation will occur when \(a_{t-1} = a_t\). In this last case, the wage rate and the equilibrium level of \(G\)-skills are \(\omega^*\) and \(\bar{G}^*\), respectively.

Before moving on to considering how the distance to frontier of the lagging country evolves over time, there is another important point to make. Since the labor market is competitive, the wages equal marginal products, hence there are no extra profits from innovation. However the monopoly profits in the market for intermediates depend on the productivity level. In the absence of any external distortions, thus, the technological level is maximized. Since the growth rate of technology is given by \(g_t = (A_t - A_{t-1}) / A_{t-1}\), and \(A_{t-1}\) is predetermined, the maximization of technology improvements results in output growth maximization at each point in time. In other terms, in the absence of any other distortions, the market mechanisms are enough to generate the appropriate incentives for firms and workers to allocate resources optimally (in terms of growth). We close this section with the following result:

**Proposition 2.** In the absence of migration, the market solution is growth maximizing.

**Proof.** See Section A.2 in the Appendix.
5 Convergence under autarchy

Having described the possible equilibria, in this section we analyze the evolution over time of the distance to frontier in the source country when no migration is possible. We show that the lagging country tends to grow faster than the leading one, and converging over time towards the technological frontier. Recall that the distance to frontier at time $t$ is defined as:

$$ a_t = \frac{\bar{A}_t}{A_t} \geq 1, $$

thus, as long as the growth rate of the lagging country is larger than the growth rate of the leading one, convergence towards the frontier will occur, that is $a_t$ will decrease over time.

Under autarchy, both the leading and the lagging country enjoy a growth maximizing allocation of workers across skills, that is the ratio $T_t/G_t$ that arises in equilibrium, maximizes the growth rate as shown above in Proposition 2.

The growth rate for the leading country, $\bar{g}$, is given, from (8), by

$$ \bar{g} = \frac{\bar{A}_t - \bar{A}_{t-1}}{\bar{A}_{t-1}} = T_t^\phi G_t^{1-\phi}, $$

and it only depends on the equilibrium levels of $T_t$ and $G_t$, which are independent of $a_t$ and constant over time.

To determine the evolution over time of the distance to frontier of the lagging economy, we need to compare its growth rate with the growth rate of the frontier country, $\bar{g}$. Consider first what happens when the source country is very far from the frontier and, in particular, when its distance to frontier, $a_t$, is larger than the critical value $\tilde{a}_h$. Under these circumstances, the lagging country fully specializes in imitation and its growth rate is given by $(a_{t-1} - 1)/T_t^\sigma G_t^{1-\sigma}$. Contrary to what happened in the leading country, in this case the growth rate increases with the distance to the technological frontier, as imitation is more productive the larger the technological gap. The lagging country thus grows faster than the technological leader and gets closer (at decreasing rates) to the frontier.

When the distance to frontier reaches the threshold $\tilde{a}_h$, firms in the lagging country also begin innovating, as it now proves profitable for them to do so. By combining the two activities firms maximize their productivity, and the growth rate of the lagging economy remains higher than the rate of expansion of the frontier $(\bar{g})$. The process of convergence continues until the distance to frontier reaches the level at which companies in the lagging country fully switch to innovation, i.e. until $a_{t-1} = \tilde{a}_l$. Once the lagging country has reached this threshold, it makes use of the same production function as the leading country to increase productivity: the growth rates of the two countries are now equal, and the process of convergence is completed. This is summarized by the following:

Proposition 3. In the absence of migration, the lagging country achieves convergence, and reaches the steady-state distance to frontier $\tilde{a}_l$.

\footnote{As discussed in the proof of Proposition 1 in Appendix A.1, the value of $\tilde{a}_h$ and $\tilde{a}_l$ only depend on the values of the production elasticities, and on the equilibrium levels of $\omega_G$ and $\omega_T$, that are both independent of $a_{t-1}$.}
Proof. See Section A.3 in the Appendix.

The intuition behind this result is relatively straightforward: recall that firms always choose the composition of innovation/imitation that maximizes the rate of productivity growth, indeed, we know from Proposition 2 that in the absence of migration the market outcome is growth maximizing. When a country is lagging away from the frontier, i.e. when the distance-to-frontier parameter $a_{t-1}$ is larger than $\tilde{a}_l$, it is advantageous for firms to perform at least some imitation and not to fully specialize in innovation: firms exploit the higher productivity of imitation (away from the frontier) relative to innovation, for any given level of the relative supply of skills. Thus, as long as there is some gains to be earned by imitating, the average productivity (and hence the growth rate) will be higher for the lagging country than for the leading country. At any distance from the frontier larger than $\tilde{a}_l$, the lagging country has a growth rate higher than $\bar{g}$ and the technological distance that separates it from the frontier tends to decrease.

This catching-up effect, reminiscent of similar effects in the technology diffusion literature (see, e.g. Barro and Sala-i-Martin (2004), chpt. 8), vanishes when the technological gap disappears. When there are no longer advantages to be derived from imitation, innovation is the only means to foster productivity; full specialization occurs (see Proposition 1), and convergence to the group of technological leaders has been accomplished. Graphically, this is presented on the $(a_t, g_t)$ plane in Figure 5. The horizontal dashed line $N$, stands for the growth rate of the innovating, frontier country, $\bar{g}$. The upward-sloping line $M$ describes the growth rate reported by the economy that employs technological adoption as its only means to increase productivity. This line slopes upwards because of the increasing benefits of lagging behind the frontier, as discussed above. The solid lines with arrow represents the lagging economy’s process of convergence towards the frontier, through the three different phases of imitation-only (the straight part of the solid line), imitation-innovation (the curved part) and innovation-only, when it reaches
the distance \( a_t = \tilde{a}_t \), where convergence is complete.

6 Migration and distance to frontier

We now turn to analyzing the effects of the possibility of migration on the growth rate of the lagging country, and on its steady-state level of income.

Assume that both \( G \)-skilled and \( T \)-skilled workers in the lagging country have some non-negative, exogenous probability, \( p_G \) and \( p_T \) respectively, to migrate to the more developed leading country. We assume that migration is random, i.e. there is no possibility of screening potential migrants, and hence workers of each type face the same probability of migration. First, we study the case when the probability of migration is the same for both types of workers. Next, we analyze what happens when one type is favoured by the destination country, i.e. when agents of a given type have a higher probability of migrating.

6.1 Uniform probability of migration

Suppose that both types of skilled agents \( G \) and \( T \) have the same chances to migrate to the frontier country: i.e. having acquired their skills, workers will be able to offer their labour services abroad with probability \( p_G = p_T = p \in (0, 1) \).

The possibility of migration influences the accumulation decisions of workers only in the source country. Indeed, since wages in the lagging country are lower than in the leading one, migration proves unappealing to skilled workers from the leading country. In the lagging country, however, rational workers will take into account that with some probability they will be able to migrate to the more advanced country and obtain higher wages.

In this context, the condition for the marginal worker in the destination country reads:

\[
(p a_{t-1} \bar{\omega}_T + (1-p)\omega_T) \xi = (p a_{t-1} \bar{\omega}_G + (1-p)\omega_G) j'.
\]

Recalling the expression used in (2) to identify the indifferent worker,

\[
j' = \sqrt{1 - 2G_t},
\]

we immediately see that (15) implicitly expresses the supply of \( G \)-skilled labour in the source country, for any level of \( p \) and \( a_{t-1} \). In a graph similar to the one in Figure 4, the supply curve under migration is characterized by a lower level of \( \omega \) than the original curve \( S \), for each level of \( G \). Indeed, for workers to supply any level of \( G \)-skills (smaller than \( G^* \)) the domestic relative wage has to be lower than before, given that the relative wage abroad is never lower than at home. Figure 6 presents the relative graph. Notice that in the specific case where the migration probability is the same for both types of workers, the two lines coincide when both countries specialize in innovation (at point \( \bar{E} \) in the figure).\(^{19}\)

\(^{19}\)From equation (15), it is possible to rewrite the supply of \( G \)-skills in terms of \( \omega \) as \( \omega = \omega_{\text{autarchy}} - (\bar{\omega} - \omega_{\text{autarchy}}) \frac{\omega_{\text{autarchy}}}{\bar{\omega}_G}, \) where \( \omega_{\text{autarchy}} = \frac{\xi}{\sqrt{1 - 2G}} \) expresses the wage under autarchy, for each level of \( G \). Since \( \bar{\omega} \) is the maximum value for the equilibrium level of the relative wage, it follows that indeed, in the \((G, \omega)\) plane, the supply of \( G \)-skills under migration is below \( S \) for \( a > \tilde{a}_t \), or, which is equivalent, for \( G < G^* \).
Firms’ decisions to hire workers only depend on domestic conditions: given our assumption that the share of entrepreneurs is the same in the two countries, the number of workers per firm is the same in the two countries and across regimes. Thus, the possibility of migration does not affect the firms decision in any way: firms still maximize profits taking the wage level as given.

As before, labour demand and labour supply jointly determine the equilibrium level of \( G \) and \( T \). The probability of migration influences these equilibrium levels by distorting the accumulation incentives of the workers.

To understand why, remember that while workers respond to price incentives from both countries, firms only face domestic prices. Thus, at every equilibrium there will be a wedge between the wage ratio perceived by workers and the wages faced by firms. With the exception of point \( \bar{E} \) where countries are de facto identical as refers to wage rates and technology, at every other migration equilibrium workers will perceive higher wage rates than firms. In particular the wage rate \( \omega \) perceived by workers will be higher than the one perceived by domestic firms. This is due to the fact that the alternative to working at home is to work abroad, where only innovation takes place: since generalists are more productive than technicians in innovation, they are relatively more rewarded in the leading economy. Thus, workers naturally bias their decision towards \( G \)-skilled labour. From Figure 6, it is apparent that, with the exception of point \( \bar{E} \), where only innovation occurs, every equilibrium point under migration will be characterized by a higher level of \( G \)-skills than the corresponding autarchy equilibrium. As this happens, the economy moves away from the growth-maximizing factor composition, \( T^*/G^* \) (see Proposition 2), at each level of the distance to frontier larger than \( \bar{a}_l \). Hence, the growth rate of the source economy declines, leading to the following result:

Figure 6: Demand and Supply under Migration.
Proposition 4. When migration of skilled workers is possible, the growth rate of the lagging economy is reduced for all $a \in (\tilde{a}_l, \infty)$.

Proof. In text. \qed

As before, however, when the distance to frontier is $\tilde{a}_l$, firms still specialize in innovation at point $N'$; offer a relative wage equal to $\bar{\omega}^*$; and hire $\bar{G}^*$ workers. Indeed, when the probability to migrate is the same for both types of workers, the distortionary effect of migration decreases with the level of specialization in innovation, or which is the same, it increases with the distance to frontier, see (15). When the lagging country fully specializes in innovation, the possibility of migration ceases to play any role. In this situation the education incentives for agents are identical in both countries.

Elsewhere, however, things are more complicated. For firms it will still be profitable to combine innovation and imitation for any level of the distance to frontier larger than $\tilde{a}_l$. The range (in terms of $a$) where the two activities coexist, however, is larger now than under autarchy. The supply of $G$-skills is in fact larger along the $E' - E$ line than along the $S$ line, for any $\omega$. As a consequence, at $\tilde{a}_h$ the level of $G$ (and of $T$) will differ from its optimal level $G^*$. Hence, imitation only is not productive enough at $\tilde{a}_h$ to justify full specialization in this activity. Specialization will necessarily occur at a level of $a$ larger than $\tilde{a}_h$, call it $\tilde{a}_{h1}$, given that the productivity of imitation increases with the distance to frontier. This discussion leads us to our next result:

Proposition 5. When migration of skilled workers is possible, and $p_G = p_T = p$, the lagging country converges at a steady-state distance to frontier equal to $\tilde{a}_l$.

Proof. See Section A.4 in the Appendix. \qed

The conclusion from this and from Proposition 4 is that when no type of skill is favoured by the leading country in terms of migration, the lagging country still converges to the same level of development as before the introduction of migration, but it does so at a slower rate than before.

We can summarize the effects of the probability of migration in this situation as follows: in the first place migration distorts the accumulation of human capital reducing the growth rate (from imitation). Graphically, this is presented in Figure 7, where the downward sloping line $M_1$ is below the line $M$, that represents the growth rate without migration. Second, since imitation is now less productive \textit{ceteris paribus}, firms will tend to begin innovating further away from the technological frontier. Indeed, the threshold value for innovation, $\tilde{a}_h$, shifts right to $\tilde{a}_{h1}$ in Figure 7.\footnote{This can be easily seen from expression (A.2) in Appendix A.1. The introduction of migration makes technicians scarcer and reduces the relative wage faced by firms, since $\sigma > \phi$ this signifies an increase in the threshold level $\tilde{a}_h$.} Third, the process of convergence, however, continues up to the point $\tilde{a}_l$, the same one as in the no-migration case, since the distortionary effect of migration is irrelevant when both countries specialize in innovation and the probability of migration is the same across workers’ type.
6.2 Non-uniform probability of migration

In the previous subsection we have shown that, if both types of human capital have the same probability of migration, the lagging country achieves full convergence and the steady-state distance to frontier is \( \tilde{a}_l \). When the probabilities of migration differ across skill types, however, workers’ incentives to accumulate skills are distorted to an even larger extent.

The analysis here parallels the analysis performed above for the case of uniform probabilities, with the exception that we assume that \( G \)-skilled workers, being more productive in innovation, will be more demanded in the frontier country than \( T \)-type workers and will accordingly face a higher probability of migration, i.e. we assume that \( p_G > p_T \). Let us first rewrite (15), allowing for different probabilities of migration in different sectors:

\[
(p_T a_{l-1} \tilde{\omega}_T l + (1 - p_T) \tilde{\omega}_T l) \xi = (p_G a_{l-1} \tilde{\omega}_G l + (1 - p_G) \tilde{\omega}_G l)' \tag{16}
\]

As in the previous case, the probability of migration distorts the accumulation of human capital and reduces the growth rate, all else equal. Here, however, the fact that \( p_G \) is larger than \( p_T \) increases the expected value of accumulating \( G \)-skills to a larger degree. One of the consequences is that, in this case, the distortion affects the accumulation of skills also when the distance to frontier equals \( \tilde{a}_l \). Indeed, from equation (16), it is apparent that, even when workers face the same wages per unit of effective labour both at home and abroad (\( \tilde{\omega}_G \) and \( \tilde{\omega}_T \)), the relative wage perceived by potential migrants is higher than \( \tilde{\omega}^* \), causing an over-supply of \( G \)-skills.

Moreover, the conclusions from Proposition 4 also hold in this case, and are further reinforced by the positive difference between \( p_G \) and \( p_T \). Thus, the growth rate decreases further relative to the case where the probability of migration is expressed by the common \( p \). From these two observation we can derive the following:

**Proposition 6.** When migration of skilled workers is possible, and \( p_G > p_T \), the steady-
state distance to frontier of the sending economy increases. Moreover, complete specialization in innovation is never achieved by the lagging country.

Proof. See Section A.5 in the Appendix.

We use Figure 8 to complete the discussion of this case. The line $M_2$ in the figure lays strictly below the $M$ line, which represents the growth rate without migration possibilities. However, based on our discussion above, we know that this line also lies below the line, $M_1$, that we used in Figure 7 to illustrate the case of common $p$.

\[ \begin{align*}
\text{Figure 8: No convergence with migration, and } p_G &\neq p_T. \\
\end{align*} \]

The mechanism at work is the same as before: far away from the frontier it pays to concentrate skills in imitation, since this is the most profitable activity. The decrease in the distance to frontier, however, reduces the productivity gap between imitation and innovation. When the productivity has decreased enough, we observe a switch away from pure imitation. This happens for a value of $a$ equal to $\tilde{a}_{h2} > \tilde{a}_{h1} > \tilde{a}_h$, given the existence of larger distortions in this context (the possibility of migrating and the different probabilities of doing so).

That the economy is distorted to a larger degree, finally, is evident from the fact that the lagging country experiences a development trap in this case. The fact that generalists are more favoured in migration means that technical skills become scarcer in the source country also at $\tilde{a}_l$. Thus the growth performance of the lagging country cannot exceed the growth performance of the leading country (which is the speed of expansion of the frontier) at $\tilde{a}_l$. By the continuity of the function expressing the growth rate, and the fact that it increases with the distance to frontier (see the Appendices for the details) we conclude that the process of convergence towards the frontier must stop short of $\tilde{a}_l$. We identify this long-run rest-point of the system by $\tilde{a}_{\text{trap}}$ in the Figure, to emphasize the suboptimal nature of this outcome. Despite having the potential to reach the other countries at the frontier, the distortions induced by the workers’ migration prospects lock
the country in a vicious circle of inappropriate accumulation of skills, lower economic growth (relative to potential) and persistently larger distance from the frontier.

7 The role of subsidies in the process of development

In the previous sections we have shown the effects of human capital’s composition on the rate of economic growth and the potential convergence of a developing country. We concluded that the prospect of migration distorts the composition of human capital in the lagging countries and that, as a consequence, the brain drain translates into smaller growth rates and, potentially, into steady-states with larger gaps from the technological leaders.

A natural concern for policymakers in developing countries might then be to design policies aimed at correcting the distortions, and at adjusting the formation of human capital to the needs of local entrepreneurs. In this section we investigate one such instrument: targeted subsidies to education.

Under migration, the composition of human capital is suboptimal from the lagging country perspective, thus subsidies might be used as additional incentives to adopt the ‘appropriate’ type of skills. To off-set the negative impact of brain drain on human capital composition, policymakers in the lagging country could consider subsidizing the acquisition of technical education or, which is equivalent, taxing general skills. Without loss of generality, in what follows we consider subsidies to technical education.

Formally, we present subsidies as an increase in the returns to this type of education. Workers offering $\xi$ units of technical skills on the market will receive a compensation of $w_T \xi (1 + \tau)$, where $\tau > 0$ is the subsidy rate. Our modelling of subsidies provides a rather general representation of monetary transfers, in fact, every agent of type $T$ works the same hours, and thus receives the same amount of subsidies.

To see how the subsidy to technical education corrects the distortionary effects of the possibility of migration, consider Figure 9.

In the absence of migration, this point determines the optimal supply (and composition) of skills. When migration possibilities enter the picture, however, expected wages increase for both skill types, and both schedules shift up. The wages raise from $w_G$ to $w'_G$ and $w_T$ to $w'_T$. However, since $G$-skills are relatively more rewarded abroad, the upwards shift in the sloping line is more marked, the relevant curves now cross in $B$, and the indifferent agent has a lower index: $j''$. Accordingly, the supply of $G$-skills increases, while that of $T$-skills decreases. This yields a suboptimal result in terms of the availability of skills for domestic firms. As discussed in previous sections, this results
in a reduction of the growth rate and, when \( p_G > p_T \), in an increase of the long-run distance to frontier: a development trap.

Increasing the returns to accumulating technical skills tends to correct the distortion caused by the migration prospects. The provision of a subsidy increases the wages of \( T \)-skilled workers, and raises the horizontal line in the figure further up. When the subsidy is set optimally, the indifferent agent is once again indexed by \( j' \). Since the subsidy does not distort the demand for skills, this is sufficient to repristinate optimality. For that to be the case, however, \( \tau \) must be set equal to

\[
\tau^* = \frac{\omega_T}{\omega_G} \frac{p_G a_{t-1} \bar{\omega}_G + (1 - p_G) \omega_G}{p_T a_{t-1} \bar{\omega}_T + (1 - p_T) \omega_T} - 1, \tag{17}
\]

where the ‘no-\( \mu \)’ subscript indicates the optimal wages without migration. When technical education is subsidized according to this rule, that is when \( \tau = \tau^* \), the marginal agent \( j' \) faces the same expected relative returns to accumulating skills, irrespective of the regime of international mobility.

Notice that, since the strength of the distortion increases with the distance to frontier, the optimal subsidy \( \tau^* \) is an increasing function of \( a_{t-1} \). Indeed, recall that the relative wage \( \omega_t \) decreases with the distance to frontier, going from \( \tilde{\omega}^* \) to \( \omega^* \), as can be clearly seen from Figure 4; thus, \( \omega_{T,\text{no-}\mu}/\omega_{G,\text{no-}\mu} \) increases with \( a_{t-1} \). Moreover, the relative domestic wage \( \omega_{G}/\omega_{T} \) decreases with the distance to frontier, as the productivity of \( G \)-skills decreases with the decreasing weight of innovation relative to imitation. Finally, \( p_G > p_T \). Thus, also the second ratio at the right-hand side of (17) increases with \( a_{t-1} \).

As the process of development and convergence to the frontier proceeds, the rate of the subsidy necessary to restore the optimal trajectory declines over time to satisfy (17) at

Figure 9: Effects of a subsidy (\( \tau \)) to technical education.
each instant (and at each level of the distance to frontier $a_{t-1}$). We can summarize this discussion in the following result:

**Proposition 7.** When technical skills are subsidized according to (17), the optimal accumulation of skills is restored. Moreover, the optimal subsidy rate $\tau^*$ declines over time, as the technological frontier draws nearer.

*Proof.* In the text. \[\]

Thus, subsidizing technical education when the prospects of migration might distort accumulation incentives on the part of workers corrects the incentives and restores optimality. We view this implication of our model as an interesting rationalization of the policies performed by the successful East-Asian economies that we discussed in Section 2. There the state invested in specific types of tertiary education, with an eye (and something more) to the interests of the local employers. The implications of the model, moreover, seem consistent with the evolution of the attitude of the policymakers responsible for educational policy mentioned by some observers. The shift from interventionism to *laisser-faire* is in line with our story: when the structure of the economy changes to match that of the leading economies, direct interventions in education, to regulate the structure of the supply of skills become redundant, and the market mechanisms regain center stage.

8 Conclusions

The debate on the economic effects of the brain drain has not yet reached univocal conclusions under many respects. This is particularly true for studies focusing on developing countries. Recent empirical contributions (e.g. Beine, Docquier, and Rapoport 2004) have argued that among developing countries there are both winners and losers, and concluded that more theoretical work is needed to understand this pattern.

Building on these ideas, we develop a simple theoretical model to investigate whether the prospects of migration have an influence on growth and convergence. Our contribution extends the framework of Vandenbussche, Aghion, and Meghir (2004) to incorporate the endogeneity of human capital accumulation in a model where growth is driven by technical progress, and technical progress is the result of purposive activities of imitation and of *bona fide* innovation. The main insight from the model is that, at different levels of development, different types of human capital, or rather different proportion thereof, are needed to achieve optimal growth. Thus, the key determinant of the optimal composition of skills in any given economy is its distance from the technological frontier.

By blurring the borders between economic systems at different levels of development, the possibility of migration distorts price signals, induces change in the accumulation of human capital, and ultimately proves detrimental for developing countries. We find that the brain drain reduces the growth rate of a developing country along the transition to its long-run balanced growth path. Moreover, we point at the possibility of the emergence of development traps, as the opportunities of migration might reduce the long-run income level of lagging countries.

Our theoretical contribution also provides some normative conclusions that might shed more light on the astonishing performance of the most successful East Asian economies
over the past few decades. From our positive analysis we know that some types of human capital are more important for the developing countries, but the incentives to acquire them are reduced by the prospects of migration. Hence, on the normative side, we show that a very important role can be played by government interventions, e.g. in the form of subsidies the encourage the acquisition of those particular skills which are most needed domestically. Moreover, since the distortionary impact of the migration prospects decline with the proximity to the frontier, the government’s support in favor of certain skills should taper off as the development process proceeds.

We find this story particularly useful, as it can be used to rationalize the behaviour of the successful East Asian economies over their path to development. Countries like Japan, the Republic of Korea, Taiwan and Singapore all experienced both high rates of GDP growth and of brain drain at the same time: a puzzling story, at first sight. However, they are also quoted as examples of the government’s intervention in the educational field, where policies were aimed at favouring the acquisition of technical skills over other skills. According to our story, these policies might have helped the growth performance of Japan and of the Asian Tigers by correcting the distortionary impacts induced by the brain drain. We find more support for the predictive power of our analysis in the fact that more recently the same countries are advocating a pervasive change in their educational strategies, to favour the autonomous display of market forces. This corresponds to the policy a regulator in our model would find optimal once the technological frontier has been reached.

Our analysis, however, raises a number of questions, the most obvious of which refers to the empirical relevance of the mechanisms we identify. We present some stylized facts and anecdotal evidence supporting our theory, yet, a more thorough empirical analysis is called for by our results. Among our plans for future work, finding the necessary data and testing the implications of the model necessarily plays a prominent role.

References


A Appendix

A.1 Proof of Proposition 1

Firms maximize profits choosing employment in productivity-enhancing activities, taking wages as given:

$$\max_{\{T_{mt}, T_{nt}, G_{mt}, G_{nt}\}} \pi_t = \zeta \left[ 1 + T_{nt}^\phi G_{nt}^{1-\phi} + (a_{t-1} - 1) T_{mt}^\sigma G_{mt}^{1-\sigma} \right] +$$

$$- \omega_G (G_{mt} + G_{nt}) - \omega_T (T_{mt} + T_{nt}), \quad \text{(A.1)}$$

s.t. $T_{mt} \geq 0$, $T_{nt} \geq 0$, $G_{mt} \geq 0$, $G_{nt} \geq 0$.

Where we have normalized the expression for profits using the distance to frontier $A_t - 1$, letting $a_{t-1} \equiv \tilde{A}_{t-1}/A_{t-1}$. From the first-order conditions of this problem, we know that, for every level of the relative wages, the relative demand for skills in innovation and imitation must satisfy

$$T_{nt} G_{nt} = \left( \phi \frac{\omega_G}{1 - \phi \omega_T} \right)^\phi G_{nt} + (a_{t-1} - 1) \left( \frac{\sigma}{1 - \sigma} \frac{\omega_G}{1 - \sigma \omega_T} \right)^\sigma G_{mt} - \omega_G (G_{nt} + G_{mt}) +$$

$$\omega_T \left( \frac{\phi}{1 - \phi \omega_T} G_{nt} + \frac{\sigma}{1 - \sigma} \frac{\omega_G}{1 - \sigma \omega_T} G_{mt} \right).$$

Plugging these back into (A.1), we obtain the following alternative expression:

$$\pi_t = \zeta \left[ \left( \phi \frac{\omega_G}{1 - \phi \omega_T} \right)^\phi G_{nt} + (a_{t-1} - 1) \left( \frac{\sigma}{1 - \sigma} \frac{\omega_G}{1 - \sigma \omega_T} \right)^\sigma G_{mt} \right] - \omega_G (G_{nt} + G_{mt}) +$$

$$\omega_T \left( \frac{\phi}{1 - \phi \omega_T} G_{nt} + \frac{\sigma}{1 - \sigma} \frac{\omega_G}{1 - \sigma \omega_T} G_{mt} \right).$$

The necessary conditions for a maximum read:

$$(1 - \phi) \left( \frac{\phi}{1 - \phi \omega_T} \right)^\phi \omega_G \leq \omega_G,$$

$$\left[ (1 - \phi) \left( \frac{\phi}{1 - \phi \omega_T} \right)^\phi - \omega_G \right] G_{nt} = 0;$$

$$(a_{t-1} - 1)(1 - \sigma) \left( \frac{\sigma}{1 - \sigma \omega_T} \omega_G \right)^\sigma \leq \omega_G,$$

$$\left[ (a_{t-1} - 1)(1 - \sigma) \left( \frac{\sigma}{1 - \sigma \omega_T} \omega_G \right)^\sigma - \omega_G \right] G_{mt} = 0.$$ 

An interior solution for this problem obtains when the left-end sides of both inequalities above equal $\omega_G$. Thus, both activities occur in equilibrium whenever

$$a_{t-1} = 1 + \frac{1 - \phi}{1 - \sigma} \left( \frac{\phi}{1 - \phi \omega_T} \right)^\phi \omega_G^{\phi - \sigma} \equiv \tilde{a}(\omega_t). \quad \text{(A.2)}$$

That is, for every value of $\omega_t \equiv \omega_G/\omega_T$, there exists a unique value of $a_{t-1}$, such that an interior solution obtains. In other terms, the solution is characterized as follows:

$$\begin{align*}
\text{if } a_{t-1} &< \tilde{a}(\omega_t) \Rightarrow \text{innovation only;} \\
\text{if } a_{t-1} &\geq \tilde{a}(\omega_t) \Rightarrow \text{innovation and imitation;} \\
\text{if } a_{t-1} &> \tilde{a}(\omega_t) \Rightarrow \text{imitation only.}\end{align*} \quad \text{(A.3)}$$

However, from the discussion of the equilibria in Section 4, we know that at any equilibrium, the wage rate must lay in the interval $[\omega^*, \bar{\omega}^*]$. This implies bounds for the range
of the values of \(a_{t-1}\) for which interior solutions may occur in the equilibrium. Let the lower bound of the interval be \(\tilde{a}_l = \tilde{a}(\omega^*)\) and the upper bound be \(\tilde{a}_h = \tilde{a}(\omega^*)\).

Then, from (A.3), we can conclude that when \(a < \tilde{a}_l\), only innovation occurs. While \(a_{t-1} > \tilde{a}_h\), implies that firms only resort to imitation. For the intermediate range, \(a_{t-1} \in (\tilde{a}_l, \tilde{a}_h)\), the equilibrium is characterized by firms performing both imitation and innovation.

\[A.2\] Proof of Proposition 2

Recall equation (7),

\[
A_t = A_{t-1} + A_{t-1}T_{nt}^\phi c_{nt}^{1-\phi} + (\tilde{A}_{t-1} - A_{t-1})T_{mt}^\sigma c_{mt}^{1-\sigma};
\]

the growth rate is:

\[
g_t \equiv \frac{A_t - A_{t-1}}{A_{t-1}} = T_{nt}^\phi c_{nt}^{1-\phi} + (a_{t-1} - 1)T_{mt}^\sigma c_{mt}^{1-\sigma}. \tag{A.4}
\]

The market solution implies that wages equal the marginal products of the two types of skills in both activities, using \(G_{nt} + G_{mt} = G_t\) and \(T_{nt} + T_{mt} = T_t\), we write this as:

\[
\omega_{Gt} = (1 - \phi)T_{nt}^\phi G_{nt}^{-\phi} = (a_{t-1} - 1)(1 - \sigma)(T_t - T_{nt})^\sigma(G_t - G_{nt})^{-\sigma};
\]

\[
\omega_{Tt} = \phi T_{nt}^{\phi-1} G_{nt}^{1-\phi} = (a_{t-1} - 1)\sigma(T_t - T_{nt})^{\sigma-1}(G_t - G_{nt})^{1-\sigma}.
\]

Focusing on imitation, this implies:\[21\]

\[
\frac{\omega_{Gt}}{\omega_{Tt}} = \frac{1 - \sigma}{\sigma} \left( \frac{T_t - T_{nt}}{G_t - G_{nt}} \right).
\]

In the absence of migration, this ratio must equal (3) in equilibrium, i.e.

\[
1 - \frac{\sigma}{1 - 2G_t} \left( \frac{T_t - T_{nt}}{G_t - G_{nt}} \right) = \frac{\xi}{\sqrt{1 - 2G_t}}. \tag{A.5}
\]

Let \(G_t^*\) be the growth maximizing value of \(G_t\):

\[
G_t^* = \arg \max g_t = \arg \max T_{nt}^\phi c_{nt}^{1-\phi} + (a_{t-1} - 1)(T_t - T_{nt})^\sigma(G_t - G_{nt})^{1-\sigma};
\]

since \(g_t\) is strictly concave and continuous, and \(T_t\) is a function of \(G_t\) according to (4), the sufficient condition for a maximum reads:

\[
1 - \frac{\sigma}{1 - 2G_t} \left( \frac{T_t - T_{nt}}{G_t - G_{nt}} \right) = -\frac{\partial T(G_t)}{\partial G_t}. \tag{A.6}
\]

From (4), it is straightforward that

\[
-\frac{\partial T(G_t)}{\partial G_t} = \frac{\xi}{\sqrt{1 - 2G_t}}.
\]

Hence, (A.6) becomes

\[
1 - \frac{\sigma}{1 - 2G_t} \left( \frac{T_t - T_{nt}}{G_t - G_{nt}} \right) = \frac{\xi}{\sqrt{1 - 2G_t}}.
\]

Since this expression is identical to (A.5), we conclude that the market outcome is growth maximizing.\[21\] This is without loss of generality, focusing on innovation yields equivalent results.
A.3 Proof of Proposition 3

From Proposition 1 we know that the lagging country can be in any of three situations: it can be performing innovation only (when \(a \leq \tilde{a}_l\)), it can be engaging in both innovation and imitation (when \(a \in (\tilde{a}_l, \tilde{a}_h)\)), or it can be fully specialized in imitation (when \(a \leq \tilde{a}_h\)). Under the innovation-only regime, the lagging country is, by symmetry, identical to the technological leader, and its growth rate will then be \(\bar{g}\), a constant. If imitation and innovation co-exist at the optimum, the growth rate will be given by (A.4). Proposition 2 shows that for each level of \(a_{t-1}\), this function is maximized by the solution of our model. Moreover, the function in (A.4) is continuously differentiable in \(a \in (\tilde{a}_l, \tilde{a}_h)\). Thus, applying the envelope theorem yields:

\[
\frac{\partial g_t}{\partial a_{t-1}} = T_{mt}^\sigma g_{mt}^{1-\sigma} > 0
\]

In the imitation-only case, equation (A.4) reduces to

\[
g_t = (a_{t-1} - 1)T_t^\sigma G_t^{1-\sigma}, \quad (A.7)
\]

and the same reasoning goes through. We can conclude that the growth rate of the lagging economy increases with the distance to frontier and is higher than the rate of frontier expansion at each point where \(a > \tilde{a}_l\).

\[\square\]

A.4 Proof of Proposition 5

First notice that when \(p_G = p_T = p\) the labour supply of the lagging country, implicitly defined by (15), coincides with the one for the leading economy whenever \(\omega = \bar{\omega}\). In this case, by symmetry, the two economies are identical and their growth rates are also equal. Hence when the lagging country, has a distance to frontier equal to \(\tilde{a}_l\), it specializes in innovation, and will grow at the same rate as the leading economy: \(g_t = \bar{g}\).

Whenever \(a_t \in (\tilde{a}_l, \tilde{a}_h)\), both innovation and imitation occur at the same time. The growth rate of the economy is then:

\[
g_t() = T_{nt}^\phi G_{nt}^{1-\phi} + (a_{t-1} - 1)T_{mt}^\sigma g_{mt}^{1-\sigma}.
\]

In the presence of migration, the equilibrium level of \(G\) and \(T\), and the split thereof, will not be the same as without migration, hence the growth rate will not be maximized.

Differentiating the above expression with respect to \(a_{t-1}\) yields:

\[
\frac{d g_t}{da_{t-1}} = \phi T_{nt}^{\phi-1} G_{nt}^{1-\phi} \frac{\partial T_{nt}}{\partial a_{t-1}} + (1 - \phi)T_{nt}^{\phi} G_{nt}^{-\phi} \frac{\partial G_{nt}}{\partial a_{t-1}} + (a_{t-1} - 1)\sigma T_{mt}^{\sigma-1} G_{mt}^{1-\sigma} \frac{\partial T_{mt}}{\partial a_{t-1}} + (a_{t-1} - 1)(1 - \sigma)T_{mt}^\sigma G_{mt}^{-\sigma} \frac{\partial G_{mt}}{\partial a_{t-1}} + T_{mt}^\sigma G_{mt}^{1-\sigma}. \quad (A.8)
\]

At any interior equilibrium it must be the case that \(\phi T_{nt}^{\phi-1} G_{nt}^{1-\phi} = (a_{t-1} - 1)\sigma T_{mt}^{\sigma-1} G_{mt}^{1-\sigma} = \omega T_{Gl}\), and \((1 - \phi)T_{nt}^{\phi} G_{nt}^{-\phi} = (a_{t-1} - 1)(1 - \sigma)T_{mt}^{\sigma} G_{mt}^{-\sigma} = \omega G_{l}\). Moreover, since \(T = T_{nt} + T_{mt}\), it follows that \(\partial T_{nt}/\partial a_{t-1} = \partial T_{nt}/\partial a_{t-1} - \partial T_{mt}/\partial a_{t-1}\); a similar expression holds for \(G_{t}, G_{mt}\) and \(G_{nt}\). Using these facts into (A.8), we obtain,

\[
\frac{d g_t}{da_{t-1}} = \omega T_t \frac{\partial T_t}{\partial a_{t-1}} + \omega G_t \frac{\partial G_t}{\partial a_{t-1}} + T_{mt}^\sigma G_{mt}^{1-\sigma}.
\]
Recalling the expression linking the supply of $T$-skills to $G_t$, from equation (4), we can rewrite $\frac{\partial T_t}{\partial a_{t-1}}$ as,

$$\frac{\partial T_t}{\partial a_{t-1}} = -\frac{1}{\sqrt{1-2G_t}} \frac{\partial G_t}{\partial a_{t-1}}$$

Plugging this into the expression for $\frac{dg_t}{da_{t-1}}$ finally gives us,

$$\frac{dg_t}{da_{t-1}} = \left[ \omega_{Gt} - \frac{\omega_{Tt}}{\sqrt{1-2G_t}} \right] \frac{\partial G_t}{\partial a_{t-1}} + T_{mt} G_{mt}^{1-\sigma} > 0, \quad (A.9)$$

since both terms are always positive. Indeed, as discussed in section 6.1, the term in square brackets is always negative since the supply curve under migration, and hence the equilibrium value of the wage rate $\omega$, lies below the supply curve relative to the no migration case. Recalling footnote 19 this implies $\omega_t < \omega_{no-m}$, or $\omega < \xi/\sqrt{1-2G_t} < 1/\sqrt{1-2G_t}$. Since $\partial G_t/\partial a_{t-1} < 0$ because the equilibrium level of $G_t$ decreases with $\omega_t$, while $\omega_t$ decreases with the distance to frontier, the first term is always positive. The positivity of the second term at the right-hand side, on the other hand, is trivial.

To conclude the proof notice that when only imitation occurs by a similar reasoning we immediately get (A.9), by setting $T_{mt} = T_t$ and $G_{mt} = G$.

Thus, we have shown that the lagging economy grows faster than the leading one for each level of the distance to frontier larger than $\tilde{a}_l$, while it grows just at the same rate as the frontier when full specialization in innovation (i.e. convergence) is finally achieved.

**A.5 Proof of Proposition 6**

Notice that the proof in Proposition 5 that the growth rate of the lagging economy increases with the distance to frontier holds irrespective of the values of $p_G$ and $p_T$. Hence, also in this case we can conclude that $g$ increases monotonically with $a$. Thus, there is a tendency for the technologically lagging country to converge.

To prove that the process stops prematurely when the probability of migration is different for workers with different types of skills, recall (16). If the distance to frontier were $\tilde{a}_l$ and firms in the lagging country were to specialize in innovation, by symmetry, domestic wages would equal foreign ones. Contrary to before though, the two economies are not identical, given the wedge induced by the different probabilities of migration. Thus the accumulation of human capital is distorted. From Proposition 2, the growth rate is maximized in the leading country. Since $g$ is strictly concave and continuous, it follows that the growth rate that can be obtained for $\tilde{a}_l$ by the lagging country can only be smaller than $\bar{g}$.

Since the growth rate increases monotonically with $a$, it follows immediately that the lagging country will stop converging towards the frontier at a level of the distance to frontier $a_{trap} > \tilde{a}_l$.

Thus, the steady-state distance to frontier is increased by the possibility of migration and that no specialization in innovation is possible in this case.