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Advanced Distribution Network Modelling with Distributed Energy Resources

Alison O’Connell, B.E.

A thesis submitted to University College Dublin in fulfilment of the requirements for the degree of

Philosophiæ Doctor

College of Engineering and Architecture
School of Electrical and Electronic Engineering

Supervisor and Head of School: Dr. Andrew Keane
Co-supervisor: Dr. Damian Flynn
Nominating Professor: Prof. Mark O’Malley

September 2015
Abstract

The addition of new distributed energy resources, such as electric vehicles, photovoltaics, and storage, to low voltage distribution networks means that these networks will undergo major changes in the future. Traditionally, distribution systems would have been a passive part of the wider power system, delivering electricity to the customer and not needing much control or management. However, the introduction of these new technologies may cause unforeseen issues for distribution networks, due to the fact that they were not considered when the networks were originally designed.

This thesis examines different types of technologies that may begin to emerge on distribution systems, as well as the resulting challenges that they may impose. Three-phase models of distribution networks are developed and subsequently utilised as test cases. Various management strategies are devised for the purposes of controlling distributed resources from a distribution network perspective. The aim of the management strategies is to mitigate those issues that distributed resources may cause, while also keeping customers’ preferences in mind.

A rolling optimisation formulation is proposed as an operational tool which can manage distributed resources, while also accounting for the uncertainties that these resources may present. Network sensitivities for a particular feeder are extracted from a three-phase load flow methodology and incorporated into an
Abstract

optimisation. Electric vehicles are the focus of the work, although the method could be applied to other types of resources. The aim is to minimise the cost of electric vehicle charging over a 24-hour time horizon by controlling the charge rates and timings of the vehicles. The results demonstrate the advantage that controlled EV charging can have over an uncontrolled case, as well as the benefits provided by the rolling formulation and updated inputs in terms of cost and energy delivered to customers.

Building upon the rolling optimisation, a three-phase optimal power flow method is developed. The formulation has the capability to provide optimal solutions for distribution system control variables, for a chosen objective function, subject to required constraints. It can, therefore, be utilised for numerous technologies and applications. The three-phase optimal power flow is employed to manage various distributed resources, such as photovoltaics and storage, as well as distribution equipment, including tap changers and switches. The flexibility of the methodology allows it to be applied in both an operational and a planning capacity.

The three-phase optimal power flow is employed in an operational planning capacity to determine volt-var curves for distributed photovoltaic inverters. The formulation finds optimal reactive power settings for a number of load and solar scenarios and uses these reactive power points to create volt-var curves. Volt-var curves are determined for 10 PV systems on a test feeder. A universal curve is also determined which is applicable to all inverters. The curves are validated by testing them in a power flow setting over a 24-hour test period. The curves are shown to provide advantages to the feeder in terms of reduction of voltage deviations and unbalance, with the individual curves proving to be more effective. It is also shown that adding a new PV system to the feeder only requires analysis for that system.

In order to represent the uncertainties that inherently occur on distribution systems, an information gap decision theory method is also proposed and integrated into the three-phase optimal power flow formulation. This allows for robust network decisions to be made using only an initial prediction for what the
uncertain parameter will be. The work determines tap and switch settings for a test network with demand being treated as uncertain. The aim is to keep losses below a predefined acceptable value. The results provide the decision maker with the maximum possible variation in demand for a given acceptable variation in the losses. A validation is performed with the resulting tap and switch settings being implemented, and shows that the control decisions provided by the formulation keep losses below the acceptable value while adhering to the limits imposed by the network.
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Appendix B EPRI Internship ................................ 129
I would like to extend my sincere thanks to all who contributed to this thesis and encouraged me over the past four years. The following people deserve a special mention for their commitment and support.

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Publications Arising From This Thesis

In Review


Journal Publications


Conference Publications


O’Connell, A., Richardson, P., Flynn, D. and Keane, A., ”Controlled Charging of Electric Vehicles in Residential Distribution Networks”, in Innovative Smart Grid Technologies Europe, Berlin, Germany, October 2012
### Technical Acronyms

<table>
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<th>Description</th>
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<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>BSOC</td>
<td>Battery State of Charge</td>
</tr>
<tr>
<td>CHP</td>
<td>Combined Heat and Power</td>
</tr>
<tr>
<td>CPOC</td>
<td>Customer Point of Connection</td>
</tr>
<tr>
<td>CSP</td>
<td>Concentrated Solar Power</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DER</td>
<td>Distributed Energy Resource</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generation</td>
</tr>
<tr>
<td>DSO</td>
<td>Distribution System Operator</td>
</tr>
<tr>
<td>EV</td>
<td>Electric Vehicle</td>
</tr>
<tr>
<td>FACTS</td>
<td>Flexible AC Transmission System</td>
</tr>
<tr>
<td>FCIM</td>
<td>Four Conductor Current Injection Method</td>
</tr>
<tr>
<td>HV</td>
<td>High Voltage</td>
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<tr>
<td>IGDT</td>
<td>Information Gap Decision Theory</td>
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<tr>
<td>LTC</td>
<td>Load Tap Changer</td>
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<tr>
<td>LV</td>
<td>Low Voltage</td>
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<tr>
<td>MAPE</td>
<td>Mean Absolute Percentage Error</td>
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<tr>
<td>MV</td>
<td>Medium Voltage</td>
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<tr>
<td>NLP</td>
<td>Non-Linear Programming</td>
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<tr>
<td>OPF</td>
<td>Optimal Power Flow</td>
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<tr>
<td>OLTC</td>
<td>On-Load Tap Changer</td>
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<td>PV</td>
<td>Photovoltaics</td>
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RMSE      Root Mean Squared Error
SMP       System Marginal Price
SQP       Sequential Quadratic Programming
TOPF      Three-Phase Optimal Power Flow
TSO       Transmission System Operator

Organisation Acronyms

EPRI      Electric Power Research Institute
ESBN      Electricity Supply Board Networks
EU        European Union
IEEE      Institute of Electrical and Electronic Engineers
NREL      National Renewable Energy Laboratory
SEMO      Single Electricity Market Operator
UN        United Nations
Nomenclature

Notation shown in bold in this thesis represents phasors, while non-bold notation indicates scalar values.

Sets

- \( \Omega_d \) Set of phases \( \{a, b, c\} \)
- \( \Omega_p \) Set of phases \( \{a, b, c, n\} \)
- \( \Omega_k \) Set of all buses
- \( \Omega_{k_i} \) Set of all buses connected to bus \( k \)
- \( \Omega_{k_s} \) Set of slack buses
- \( \Omega_{k_v} \) Set of voltage controlled buses
- \( \Omega_{k_T} \) Set of transformer buses
- \( \Omega_{k_R} \) Set of regulator buses
- \( \Omega_{k_{EV}} \) Set of buses with an electric vehicle connected
- \( \Omega_{k_{PV}} \) Set of buses with a photovoltaic system connected
- \( \Omega_h \) Set of all time steps
- \( \Omega_{h_{EV_k}} \) Set of charging time steps for an electric vehicle at bus \( k \)
- \( \Omega_m \) Set of scenarios
- \( \Omega_f \) Set of configurations
Variables

\[ V_d^k = V_{Re}^d + jV_{Im}^d \]  Real and imaginary voltage phasor at bus \( k \) phase \( d \)

\[ I_d^k = I_{Re}^d + jI_{Im}^d \]  Real and imaginary current phasor at bus \( k \) phase \( d \)

\[ \Delta V_d^k = \Delta V_{Re}^d + j\Delta V_{Im}^d \]  Real and imaginary voltage mismatch phasor at bus \( k \) phase \( d \)

\[ \Delta I_d^k = \Delta I_{Re}^d + j\Delta I_{Im}^d \]  Real and imaginary current mismatch phasor at bus \( k \) phase \( d \)

\[ S_d^G_k = P_d^{G_k} + jQ_d^{G_k} \]  Active and reactive power generation at bus \( k \) phase \( d \)

\[ S_d^D_k = P_d^{D_k} + jQ_d^{D_k} \]  Active and reactive power demand at bus \( k \) phase \( d \)

\[ S_d^k = P_d^k + jQ_d^k \]  Active and reactive power injection at bus \( k \) phase \( d \)

\[ I_{kl}^d = I_{Re}^{d_k} + jI_{Im}^{d_k} \]  Real and imaginary current flow through branch between bus \( k \) phase \( d \) and bus \( i \) phase \( d \)

\[ S_{kl}^d = P_d^{kl} + jQ_d^{kl} \]  Active and reactive power flow through branch between bus \( k \) phase \( d \) and bus \( i \) phase \( d \)

\[ P_{d,k,I_k,Z_k} \]  ZIP model components of active power at bus \( k \) phase \( d \)

\[ Q_{d,k,I_k,Z_k} \]  ZIP model components of reactive power at bus \( k \) phase \( d \)

\[ \sigma_{ki}^{st} = \delta V_{Re}^{s} \left/ \delta I_{Im}^{i} \right. \]  Sensitivity of real voltage at bus \( k \) phase \( s \) to change in imaginary current at bus \( i \) phase \( t \)

\[ \zeta_{ki}^{st} = \delta V_{Re}^{s} \left/ \delta I_{Re}^{i} \right. \]  Sensitivity of real voltage at bus \( k \) phase \( s \) to change in real current at bus \( i \) phase \( t \)

\[ \mu_{ki}^{st} = \delta V_{Im}^{s} \left/ \delta I_{Im}^{i} \right. \]  Sensitivity of imaginary voltage at bus \( k \) phase \( s \) to change in imaginary current at bus \( i \) phase \( t \)

\[ \gamma_{ki}^{st} = \delta V_{Im}^{s} \left/ \delta I_{Re}^{i} \right. \]  Sensitivity of imaginary voltage at bus \( k \) phase \( s \) to change in imaginary current at bus \( i \) phase \( t \)

\[ P_{d, EV_k} \]  Electric vehicle active power demand at bus \( k \) phase \( d \)

\[ r_{ki}^d \]  Turns ratio between bus \( k \) phase \( d \) and bus \( i \) phase \( d \)

\[ Q_{d, inv_k} \]  Inverter reactive power at bus \( k \) phase \( d \)

\[ P_{d, PV_k} \]  Photovoltaic system active power output at bus \( k \) phase \( d \)

\[ P_{d, stor_k} \]  Rate of charge/discharge of storage device at bus \( k \) phase \( d \)

\[ E_{d, stor_k} \]  Energy stored in storage device at bus \( k \) phase \( d \)
Nomenclature

\[ P_{L_{ki}} \]  Active power loss between bus \( k \) phase \( d \) and bus \( i \) phase \( d \)

\[ V_{unb_k} \]  Voltage unbalance at bus \( k \) phase \( d \)

\( \alpha \)  Radius of uncertainty

Parameters

\[ G^{st}_{kk}, G^{st}_{ki} \]  Conductance from nodal admittance matrix

\[ B^{st}_{kk}, B^{st}_{ki} \]  Susceptance from nodal admittance matrix

\( C_h \)  Cost of electricity at time step \( h \)

\( V_{min} \)  Minimum allowable voltage

\( V_{max} \)  Maximum allowable voltage

\( V_d^{init}_k \)  Initial voltage at bus \( k \) phase \( d \)

\( P_{min}^{EV} \)  Minimum electric vehicle charge rate

\( P_{max}^{EV} \)  Maximum electric vehicle charge rate

\( E_d^{max}_{EV_k} \)  Maximum energy capacity of electric vehicle at bus \( k \) phase \( d \)

\( E_d^{max}_{EV_k} \)  Initial state of charge of electric vehicle at bus \( k \) phase \( d \)

\( I_{ki}^{max} \)  Current magnitude rating of branch between bus \( k \) phase \( d \) and bus \( i \) phase \( d \)

\( S^{max}_{ki} \)  Apparent power rating of branch between bus \( k \) phase \( d \) and bus \( i \) phase \( d \)

\( S^{max}_{SC} \)  Apparent power rating of service cables

\( V_d^{spec}_{slack} \)  Real and imaginary voltage phasor at slack bus

\( V_d^{spec}_k \)  Specified voltage magnitude at bus \( k \) phase \( d \)

\( S^{d}_{inv_k} \)  Apparent power rating of inverter at bus \( k \) phase \( d \)

\( E_d^{max}_{stor_k} \)  Maximum energy capacity of storage device at bus \( k \) phase \( d \)

\( V_d^{opt_k} \)  Optimal voltage set-point at bus \( k \) phase \( d \)

\[ |S_d^{D_k}| \]  Predicted value of apparent power demand at bus \( k \) phase \( d \)

\( \bar{P}_L \)  Predicted optimal active power losses

\( P_{Lc} \)  Tolerable loss limit

\( \beta \)  Tolerable loss variation
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1.1 Background

Nations the world over are expressing increasing concern over the same impending issue: climate change. It is becoming more and more evident that human behaviour is having an increasingly negative influence on the earth’s atmosphere. Excessive burning of fossil fuels, eradication of rainforests, and increasing levels of agricultural activity have all led to a considerable growth in the amount of greenhouse gases that are filling our atmosphere. The primary result of a build up of greenhouse gases is a steady rise in the earth’s temperature, otherwise known as global warming. According to climate change researchers, a global temperature rise of even 1°C could have a disastrous effect on the environment (Smith et al., 2009).

Although the adverse impacts of human activity on climate change have been known for some time, it is only in recent times that countries have begun to take a proactive approach in providing a solution. In 2005, the United Nations (UN) enforced the Kyoto Protocol, which is an international agreement between committed countries to reduce greenhouse gas emissions, through the
implementation of individual national targets (United Nations, 1998). The United States has recently adopted some of the Kyoto policies as part of its Clean Power Plan (United States Environmental Protection Agency, 2015) and is aiming to reduce carbon emissions from the power sector by 32% below 2005 levels, by 2030. In 2014, European Union (EU) leaders agreed on an even more stringent plan, with a framework for 2030 targets which aim to: reduce EU greenhouse gas emissions by 40% from 1990 levels, raise the share of EU energy consumption produced from renewable resources to at least 27%, and improve the EU’s energy efficiency by at least 27% (European Commission, 2013).

Various measures are being introduced in order to reach the challenging goals that have been put in place. Strategies such as the introduction of carbon taxes, carbon capture and storage, consumer energy efficiency education, building energy ratings, energy device labelling, reforestation, etc. are being proposed and implemented in countries worldwide. However, the element at the core of most climate change mitigation measures is the integration of low carbon, renewable energy resources. Conventionally, in sectors such as electricity generation, transport, and water/space heating, fossil fuels have dominated. However, as well as contributing considerably to the level of greenhouse gases in the atmosphere, fossil fuels such as coal, oil, and gas are finite resources which are steadily running out (Shafiee and Topal, 2009).

Renewable energy resources are defined as resources which are not at risk of depletion, and are replenished naturally and in a timely fashion. Examples of these types of resources are: wind, solar, hydro, ocean, geothermal, and biomass. Similar to fossil fuels, renewable resources are geographically dependent, however, because such diversity exists in the types of renewable resources available, most regions will have at least one viable resource to draw on. The electricity sector stands to be the largest integrator of renewable resources, as large-scale fossil fuel generation plants can potentially be displaced by renewable plants. In fact, the electricity sector has already incorporated significant levels of renewable energy in various forms. Hydro energy, the harnessing of energy from falling water, is one of the oldest forms of renewable energy, with uses for electricity generation dating as
far back as the late 1800s (International Hydropower Association, 2015). Since then, hydro energy has become one of the most prominent types of renewable energy, primarily in the form of dams and pumped storage plants.

Like hydro, the potential of wind for energy purposes is not a recent discovery. Wind has been harnessed in the transport sector, in the form of sailing, for thousands of years; windmills were employed to pump water, grind grain, and cut wood as early as 200 B.C.; and wind turbines have been used to generate electricity since the late 1800s. Low oil prices in the 1950s meant that wind power was sidelined for a time (Wind Energy Foundation, 2015), however, recent concerns over climate change and sustainability have led to a resurgence in wind power. Many countries, particularly coastal or mountainous countries, have a considerable wind resource available and a number of these countries are currently exploiting this resource in order to meet their renewable energy and climate change targets. Europe has been the frontrunner in terms of wind energy, with countries such as Denmark having 32% and Portugal having 22% of their annual electricity demands being generated by wind in 2012 (Energy Information Administration, 2015). North America and Asia are also making significant progress with 3.4% and 1.7% of their annual demands respectively being generated by wind in 2012.

Alongside hydro and wind, solar energy looks set to be one of the foremost contributors to the renewable energy effort. Although solar power has been employed for cooking and heating for hundreds of years, its leap into electricity generation did not come about until the 1950s. Two main technologies are capable of converting sunlight to electricity, photovoltaics (PV), a direct conversion method, and concentrated solar power (CSP), an indirect method. Some notable solar installations are: the 550 MW Topaz PV farm in California, U.S.A; the 392 MW Ivanpah CSP facility, also in California, U.S.A; and the 320 MW Longyangxia Dam PV Park in the Qinghai Province, China. It is predicted that solar PV and CSP will together be the largest source of electricity generation by 2050 (International Energy Agency, 2014).
Geothermal, ocean, and biomass energy have also made significant strides in recent years. Geothermal energy utilises heat from deep within the earth for the purposes of space/water heating, via heat pumps, and electricity generation, via vapour and water plants. Ocean energy can refer to both tidal and wave energy, and is usually employed in an electricity generation capacity. Technologies for ocean energy are mostly still in their infancy, due to the difficulty in designing devices that can withstand the harsh environment of the ocean, without requiring an excessive amount of maintenance. Biomass primarily refers to plants or plant-based materials which can be employed directly to produce heat or electricity, or converted to biofuels or biogas which can be utilised to generate electricity, heat, and transportation fuels.

Although it may seem that renewable resources are the answer to the climate change issue, there are some key barriers to overcome. There is a degree of public opposition and debate surrounding climate change and certain renewable resources. Certain groups exist that dismiss the scientific consensus on anthropogenic climate change. Significant opposition also exists in communities where renewable resources are planned to be built, in particular wind farms. There are disputes over the economic viability of renewables, and whether the return is worth the initial investment cost. Variability inherently affects a number of renewable resources, namely wind, solar, and ocean energy. It would not be feasible for a region to rely solely on these variable resources, without significant storage resources. Furthermore, electricity system infrastructure will require significant upgrading to accommodate new renewable resources, particularly since the most suitable locations for these resources are not always convenient from an electricity system standpoint.

1.2 The Electricity System

Undoubtedly, the advent of renewable resources will have the most significant impact on the electricity sector. On the surface, it may seem that the only required change is that conventional generation should be replaced by renewable
generation. However, the solution is not that simple, due, in part, to the electricity system.

The electricity system is the complex underlying network that essentially connects generation to demand. The development of the electric power system that we use today began in the late 1800s. Thomas Edison designed the first direct current (DC) electricity system in New York in 1882. However, the DC design limited the distance that the power could travel. Not long after, an alternating current (AC) power system was developed by George Westinghouse and William Stanley in Massachusetts (Glover et al., 2011). In the years that followed, a weighty battle emerged between DC and AC power systems with AC eventually winning out. AC power systems have continued to be the standard since then, however recent research has investigated the potential benefits of DC systems, particularly in the area of offshore wind (Robinson et al., 2010). A simple overview of the modern electricity system is given in Fig. 1.1.

1.2.1 Transmission System

Oftentimes, the locations of electricity generating stations can be geographically distant to the locations of the primary demand centres, and therefore, there is a need to transport the electric power over large distances. This is the task of the transmission system, which is the area given in yellow in Fig. 1.1. The transmission system is a network of high voltage (HV) three-phase power lines, cables, and other equipment which are used to transmit power from generating stations to substations at load centres. HV is utilised for the transmission system,
as it reduces the current flowing and thus the amount of power that is lost through the transmission process. Typically, the transmission system voltage is defined as any voltage above 35 kV (IEC, 2009), however, this is very much country and transmission system operator (TSO) dependent. All conventional generation connected to the transmission system rotates at the same electrical speed or frequency, therefore, virtually every electricity system operates as one synchronous system. Nominal frequency values are usually either 50/60 Hz depending on location.

Although the transmission system is a three-phase system, the aggregation of the load means that each phase carries approximately the same level of power. As a result, the transmission system can be adequately modelled using only a single phase representation, with the results being applicable to all three phases. This greatly simplifies the modelling and analysis of the transmission system for planning and operational purposes, which is of great use to TSOs.

1.2.2 Distribution System

The distribution system is the electricity delivery network that connects the transmission system to individual customers. The transmission system terminates at substations where the voltage is stepped down to distribution level via transformers, as shown by the transition from the yellow area to the green area in Fig. 1.1. Lower voltages are utilised for the distribution system because the distances covered are much shorter than those of the transmission system, and the amount of power being transported is significantly less. Distribution voltages are classified as medium voltage (MV) and low voltage (LV), with MV usually referring to nominal voltages between 1 kV and 35 kV (IEC, 2009), and LV being any voltage below 1 kV. MV is used for delivery from the transmission system to step-down distribution transformers, while LV is employed between distribution transformers and customer connection points. LV is the voltage level that is utilised by most customers in their homes, businesses, etc. Typical nominal voltages of LV systems are 120/240 V and 230/400 V (line-ground/line-line) depending on location (IEC, 2009).
The structure of the distribution system varies from region to region. For example, in Europe, the MV distribution system connects to large distribution transformers which serve LV feeders that deliver power to hundreds, or sometimes thousands, of customers. Whereas, the North American distribution system consists of the MV system connecting to a larger number of small distribution transformers, that connect directly to a few ($\approx 10$) customers (Short, 2004). References to distribution systems in this thesis will primarily be concerned with the European structure, in particular LV feeders. Distribution systems can either be radial or meshed, where radial refers to one supply point or distribution transformer providing power downstream through sequential wires or cables, and meshed describes a system with multiple supply points that are interconnected at one or more locations downstream. Radial is typically the more common distribution configuration.

The design of distribution systems is primarily influenced by customer load (Willis, 2004). The layout, ratings, and required equipment are all based on the characteristics of the load that will be connected. The nature of human activity means that individual customer load is constantly changing. Therefore, over the course of a particular time period, a customer will have a non uniform load curve. Each customer on a feeder will have their own unique load curve, so distribution feeders tend to experience a significant level of load diversity. This diversity can be advantageous in that customers’ demand peaks are not likely to coincide, therefore the peak demand of the feeder will be significantly less than the sum of the individual customer demand peaks. Demand diversity and peak demand are the main characteristics that are considered when determining equipment ratings, in particular transformer ratings. However, in order to account for load growth, and to ensure that distribution systems can be operated passively, transformers and other equipment are usually significantly oversized.

Similar to the transmission system, the MV distribution system is a three-phase system, which usually has similar load on each phase, therefore, for modelling purposes a balanced representation is often adequate. LV distribution systems however, are somewhat different. Most residential and commercial
customers have single-phase load and are connected to the LV system through a single-phase wire. Two and three-phase customer connections are also utilised for customers with larger demand requirements. Due to the presence of these loads, single, two, and three-phase lines all exist in LV feeders. Furthermore, unlike transmission system lines, distribution lines are untransposed (order of phases is not switched) (Kersting, 2002). The combination of these attributes makes LV feeders an inherently unbalanced part of the electricity system as a whole. Subsequently, the balanced modelling that can be applied to HV transmission and MV distribution systems, is not sufficient for modelling LV distribution feeders.

1.3 Distributed Energy Resources and the Distribution System

Distributed energy resources (DER) are defined as small or medium scale generation and storage resources that are connected directly to the distribution system. Although DER typically refers to capacities up to 10 MW in size, the types of DER that will be discussed in this thesis are connected either at the customer’s premises or directly to the LV feeder. In terms of generation, DER are generally renewable resource based such as PV or micro-wind. DER storage devices consist of fuel cells, flywheels, and batteries. Other technologies such as EVs, micro combined heat and power (CHP), and heat pumps are also classed as types of DER.

Having these resources located close to the load has the potential to be advantageous in a number of ways, e.g. peak demand reductions, increased flexibility, and reduced losses, however, the emergence of DER also presents some challenges for distribution systems (Ochoa and Mancarella, 2012). Conventionally, distribution networks were not designed with these types of technologies in mind. Subsequently, the addition of new resources may push existing feeders outside of their safe technical operating limits. Different resources will present different issues, therefore impact assessments of DER are critical in order to develop effective solutions.
1.3.1 Photovoltaics

Solar PV systems look set to become the most common form of distributed generation (DG) that LV feeders will encounter, both in the form of small-scale customer PV, as well as larger scale utility installations. The potential impacts that PV could have on distribution systems have been studied extensively in the literature (Jung et al., 2015). The authors in Tonkoski et al. (2012) have modelled a residential feeder in PSCAD, and assessed the impacts of increasing PV penetration levels, up to 75%, on the feeder voltage profile. They concluded that voltage rise is dependent not only on the penetration level of PV but also the specific feeder characteristics, such as length and transformer impedance. The work in Ari and Baghzouz (2011) examines the effects that a 20% penetration level of PV has on transformer tapping and voltage flicker for a portion of a distribution substation. The results showed that voltage flicker was not a concern, however, a significant increase in tap changes was observed in comparison to a scenario with no PV. The authors of Baran et al. (2012) investigate how system protection and voltage variation are impacted by PV, utilising an actual distribution circuit modelled in PSCAD. They observe that although protection devices are not significantly affected by high PV penetration, voltage variations can be excessive. In Gonzalez et al. (2012) power quality issues are analysed in the form of over and under voltages and unbalance, for four feeders in Belgium. The authors note that although the addition of PV may be beneficial to certain feeders, power quality issues occur on others, therefore different feeders may require different solutions. The results of the various works discussed here demonstrate that a wide variety of issues can be caused by PV, therefore, it may be most beneficial to analyse feeders on an individual basis.

1.3.2 Electric Vehicles

Electric vehicles (EVs) are a technology that have become increasingly popular in recent times. EVs have the potential to considerably reduce dependence on fossil fuels in the transport sector, however, high penetration levels of EVs
will substantially increase electricity demand, particularly at the distribution
level. The work in Richardson et al. (2010) examines the impacts of increasing
penetration levels of EVs on LV feeder voltages and asset loading. The results
indicate that the feeder under examination breaches limits for both voltage
and loading at penetration levels between 20-40%. The location and phase
allocation of EVs is also shown to have a significant effect. The authors in
Papadopoulos et al. (2012) use a probabilistic approach to model EV location,
time, and duration of charging. Impacts on voltages, distribution transformer,
cables, and losses for a generic UK distribution feeder are assessed for high and
low EV penetration levels. Both high and low penetration levels caused cable and
transformer overloads, with the voltage and losses being significantly impacted
at the high penetration level. The approach in Taylor et al. (2009) examines EV
impacts on thermal loading, voltage regulation, transformer loss of life, unbalance,
losses, and harmonic distortion levels. Multiple distribution systems are analysed
using both deterministic and stochastic methods with the OpenDSS simulation
software. The authors conclude that different feeders experience different issues
and that utilities need to analyse the impacts of EVs on their feeders. The work
in Pieltain Fernandez et al. (2011) determines the impact of different levels of
EV penetration on the distribution network’s investment and energy losses, for
two different large-scale distribution areas. The results showed that the cost of
required network reinforcements could be 19% higher than the network costs for
the case without EVs. Results also showed that losses could increase by up to
40% in a high penetration (62%) scenario.

1.3.3 Combined Heat and Power

CHP, or co-generation, is a technology that produces electricity and heat through
a single process. Conventional electricity plants produce waste heat as they
generate electricity. CHP plants capture this heat and subsequently employ it for
space and water heating. Micro-CHP plants perform the same task on a smaller
scale for use in the residential and commercial sectors. In a similar fashion to
PV, electricity generated by micro-CHP may present some challenges for the
distribution system. The work in Mancarella et al. (2011) assesses the impact of CHP units and heat pumps on various LV networks using models for both electricity and heat demand/generation. The authors note that CHP alone can lead to voltage increases, particularly at high penetration levels, however the addition of heat pump load to the feeder can mitigate these rises. In Quezada et al. (2006) the impact that different forms of DG could have on annual distribution system losses is analysed. The types of DG examined are wind, PV, and CHP. The results indicate that out of the three types of DG, CHP results in the least loss increase, however high penetration levels still produce considerably higher losses than the case with no DG. The authors in Zhang et al. (2013) examine the effects of CHP on electrical and water distribution systems. The results show that CHP could cause congestion in both the electricity and water networks.

1.3.4 Heat Pumps

Heat pumps are devices that exchange heat between heat sources and sinks. Common sources for heat pumps include ground, air, and water. Heat pumps use electrical power to perform the exchange of heat between the source and sink and are utilised primarily for the purposes of space and water heating. Due, in part, to their efficiency over conventional boilers, heat pumps are becoming a popular means of heating in the residential and commercial sectors. With heat pump capacities ranging from a few kW to hundreds of kW, the addition of large quantities of heat pumps would result in considerable additional load. Recent work in Akmal et al. (2014) investigates the static and dynamic effects of heat pumps on a generic UK low voltage network. The authors model the feeder in DlgSILENT using actual heat pump data to create an accurate model. A notable result is the effect that start-up transients have on voltage, with the voltage falling below 90% of nominal values in some cases. Mancarella et al. (2011) note that heat pumps could result in dramatic voltage drops at high penetrations, as well as significant transformer overloading. The authors in Li and Crossley (2014) perform a Monte Carlo study to determine how EVs and heat pumps will affect LV voltage levels. The results show that the location of the devices on the feeder
significantly impacts voltage levels. Time of use tariffs are also examined and are shown to reduce voltage violations at peak times, however, violations are subsequently shifted to off-peak times.

### 1.3.5 Storage

At present, storage at distribution level is not commonplace. However, more recently, distributed and small-scale storage has been introduced for consumer use (Tesla, 2015). As previously discussed, the introduction of some forms of DER to distribution feeders may prove challenging to these feeders and their equipment. Storage is a way of mitigating some of these issues, particularly if DG is present. The research in Kleinberg et al. (2014) investigates the impacts of using energy storage with distributed solar PV. The use of storage is shown to reduce line congestion and smooth out the net daily demand and voltage profiles. Work in Kwhannet et al. (2010) consists of modelling a micro-grid that has a small biomass plant as well as a battery energy storage system. The micro-grid is analysed for cases with and without the battery system. The results with storage present show a loss reduction of 7% over the case without storage present.

### 1.4 Distribution System Management

As illustrated in the previous section, large penetration levels of DER have the potential to cause problems for the distribution system in the form of considerable voltage deviations, increased losses, increased tap operations, and equipment overloading. One possible solution is for distribution system operators (DSOs) to upgrade their existing feeders and equipment. A more long term solution, however, is to introduce a degree of control and management to distribution systems.

At present, unlike the transmission system, the distribution system is operated in a relatively passive manner. Conventionally, the task of the distribution system was to deliver electricity to the customer, therefore most of these networks have been designed with this sole purpose in mind (Kersting, 2002). Equipment is rated
sufficiently high such that it is able to cope with substantial demand growth without the need for control or management. The result is that measurement devices are sparse which provides little to no observability. Furthermore, although control is somewhat coordinated, it is limited and primarily autonomous, e.g. transformer/regulator tap controls and capacitor switching controls. The integration of DER may require a more active approach, with greater awareness of distribution system conditions in real time.

Smart electricity meters are being proposed as part of many climate change strategies (U.S. International Trade Commission, 2014), and may provide the insight that is vital for effective distribution system management. These meters are installed at the customer’s premises and record real time measurements of the consumption for that customer. Depending on the system in place, the frequency of these measurements could range from every second to every 15 minutes. Smart meters have the potential to be beneficial for distribution systems in a number of ways. Incorporating smart meters with in home displays could provide a degree of awareness for customers and could allow them to manage their electricity usage more effectively. In particular, if time of use tariffs were introduced to replace fixed rate pricing, customers could schedule certain loads to run at low cost times. This would not only be economically beneficial for the customer, but on an aggregate scale could also reduce system demand peaks. The combination of time of use tariffs and DER could significantly alter the conventional system demand curve as well as the diversity of demand on distribution systems. The result may be that demand peaks are shifted and even reduced, which could change the way that distribution systems are designed. Detailed records of customer usage could play a valuable role in future distribution system planning and design. However, the fundamental advantage that smart meters could provide, from a distribution perspective, is real time observability. Although not all smart meters have the capability to report real time measurements at present, widespread real time reporting is envisioned in the future. A smart meter with such capabilities at every customer point of connection (CPOC) could provide system operators with a measurement of current distribution system conditions. These measurements
could subsequently play a crucial role in any future distribution management schemes.

1.4.1 Modelling

One of the most critical aspects of developing management strategies for distribution feeders is creating accurate models. Models of the distribution system itself, of the different components that currently exist on distribution feeders, as well as the technologies that may begin to emerge in the future. There are numerous ways of modelling electricity systems, depending on what information is needed, such as transient studies, harmonics analysis, short circuit analysis, unit commitment, and economic dispatch. The most common way of modelling electricity networks, however, is using load flow analysis, also known as power flow analysis. This simulation technique calculates the operating points that an electricity system would experience for a particular set of conditions, i.e. load and generation. The solution shows how the system would react under normal steady-state operation. Most power flow methods employ numerical methods, such as Newton-Raphson (Tinney and Hart, 1967) or Gauss-Seidel (Glimm and Stagg, 1957), for the purposes of solving the set of non-linear power flow equations.

As previously discussed, the distribution system is unbalanced in nature, therefore, in order to ensure accurate modelling, it should be represented as such. Accordingly, balanced power flow formulations have evolved into three-phase unbalanced power flow. The work in Chang et al. (2007) details a forward-backward sweep formulation that is capable of solving power flow for three-phase radial feeders. In Cheng and Shirmohammadi (1995) a compensation based method for radial and weakly meshed three-phase systems is presented. The method introduces appropriate break points to meshed networks which convert them to radial networks that can be solved using the forward-backward sweep technique. Another approach in Teng (2003) develops two matrices which are used directly to solve radial three-phase load flow. The authors also extend the formulation to include weakly meshed feeders. The authors in Penido et al. (2008)
describe a Newton-Raphson based current injection method for solving power flow in four wire systems. The formulation is suitable for radial and meshed systems.

These load flow formulations form the basis of many control and management applications for distribution systems. In order to achieve the objective of controlling distribution networks resources, optimisation is often employed alongside power flow. The fusion of these two methods is known as optimal power flow.

1.4.2 Optimal Power Flow

One of the most widely used types of algorithms for planning and operation of transmission systems is the optimal power flow (OPF) algorithm which uses optimisation methods to find solutions for power system variables. The earliest major contributions to the area came from Carpentier (1962) and Dommel and Tinney (1968). Both used non-linear programming (NLP) methods, with Carpentier (1962) solving the economic dispatch problem, and Dommel and Tinney (1968) minimising fuel costs and power losses with the inclusion of penalty functions. Since then, OPF methods have been applied to a number of problems faced by transmission networks, including economic dispatch (Barcelo et al., 1977; Momoh and Zhu, 2001), reactive power and/or voltage control (Yoshida et al., 2000; Cuffe et al., 2012), congestion management (Padhy, 2004), optimal allocation (Saravanan et al., 2005) and contingency analysis (Alsac and Stott, 1974). Although some transmission systems problems can be solved using a simplified DC OPF, which calculates only active power flows, most of the aforementioned works describe AC OPF approaches.

The unbalanced nature of distribution networks means that typical balanced OPF methods are not sufficient to represent, analyse, and control distribution networks. Furthermore, the DC simplification cannot be used at distribution level. The expansion of traditional OPF methods has led to new research into the area of three-phase or unbalanced OPF. The authors in Cao et al. (2013) have proposed a chance-constrained optimisation-based unbalanced OPF for use on radial distribution networks with DG present. Chance-constraints are used to
represent the feeder limits, and are implemented by ensuring that the probability that the limits are adhered to is above a predefined value. The objective function in this case is multi-objective and aims to minimise active power losses of lines, overloading of lines, and voltage violations. The power flow formulation is in sequence components with purely the positive sequence component being solved. The work in Bruno et al. (2011) presents a three-phase OPF (TOPF) for smart grids. The method is implemented using an optimisation program communicating with an unbalanced load flow formulation. The method was tested for five different cases, which examine load control to alleviate equipment overloads, and reactive power control to minimise losses and unbalance. Carpinelli et al. (2006) performs a voltage stability analysis, using three-phase power flow equations as equality constraints in an optimisation formulation. The method is implemented using a the sequential quadratic programming method, and the objective is to maximise the load at a particular bus. The authors in Paudyal and Dahal (2011) have utilised ABCD parameters to model series components in a distribution network. The method focuses largely on the modelling of integer variables without having to solve a mixed integer non-linear problem. The objective in this case is to minimise the energy drawn from the substation while limiting the number of switching operations performed by load tap changers (LTC) and capacitors. The authors of de Araujo et al. (2013) present a TOPF method which is solved using the primal-dual interior point method. The objective is to minimise voltage unbalances by controlling voltage regulators, capacitors and DG units. The method is tested using two of the IEEE test distribution feeders.

1.4.3 Management of Distributed Energy Resources

Controlling DER is not a trivial task. The various works discussed in Section 1.3 show that each form of DER introduces a different range of distribution issues, and therefore approaches for determining solutions can vary widely depending on the type of technology that is under examination. Developing solutions that are applicable to multiple forms of DER can be challenging as different forms of DER may have conflicting requirements. Furthermore, considering that many of
these resources will be customer owned, customer needs and preferences should also be taken into consideration.

Various authors have proposed methods for controlling charging of EVs from both a system point of view (Shortt and O’Malley, 2012; Mitra and Venayagamoorthy, 2010) and a distribution network point of view (Gong et al., 2012; Paudyal and Dahal, 2011; Richardson et al., 2012b). The work in Petrou et al. (2015) develops a centralised control algorithm using a P-controller which controls the connection status of EVs in order to avoid transformer and cable congestion. The results show that although charging times are extended, particularly in high EV penetration scenarios, asset congestion is prevented using a relatively straightforward controller algorithm. In Richardson et al. (2012a) the authors used linear programming to maximise the energy delivered to EVs, subject to network constraints. Network sensitivities to the addition of EV load were then used to optimise EV charging rates. Another method proposed by Clement-Nyns et al. (2010) used both quadratic and dynamic programming to minimise distribution network losses, based on when customers need their EV to be fully charged. In Deilami et al. (2011) a combination of tariffs and customer preferences were used to perform a maximum sensitivities selection optimisation, with the objective of minimising system losses, which would ultimately lower the cost of EV charging. The work in Pecas Lopes et al. (2009) compared dumb charging, charging using a dual tariff policy and smart charging. The objective was to maximise the integration level of EVs on a MV grid, by increasing the number of charging EVs in a stepwise manner until the grid became saturated. The authors in Acha et al. (2010) examine multiple case studies, which incorporate varying penetration levels of both plug-in hybrid EVs and CHP units as well as different charging schedules for the EVs. The method used is a time coordinated optimal power flow which aims to minimise both tap changing and energy losses subject to network and EV constraints. Optimal allocation of CHP is examined in Zhang et al. (2014). The authors investigate integrated electricity, gas, and water distribution networks with the objective of maximising the CHP output.
Numerous works detail methodologies for the management of distributed PV. The work in Farivar et al. (2012) aims to mitigate voltage fluctuations through the use of an OPF. The system under investigation has one large-scale PV system and its inverter’s reactive power output is optimised to minimise losses and energy consumption. In Navarro-Espinosa and Ochoa (2015) the potential for improving LV feeder hosting capacity for PV, using on-load tap changer (OLTC) transformers, is analysed. The authors use a probabilistic approach to account for the uncertainties associated with PV. The results indicate that PV hosting can reach up to 100% depending on the OLTC strategy employed. The authors in Weckx et al. (2014) present a combination of both centralised and local control to manage both the active and reactive power output of PV on a test network in Belgium. Simulations are performed for a Summer week. The addition of storage to PV systems can be advantageous for the purposes of storing the excess energy that is generated when demand is low, as well as mitigating feeder voltage rises. The research detailed in Teng et al. (2013) determines optimal charge/discharge schedules for PV battery systems, using a genetic algorithm, with the aim of minimising losses. In Liu et al. (2012), a real time digital simulator, with hardware in the loop, is used to model a test distribution network with PV and energy storage systems. The aim of the work is to coordinate the storage with the transformer tap changer to reduce losses and introduce peak shaving. The hardware in the loop results are validated against simulation results, and the objectives of reducing peak load and losses are achieved with the addition of storage.

In Pecas Lopes et al. (2006), control strategies for a microgrid are assessed. The microgrid consists of an LV network, fixed and flexible loads, fuel cells, microwind, PV, and storage. The results demonstrate the importance of storage in a microgrid environment. The work in Ziadi et al. (2014) also considers multiple forms of DER and proposes an optimal scheduling method. The elements examined in the work are PV, heat pumps, EVs, batteries, and the distribution transformer tap changer. The authors use particle swarm optimisation to determine a schedule which will minimise distribution losses.
1.4.4 Voltage Optimisation

Voltage is a variable that can have a significant effect on the overall quality and efficiency of a distribution system. DSOs usually enforce voltage limits on their networks for the purposes of ensuring safe and reliable operation for customers and their appliances. Limits tend to be within a certain tolerance range of the nominal voltage. For example, in Ireland, the voltage tolerance is \( \pm 10\% \) of the nominal value of 230 V (ESB Networks, 2007), which gives minimum and maximum allowable voltages of 207 V and 253 V respectively.

Typical approaches for maintaining these limits include the use of tap changers at distribution transformers, downstream voltage regulators, and capacitor banks. These measures allow a degree of control over the voltage and are primarily used to uphold voltage limits.

Voltage not only affects the supply that customers receive but can also have a significant effect on feeder load and loss levels. Most forms of load are somewhat voltage dependent, meaning that varying the supply voltage will change the power that the load is drawing. Certain loads, such as incandescent lighting, cooking and heating loads, act in a purely resistive manner, which means that they are squarely proportional to voltage. The study of such loads and their voltage dependency brought about the concept of conservation voltage reduction (CVR). CVR is the principle that reducing voltage levels to some value below nominal can reduce overall energy consumption, as well as reduce demand at peak times. This was the first form of voltage optimisation that emerged at distribution level. The idea of CVR is well established and dates back to the late seventies and eighties when a lot of research was carried out in the US (Preiss and Warnock, 1978; Lauria, 1987; Kirshner, 1990). Although energy reductions were noted, widespread adoption of CVR technology did not occur due to uncertainties surrounding the conditions under which CVR could be beneficial. For example, the authors in Kennedy and Fletcher (1991) conclude that it is extremely difficult to determine the cause of the energy reductions observed in their trial without detailed end-use data. They also note that the savings seen vary significantly
from circuit to circuit. Another concern raised by CVR studies was how to ensure that voltages would be kept within acceptable limits on long radial feeders. The findings in Krupa and Asgeirsson (1987) state that during their trial, voltages below the ANSI threshold of 114 V were recorded, and numerous complaints were received.

Recently, however, CVR has gained renewed interest as a possible stepping stone in the bid to improve energy efficiency, by potentially reducing overall customer demand as well as distribution system losses (Wilson, 2010; Triplett and Kufel, 2012; Sunderman, 2012). The work in Lefebvre et al. (2008) details a CVR pilot project that took place in Canada. The project was operated on a day on day off schedule, where the on day had the voltage reduced and the off day had the voltage at its nominal value. Average energy savings of 0.4% for each 1% voltage decrease are noted. In Lamberti et al. (2013) a trial is not discussed, the work instead details a simulated assessment of the load response to voltage changes. An actual distribution feeder is modelled and time varying ZIP components of load are applied with various voltages at the head of the feeder. The authors in Diskin et al. (2012) ran a trial on a number of urban and rural feeders. The preliminary findings give an average active energy saving of 0.8% for a 1% decrease in voltage, with the urban feeder giving the highest reduction. Reactive power reductions are even higher with an average saving of 6.3%. The results show that a significant saving in load can be achieved with voltage reduction, although the extent of the reductions depend on the time of day. The same trial was the focus of time spent working for the Irish DSO during the course of this thesis, a summary of which is given in Appendix A.

As described in Section 1.3, the introduction of certain forms of DER could have significant impacts on voltage. Some of these resources, however, have the capability to control voltage. Asynchronous generation devices, such as PV and wind, use power electronics to connect to the power system. Oftentimes, the power electronic devices are capable of injecting or absorbing reactive power, which can be employed to perform voltage optimisation in a similar way to capacitor banks or FACTS devices. Voltage optimisation using PV inverters has
been discussed in the previous subsection in the work of Farivar et al. (2012) and Weckx et al. (2014). The authors in Di Fazio et al. (2013) developed a decentralised formulation for DG reactive power control that aims to improve the voltage profile of the distribution feeder. This objective is achieved by each DG finding the optimal voltage set-point for their bus, and injecting/absorbing the appropriate amount of reactive power. The research in Pham et al. (2015) examines probabilistic approaches for both centralised and local control of wind generator reactive power for the purposes of voltage control. The centralised control is shown to perform better than the local in terms of the minimum loss objective, however voltage is pushed closer to its limits which could result in increased tap operations.

1.4.5 Uncertainty

The level of uncertainty that exists in power systems is substantial. From demand and generation forecasts, to equipment outages and electricity prices, the planning and operation of the power system is made considerably more challenging by these uncertain parameters. Renewable resources are only exacerbating the issue, with the availability of most of these resources being weather dependent. At distribution level, uncertainty becomes even more significant as individual customer behaviour is quite difficult to predict. Various methods exist for dealing with the uncertainties in distribution networks (Soroudi and Amraee, 2013). The authors in Kelly et al. (2009) assess the impacts of EV charging on distribution networks in British Columbia, using Monte-Carlo simulations. The work shows that transformer overloading is likely to occur on feeders that have a large underlying demand. In Das (2006) an interval arithmetic based unbalanced power flow is developed to manage uncertainties in both load and feeder parameters. The formulation is used to assess the minimum and maximum possible values for voltage and power, for a number of feeders. The work in Caramia et al. (2007) presents a probabilistic unbalanced load flow using a Monte-Carlo approach. The method is used on a distribution system that has wind turbines connected, taking the wind speed, and active and reactive load as uncertainties. Other
works have made further progress with the incorporation of optimisation. In Viswanadha Raju and Bijwe (2008) a fuzzy unbalanced power flow with a genetic algorithm is used to control the reactive power output of capacitors as well as tap changer settings with the real and reactive power load being considered as uncertain. The objectives examined are minimisation of real power loss and real power demand. The work in Soroudi and Ehsan (2013) produces an energy procurement strategy for a distribution network which has distributed generation present, while also taking account of network constraints. The information gap decision theory (IGDT) method is used to examine uncertainties in both the electricity pool price and the demand level.

1.5 Thesis Contributions and Overview

It is clear that the integration of DER will present some major challenges for future distribution systems. Accordingly, the key objective of this thesis is to develop techniques for the management of distribution networks and DER. Previous works have made significant strides in the area, however, oftentimes, crucial elements are not considered. This thesis proposes solutions that: can be applied to various forms of DER and distribution issues, employ three-phase modelling of distribution systems, use optimisation to provide the most beneficial results, and incorporate methods capable of dealing with uncertainty. Considering the unbalanced nature of distribution systems, and the desire to model these systems as accurately as possible, three-phase unbalanced power flow is at the heart of the thesis and forms the basis for the proposed solutions. Optimisation provides another core element and is employed heavily throughout the research. In order to account for the significant uncertainty associated with distribution networks and DER, robust formulations are also developed. Both planning and operational aspects of distribution systems are analysed using the developed solutions. DER technologies that are likely to cause the most severe impacts, such as EVs and PV, are also examined. The combination of these attributes provides substantial and novel contributions to the research area.
Chapter 1. Introduction

The solutions also provide a significant contribution to industry. These solutions can be incorporated by DSOs, or other entities that may manage distribution networks in the future, in order to assess the impacts of certain technologies, determine optimal management schemes for these technologies, as well as plan and operate their existing networks in more efficient ways. It is worth noting, however, that the implementation of these solutions, particularly in an operational capacity, could present some challenges, particularly in the area of monitoring and communication. At present, no monitoring or communication infrastructure exists at LV level, therefore establishing that infrastructure would be a significant undertaking. However, if implemented, the solutions described in this thesis could provide long term management and control to an evolving distribution system.

Chapter 2 presents a multi-period, unbalanced load flow and rolling optimisation method. The aim of the formulation is to control the rate and times at which EVs charge over a 24-hour time horizon, with a minimum cost objective, subject to certain constraints. The reasoning behind the minimum cost objective is to incentivise customers to allow their EV charging to be controlled. The method would also be applicable to other forms of DER. The Jacobian matrix is extracted from the load flow and inverted to give a network sensitivity matrix. The sensitivity matrix is subsequently employed in generating network constraints for the optimisation. Inputs are updated in real time, and a load flow and optimisation is performed for the new time window every 30-min so that deviations from the initial forecast are accounted for.

Expanding on the load flow and optimisation concept in Chapter 2, the work described in Chapter 3 introduces a multi-period, three-phase, unbalanced optimal power flow (TOPF) method. The method has the capability to provide optimal solutions for distribution system control variables, for a chosen objective function, subject to required constraints. The method is validated with the Institute of Electrical and Electronic Engineers (IEEE) 123-node test feeder, and a practical test feeder with the addition of PV systems is utilised to demonstrate the multi-period TOPF capabilities. The results show the benefits of adding
storage to PV systems when combined with the TOPF formulation described here.

The work in Chapter 4 employs the TOPF from Chapter 3 to find optimal volt-var curves for grid-connected rooftop PV inverters. A range of scenarios are analysed using the TOPF, and the resulting optimal reactive power settings are utilised to determine the optimal volt-var curve for each PV system on a test feeder. The aim is to minimise voltage deviations from a predefined optimal set-point. The resulting curves are then implemented with a test scenario for the purposes of validation.

In order to incorporate uncertainty into the TOPF from Chapter 3, Chapter 5 presents an IGDT based TOPF method. Assuming that the demand is uncertain, the aim is to provide optimal and robust tap settings and feeder switch decisions over a 24-hour period, while ensuring that the network is operated safely, and that losses are kept within an acceptable range. The formulation is tested on a section of a realistic low voltage distribution network with switches and tap changers present.

Thesis conclusions are discussed in Chapter 6. This chapter also includes a description of potential future work.
2.1 Introduction

EVs are a technology that has been gaining increasing interest in recent times. Many governments have set ambitious targets for penetration levels of EVs in an effort to reduce greenhouse gas emissions (The European Parliament and the Council of the European Union, 2009). For example, in Ireland there is a targeted 10% EV penetration level by 2020 for passenger vehicles Department of Communications, Energy and Natural Resources (2009), which will be the equivalent of approximately 230,000 vehicles Department of Transport (2009). The charging of these vehicles could result in a significant increase in system demand at certain times, which may in turn cause a major strain on power systems and in particular distribution networks. Furthermore, when a typical working day is considered, it may be the case that EV charging coincides with the existing daily demand peak, resulting in the need for expensive peaking plant to come online. In order to cope with this additional load there would be a need to invest in major network reinforcement, encourage EV users to charge at off
peak times or to devise controlled charging schemes, or some combination of the above.

Methods for controlling EVs in a distribution environment have been developed in Petrou et al. (2015); Richardson et al. (2012a); Clement-Nyns et al. (2010); Deilami et al. (2011); Acha et al. (2010). Some methods assume a perfect forecast of both customer activity and system conditions, which is unrealistic. Human behaviour can be predicted quite well on an aggregate scale, e.g. total system demand, however, on an individual basis, it is much more difficult to achieve. The challenges associated with predicting individual customer needs make the task of controlling EV load more complex, but one that needs to be addressed nonetheless. Probabilistic methods or robust optimisation techniques can be employed where uncertainties are a concern, as in Kelly et al. (2009); Li and Zhang (2012); Papadopoulos et al. (2012); Hajimiragha et al. (2011). However, as the aim of this work is to provide a method which calculates the optimal daily charging schedule for a particular network, a deterministic approach is taken here.

The method presented in this chapter recognises both system and customer uncertainties by implementing a multi-period, rolling optimisation technique. Three-phase, unbalanced load flows are performed to generate network sensitivity matrices, which are subsequently used in the definition of network constraints in the multi-period optimisation. The objective of the optimisation, for the purposes of the work presented here, is to minimise the total cost of charging EVs on a test network. It should be noted, however, that the method allows for any desired objective to be implemented. An initial load flow and optimisation are performed for a 24-hour time horizon, followed by a 12-hour load flow and optimisation every 30 minutes, based on updated input forecasts, as available, such as residential base load and customer EV availability. This rolling method reduces the cost of EV charging for customers and DSOs alike, while ensuring that both safe operation of the network and customer satisfaction, in the form of fully charged vehicles, are maintained. Coordination of the unbalanced load flow, the multi-period minimum
cost optimisation and the rolling method provide a unique solution to the problem of controlling EV charging.

Section 2.2 outlines the load flow, the optimisation, and the rolling optimisation technique, Section 2.3 describes the test network and simulation case, Section 2.4 presents the results and discussion, and Section 2.5 outlines the main conclusions.

2.2 Methodology

Implementation of a load flow method allows all of the intermediary process steps to be seen and potentially employed elsewhere: in the definition of an optimisation problem, for example. For the method described in the following subsection, a certain level of knowledge of the specific network is necessary, i.e. network topology, line impedances/admittances, nominal voltages, and loads.

2.2.1 Unbalanced Load Flow

At each iteration of the three-phase unbalanced load flow method proposed by the authors in Penido et al. (2008), a Jacobian matrix is generated using the nodal admittance matrix of the system and updated inputs from generators and loads. The Jacobian, along with calculated current mismatches, is then used to compute the voltage mismatches. A simplified version of the mismatch equation is presented in (2.1), where $\Delta I$ is the current mismatch matrix, $\Delta V$ is the voltage mismatch matrix and $J$ is the Jacobian matrix. Although equation (2.1) is linear, the Jacobians are not constant, and the non-linear equations are solved iteratively.

$$\Delta I = J \times \Delta V \quad (2.1)$$

Inversion of the Jacobian results in a matrix which relates changes in real and imaginary current at each bus to changes in real and imaginary voltage at all buses including itself. The inverted Jacobian is essentially a network sensitivity matrix and provides the ability to predict variations in voltage for specific changes.
in current. Equations (2.2) and (2.3) show the structure of the inverse Jacobian matrix, and a more in depth view of one of the blocks of the matrix respectively.

\[
J^{-1} = \begin{bmatrix}
\sigma_{st}^{11} & \zeta_{st}^{11} & \sigma_{st}^{12} & \zeta_{st}^{12} & \cdots & \sigma_{st}^{1n_b} & \zeta_{st}^{1n_b} \\
\mu_{st}^{11} & \gamma_{st}^{11} & \mu_{st}^{12} & \gamma_{st}^{12} & \cdots & \mu_{st}^{1n_b} & \gamma_{st}^{1n_b} \\
\sigma_{st}^{21} & \zeta_{st}^{21} & \sigma_{st}^{22} & \zeta_{st}^{22} & \cdots & \sigma_{st}^{2n_b} & \zeta_{st}^{2n_b} \\
\mu_{st}^{21} & \gamma_{st}^{21} & \mu_{st}^{22} & \gamma_{st}^{22} & \cdots & \mu_{st}^{2n_b} & \gamma_{st}^{2n_b} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\sigma_{st}^{n_b1} & \zeta_{st}^{n_b1} & \sigma_{st}^{n_b2} & \zeta_{st}^{n_b2} & \cdots & \sigma_{st}^{n_bn_b} & \zeta_{st}^{n_bn_b} \\
\mu_{st}^{n_b1} & \gamma_{st}^{n_b1} & \mu_{st}^{n_b2} & \gamma_{st}^{n_b2} & \cdots & \mu_{st}^{n_bn_b} & \gamma_{st}^{n_bn_b}
\end{bmatrix}
\]  

(2.2)

\[
(J^{-1})_{11} = \begin{bmatrix}
\sigma_{11}^{aa} & \sigma_{11}^{ab} & \sigma_{11}^{ac} & \sigma_{11}^{an} & \zeta_{11}^{aa} & \zeta_{11}^{ab} & \zeta_{11}^{ac} & \zeta_{11}^{an} \\
\sigma_{11}^{ba} & \sigma_{11}^{bb} & \sigma_{11}^{bc} & \sigma_{11}^{bn} & \zeta_{11}^{ba} & \zeta_{11}^{bb} & \zeta_{11}^{bc} & \zeta_{11}^{bn} \\
\sigma_{11}^{ca} & \sigma_{11}^{cb} & \sigma_{11}^{cc} & \sigma_{11}^{cn} & \zeta_{11}^{ca} & \zeta_{11}^{cb} & \zeta_{11}^{cc} & \zeta_{11}^{cn} \\
\sigma_{11}^{na} & \sigma_{11}^{nb} & \sigma_{11}^{nc} & \sigma_{11}^{nn} & \zeta_{11}^{na} & \zeta_{11}^{nb} & \zeta_{11}^{nc} & \zeta_{11}^{nn} \\
\mu_{11}^{aa} & \mu_{11}^{ab} & \mu_{11}^{ac} & \mu_{11}^{an} & \gamma_{11}^{aa} & \gamma_{11}^{ab} & \gamma_{11}^{ac} & \gamma_{11}^{an} \\
\mu_{11}^{ba} & \mu_{11}^{bb} & \mu_{11}^{bc} & \mu_{11}^{bn} & \gamma_{11}^{ba} & \gamma_{11}^{bb} & \gamma_{11}^{bc} & \gamma_{11}^{bn} \\
\mu_{11}^{ca} & \mu_{11}^{cb} & \mu_{11}^{cc} & \mu_{11}^{cn} & \gamma_{11}^{ca} & \gamma_{11}^{cb} & \gamma_{11}^{cc} & \gamma_{11}^{cn} \\
\mu_{11}^{na} & \mu_{11}^{nb} & \mu_{11}^{nc} & \mu_{11}^{nn} & \gamma_{11}^{na} & \gamma_{11}^{nb} & \gamma_{11}^{nc} & \gamma_{11}^{nn}
\end{bmatrix}
\]  

(2.3)

2.2.2 Multi-Period Optimisation

The multi-period optimisation is implemented as a centralised scheme to control the charge rate of EVs on an individual feeder, or on a group of feeders.

Objective Function

The optimisation allows for minimisation (or maximisation) of any objective function \( f \). In this case the aim is to minimise the cost of charging EVs over all phases, buses, and time steps, as seen in (2.4), subject to later defined network constraints. \( P_{EV}^{d_h} \) is the EV active power at bus \( k \) phase \( d \) for time step \( h \), and \( C_{h} \) is the cost of electricity at time step \( h \). The choice of a minimum cost objective takes advantage of the multi-period capability of the optimisation, which is expressed in the summation over all time steps \( h \) in (2.4). The minimum cost objective can be viewed as a customer focused objective, as it encourages
EV users to allow the charging of their EV to be centrally controlled since their charging costs would be reduced, assuming some time of use pricing is in place. It could also be seen as advantageous from a system point of view, as a low electricity price usually correlates closely with low system demand, meaning that EV charging may be allocated to those periods of lower demand. It is assumed that the addition of EV load has been factored into the electricity price.

\[
f = \min \sum_{h \in \Omega_h} \sum_{k \in \Omega_{EV}} \sum_{d \in \Omega_d} (P_{dEV_k})_h \times C_h
\]

Constraints

Equation (2.5) shows the voltage constraint created using the network sensitivity matrix introduced in Section 2.2.1, where the voltage is limited between \(V_{min}\) and \(V_{max}\). \(V_k^{d_{init}}\) represents the initial real and imaginary voltage computed by the load flow for the base case.

\[
V_{min} \leq \left| V_k^{d_{init}} - \sum_{i \in \Omega_k} \sum_{t \in \Omega_d} \left( \left( \sigma_{di}^{kt} + j \mu_{di}^{kt} \right) I_{lm_i}^t + \left( \zeta_{di}^{kt} + j \gamma_{di}^{kt} \right) I_{Re_i}^t \right) \right| \leq V_{max}
\]

EV charging rates are considered to be continuous and variable, and are limited between inclusive lower and upper bounds, \(P_{EV_{min}}\) and \(P_{EV_{max}}\) respectively.

\[
P_{EV_{min}} \leq P_{dEV_k} \leq P_{EV_{max}}
\]

Each battery has a maximum energy capacity \(E_{EV_{k_{max}}}^{d}\) and aims to be fully charged by the end of the charging period \(\Omega_{h_{EV_k}} \subseteq \Omega_h\), while recognising charging efficiency. \(\Omega_{h_{EV_k}}\) consists of those time steps for which the EV at bus \(k\) is available to be charged, while \(E_{EV_{k_{init}}}^{d}\) refers to the initial battery state of charge of the EV at bus \(k\).

\[
\sum_{h \in \Omega_h \cap \Omega_{h_{EV_k}}} (P_{dEV_k})_h = E_{EV_{max}}^{d} - E_{EV_{k_{init}}}^{d}
\]
The total apparent power flowing through each phase of the transformer cannot exceed the given apparent power rating of $S_{d_{Ki}}^{\text{rated}}$.

$$\left| S_{d_{ki}}^d \right| \leq S_{d_{ki}}^{\text{rated}} \quad k, i \in \Omega_{kr}$$  \hspace{1cm} (2.8)

The current flowing between bus $k$ phase $d$ and bus $i$ phase $d$, i.e. the line or cable current, $I_{d_{ki}}^d$, should be less than or equal to the rated current of that specific line, $I_{d_{ki}}^{\text{rated}}$. $I_{d_{ki}}^d$ is calculated using the cable impedance matrix and the predicted voltages obtained from the voltage constraint.

$$\left| I_{d_{ki}}^d \right| \leq I_{d_{ki}}^{\text{rated}} \quad i \in \Omega_{ki}$$  \hspace{1cm} (2.9)

All service cables (the single phase cables connecting each customer household to the three phase buses) have a maximum import capacity, as defined by the DSO. Accordingly, each customer’s total load must be less than this maximum capacity rating, $S_{SC}^{\text{rated}}$, as shown in (2.10) below.

$$\left| S_k^d + P_{EV_k}^d \right| \leq S_{SC}^{\text{rated}}$$  \hspace{1cm} (2.10)

The optimisation is implemented in MATLAB (MATLAB, 2015), using the NLP function $\text{fmincon}$. Alternative programming tools could also be employed for implementation of the method. The sequential quadratic programming (SQP) algorithm, which is an iterative quadratic programming method, is used to solve the non-linear optimisation problem. The non-linearity of the problem is due to the use of current as the manipulated variable, i.e. the inverse Jacobian relates voltage to current, but power is used in the definition of both the constraints and the objective function. Equations (2.11) and (2.12) convert the real and imaginary current to active and reactive power terms.

$$P_k^d = I_{Re_k}^d (V_{Re_k}^d - V_{Re_k}^n) + I_{Im_k}^d (V_{Im_k}^d - V_{Im_k}^n)$$  \hspace{1cm} (2.11)
### 2.2.3 Rolling Optimisation

Earlier work utilises the load flow and optimisation formulation described in subsections 2.2.1 and 2.2.2 to control EV charging over a 24-hour time period, for a number of objectives, without considering uncertainty. This work is detailed in O'Connell et al. (2012). The results demonstrate the negative impact that uncontrolled charging of EVs can have on a distribution network and how implementation of a control scheme can mitigate some of these issues. However, using the load flow and optimisation method alone means that a perfect forecast of both customer activity and system conditions is assumed, which is unrealistic.

In order to account for the uncertainties associated with customer behaviour, a rolling optimisation which uses updated forecasts has been developed, which could be implemented either by the DSO themselves, or by a third party aggregator. An aggregator is an entity that manages and controls groups of distributed resources, and communicates with other entities such as market operators on their behalf. At present, it is unknown which entity would be responsible for managing distribution networks and their resources, however the solutions presented are applicable regardless of who is implementing the control.

An initial load flow and optimisation are performed for a 24-hour time period, in time steps of 30 minutes. Following the initial load flow and optimisation, and at each 30 minute time step thereafter, a load flow and optimisation are performed for the subsequent 12-hour window, using real-time measured data and updated input forecasts for the customer base load, electricity price, EV availability, and EV battery state of charge (BSOC) incorporated, as available. Fig. 2.1 gives a representation of the rolling process, where the real-time interval given, hh:mm-hh:mm, refers to the interval during which each load flow and optimisation would be performed by the aggregator. Only the first 30 minute interval of a 12-hour charging schedule is ever implemented, and the inputs for this interval are the real-time measurements of the system at the time the load flow and optimisation
are performed. The forecasts may be based on historical data for that particular network and data provided by the system operator, and could be generated by the aggregator themselves or by an external forecaster. It is assumed in this work that new forecasts are available every 30 minutes, or every hour for electricity price. However, in reality, if a new forecast was not available at a particular time step, the previous forecast would be used.

![Diagram of rolling optimisation process](image)

**Figure 2.1:** Representation of rolling optimisation process

The choice of a 24-hour period stems from the natural daily cycles of customer base demand, EV availability and energy requirements, and electricity price. The charging profiles obtained from the 24-hour load flow and optimisation are utilised to determine the customers’ energy requirements for each 12-hour load flow and optimisation, and are updated as new information becomes available. If this is not performed, the earlier 12-hour optimisations may allocate EV load as soon as an EV becomes, or is forecast to become, available.

The time window for the rolling optimisations is a parameter that is easily varied, as is the 30-minute time resolution. A 12-hour window was chosen as the most suitable optimisation horizon to ensure that EV charging would be allocated to the daily minimum cost times, and not just the minimum of that particular window. A shorter horizon would not encapsulate low cost times that may occur later in the day, while a longer horizon is unnecessary and may have a longer
computation time. A 30-minute resolution was chosen as a reasonable time frame to allow for the calculation of new solutions. It is also assumed that any changes to the network demand that may occur during the first 30 minute interval of a rolling 12-hour window are not sufficiently large to adversely affect the network, and therefore, performing more frequent optimisations is unnecessary.

In order to ensure the accuracy of the Jacobian matrix, and consequently the sensitivity matrix, an assumed operating point of the network with the addition of EV load is made, and the load flow is performed with this assumed EV load included. Each load flow generates a new Jacobian matrix, so following the performance of a 12-hour (24 interval) load flow, 24 sensitivity matrices are passed to the optimisation. If the energy requirement for a particular 12-hour window cannot be met, the energy that was not delivered is added to the required energy for the subsequent window. Continuous rolling operation would require a new 24-hour load flow and optimisation to be performed every 12 hours. The flowchart in Fig. 2.2 gives an overview of the rolling optimisation process.

![Flow chart of rolling optimisation process](image-url)

**Figure 2.2:** Flow chart of rolling optimisation process
2.3 Test Case

2.3.1 Test Network

The test network consists of a LV local feeder, where a LV substation serves a total of 85 buses: 11 three-phase buses (MV-a9), and 74 single-phase customer buses. This network represents a typical suburban feeder in Ireland, and has been used as a trial network by the DSO, Electricity Supply Board Networks (ESBN), for an electric vehicle demonstration project (Richardson et al., 2013). The lack of interdependence between residential distribution feeders, i.e. very little meshing or paralleling, means that utilising this method on multiple feeders at once would be unnecessary, thus the chosen test network is deemed to be adequately sized for the purposes of testing the method. A detailed diagram of the network can be seen in Fig. 2.3. Each service cable, customer load, and EV load is modelled individually on its corresponding phase, which is given by an arrow in Fig. 2.3. A common neutral is also modelled, however it is not shown here. MV 38:10 kV tap changing transformers are controlled to ensure that the voltage at their 10 kV side is regulated to a nominal value. A voltage drop is incurred between the 10 kV bus and the LV feeder substation bus which is accounted for by assuming that the line voltage at the high side of the transformer substation is at a value of 9.7 kV. Although this voltage may fluctuate, for simplicity, it is assumed to be fixed in this work.

In Ireland, the voltage tolerance is ±10% of the nominal value of 230 V (ESB Networks, 2007), which gives minimum and maximum allowable voltages of 207 V and 253 V respectively. However, to allow for the predictive nature of the voltage constraint in (2.5), and to provide a buffer for the assumption that changes in network demand within a 30-minute period do not noticeably affect the network, the minimum and maximum voltages are defined in the optimisation as 210 V and 250 V respectively. These limits were chosen as conservative values after studying numerous scenarios, and differences between the predicted voltages from the optimisation and the actual voltages calculated.
The transformer supplying the network is rated at 400 kVA and does not have tap changing capabilities, while the mains cable has a maximum current limit of 424 A. The maximum import capacity for average domestic households, as specified by the DSO, is 12 kVA. The above apparent power ratings and the current rating are used as the constraining values in the optimisation, as seen in (2.8-2.10).

Domestic EV charging is assumed to be single-phase only. Vehicle to grid capabilities are possible but are not considered in this work, therefore, the charging rate of the EVs is always positive and can vary continuously between 0 kW and 4 kW. Although trickle charging can occur towards the end of battery charge cycles, it is not modelled in this work. It is also assumed that EV charging rates are constant within a 30-minute interval, but can vary from interval to interval. Batteries are assumed to have a 20 kWh capacity and the EV load is modelled as a constant power load (Sortomme et al., 2012).

2.3.2 Simulation Case

The method, as described in Section 2.2, was tested for a 24-hour time period, in 30-minute time steps, from 07:00 to 07:00 the following day, so that an entire charging period could be observed. The 24-hour period that was chosen, from
confidential demand data provided by the DSO, included the highest network demand time step of the year (114 kW), and thus represented a high load winter day.

Previous load flow studies on this network determined that a 50% EV penetration level would be sufficient to justify the implementation of a controlled charging scheme (Richardson et al., 2012b), as well as providing a robust test case for the method. A 50% penetration level amounts to 37 EVs on the network, or a maximum additional load of 148 kW. So that the impact of unbalanced load could be observed, a higher number of EVs was allocated to phase a, as seen in Table 2.1.

<table>
<thead>
<tr>
<th>3-Phase Node</th>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>a2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>a3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>a4</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>a5</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>a6</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>a7</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>a8</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>a9</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>16</strong></td>
<td><strong>10</strong></td>
<td><strong>11</strong></td>
</tr>
</tbody>
</table>

**Table 2.1: EV location and phase**

**Initial Inputs**

The total annual energy usage for each customer in the above network has been obtained from the DSO. Typical yearly profiles for low, medium, and high use customers have also been obtained, which were then scaled according to each individual customer's annual usage and time shifted to recognise load diversity. The coincidence factor of the customer profiles was calculated to ensure that they are realistic. The resulting value of 0.22 is reasonable when compared to values for similar networks (Willis, 2004). As the load mix for this feeder is unknown, customer base loads are modelled as 50% constant power and 50% constant impedance based on typical residential feeder load mixes (Willis, 2004).
Fig. 2.4 shows the aggregate initial values for base load apparent power for the feeder over the test day.

![Figure 2.4: Aggregate initial base load apparent power](image)

Each EV has its own randomly assigned availability period and initial BSOC. Based upon data obtained from an EV trial carried out by the DSO, a mean plug-in time of 18:00 and plug-out time of 07:00 was used to determine each EV's availability, and a mean initial BSOC of 8 kWh was applied to specify each EV's initial BSOC (Richardson et al., 2013). It is assumed that each EV only plugs in once.

The electricity price is the system marginal price (SMP) for a winter day, acquired from the Irish Single Electricity Market Operator (SEMO) at an hourly time resolution. As all other inputs are in 30-minute intervals, the electricity price for each hour is repeated for the following half-hourly time step. The inputs described here represent the initial forecasts used for the 24-hour load flow and optimisation.

**Updated Inputs**

A forecasting methodology is not included as part of this work. Instead, all of the initial inputs described previously are updated using random numbers, to model the updated forecasts an aggregator could see at each new 30-minute time step. To demonstrate the effectiveness of the method, an intra-day electricity market
has been assumed. Although at the moment in Ireland intra-day trading does not exist, proposals have been made by SEMO to facilitate intra-day trading (SEMO, 2015).

For each new 12-hour (24 intervals) time window, each customer’s previous base load forecast and the previous electricity price forecast are updated by adding or subtracting random percentage values for each 30-minute interval, the bounds of which are given in Table 2.2. The initial aggregate test network demand forecast, and final realised aggregate demand values for the test day were used to calculate a mean absolute percentage error (MAPE) of 4.77% and root mean squared error (RMSE) of 6.13%. The error bounds were chosen for the purposes of demonstrating the method and can be easily changed. The literature indicates that these values are reasonable (Sousa et al., 2009; Wu, 2007; Yasuoka et al., 2001). The MAPE and RMSE were also calculated for the electricity price for the test day, as 7.85% and 12.07% respectively, which again appear to be sensible values (Chen et al., 2009; Zareipour et al., 2010). Some of the values calculated for the simulation case are slightly larger than those seen in the literature, however this demonstrates the effectiveness of the method.

<table>
<thead>
<tr>
<th>30 Minute Interval</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Base Load (%)</td>
<td>±10</td>
<td>±12</td>
<td>±14</td>
<td>±16</td>
<td>±18</td>
<td>±20</td>
<td>...</td>
</tr>
<tr>
<td>Electricity Price (%)</td>
<td>±1</td>
<td>±2</td>
<td>±3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each customer’s availability is updated by generating two random numbers to add or subtract from the initial plug-in and plug-out times, generating a new availability period for that customer. The new plug-in time becomes available to the aggregator at the time step after which the customer plugs in. It is assumed that when a customer plugs in, they also input the time by which they desire their battery to be full, although all EVs should be fully charged by 07:00 for the purposes of this work.

Each customer’s initial BSOC is updated by adding a random percentage between ±50% of the initial BSOC forecast to the initial BSOC forecast, as
supported by EV trial data. The updated initial BSOC also becomes known at the time step after which the customer plugs in.

2.4 Results and Discussion

The following subsections present results of the method for the minimum cost simulation case described in the previous section, for both a rolling and non-rolling (no updates) control approach, as well as an uncontrolled scenario whereby all EVs commence charging upon plugging in. Results are presented from both an aggregate point of view, as well as an individual customer perspective. The individual customer’s results are for the same customer in all of the following cases. This particular customer is located at the end of the feeder at bus \( a9 \) phase \( a \), in Fig. 2.3, and therefore their bus is one of the most sensitive on this network and is more likely to experience charging restrictions.

In all figures depicting voltage, the black solid line represents the minimum allowable voltage, as defined in Section 2.3 by the DSO, and the black dashed line represents the minimum allowable voltage defined in the optimisation. The voltages in the figures are the results of a validation load flow, as described in O’Connell et al. (2012).

2.4.1 Uncontrolled vs Controlled Charging

In order to demonstrate the need for controlling EV charging, the uncontrolled case is compared with the controlled charging scenario, which aims to minimise the total cost of EV charging across the 24-hour period. The details of the minimum cost objective, including the cost function, will be described in detail in the following subsection. The resulting aggregate EV charging profiles are presented in Fig. 2.5. The charging peak for the uncontrolled case occurs at 18:00, a time at which the network already has a high underlying base demand (87 kWh), while the peak for the controlled charging is at 02:00, the least expensive time of the test day, when the base load is relatively low (20 kWh). The corresponding electricity prices are presented in Fig. 2.10. The controlled charging seen at 22:00
is due to an EV that requested to have a full battery by 03:30. The remainder of this customer’s charging was scheduled between 01:00 and 03:30.

Figure 2.5: Aggregate EV charging profiles for controlled and uncontrolled cases

The impact of the charging can be observed in Fig. 2.6, which presents the corresponding CPOC voltage for the individual customer connected to bus a9 (via a service cable), located towards the end of the feeder, and Fig. 2.7, which, for each time step, shows the thermal loading of the service cable that has the maximum apparent power loading. Both figures compare the controlled and uncontrolled scenarios. It is evident that the addition of EV load at peak base load times results in unacceptable voltage levels for this customer, and others, with values below the allowable limit of 207 V between 17:00 and 18:30, and reaching a minimum of almost 204 V. The controlled case, however, schedules the load such that this customer’s voltage, as well as all other customers, is maintained at, or above, 210 V. Fig. 2.6 demonstrates how controlled charging not only reduces the risk of unsafe network operation, but, by incorporating the network sensitivity matrix, also has the potential to increase the EV hosting capacity of the network.

The thermal loading values for the most heavily loaded service cable at each time step are shown in Fig. 2.7, while the thermal loadings for the transformer feeding the network and the mains cable connecting the substation to the feeder are given in Fig. 2.8. All values are below the upper limit for both the controlled and uncontrolled cases, which would indicate that thermal loading is not the
binding constraint for this particular network, and therefore the focus for the following results will be on the voltage levels.

Fig. 2.9 shows the voltages at the three-phase bus a9, located at the end of the feeder, for phases a, b, and c. The impact of the additional EVs allocated to phase a is evident, with the phase a voltage nearing the lower bound of 210 V, while phase b and phase c voltages have a minimum level of 214 V. This result also highlights the importance of modelling unbalanced networks, as a balanced representation would not have captured the lower voltages on phase a, only an
average voltage value, suggesting, incorrectly, that additional EV load could be introduced without breaching limits.

Figure 2.9: Individual phase voltages at bus a9 for controlled EV charging scenario

2.4.2 Minimum Cost Objective

Implementation of a multi-period optimisation allows for the use of a minimum cost objective. A multi-period approach also affords a broader choice of objective function. Time-varying quantities, such as the electricity price, can be readily
incorporated. The forecast and realised electricity price profiles for the test day are shown in Fig. 2.10. For the realised price, the low cost times are between 01:00 and 06:00 with 02:00-03:00 being the cheapest period. Fig. 2.5 reflects this pattern, with the controlled case allocating the majority of EV charging to these times.

Taking the electricity price in Fig. 2.10 into account, the expectation might be for all available customers to be charging at the maximum rate of 4 kW at 02:00. All 37 EVs are available at 02:00, so an aggregate EV load of 148 kWh would be expected, however Fig. 2.5 shows that the EV load is only 145 kWh, with two EVs at the end of the feeder experiencing a lower charging rate. The charging restriction is due to the optimisation allocating all available EVs to charge at this low cost time, resulting in a binding voltage constraint. The binding constraint is evident when looking at the individual customer’s corresponding voltage in Fig. 2.6, which is at, or close to, the 210 V limit throughout the low cost period. The network constraints lead to a charging scenario that does not yield the absolute lowest cost, but instead the lowest possible cost subject to the limitations of the network.

Figure 2.10: Initial forecast and realised electricity price for test day

A high penetration of EVs system-wide may lead to a smoother price profile, however the method will always schedule EVs to charge at lower cost hours, even if the prices are only marginally lower, as illustrated in Fig. 2.5. It should also be
noted that the objective function can easily be adjusted, so if there comes a time when the system operator no longer wants EV charging to be driven by price, a more appropriate objective can be incorporated.

### 2.4.3 Rolling Optimisation

Two aggregate EV charge profiles are displayed in Fig. 2.11. The dashed series represents the optimised EV charging obtained following the initial input forecasts, while the solid series depicts the realised EV charging, or the final result of the rolling optimisation for that day.

![Aggregate EV charging profiles for initial input forecasts and realised inputs](image)

**Figure 2.11:** Aggregate EV charging profiles for initial input forecasts and realised inputs

The initial electricity price forecast, given in Fig. 2.10, predicted that minimum prices would occur between 04:00 and 07:00, while the realised prices have minima between 01:00 and 06:00, as seen in Fig. 2.10. The result is an overall shift in those periods when most of the EV load is occurring.

Fig. 2.12 shows the aggregate initial forecast for EV availability and the aggregate realised EV availability. The reduction in the availability of EVs in later hours, paired with updated electricity prices, can be seen in Fig. 2.11, with reductions in the realised EV load from 04:00 onwards. Vehicles that had initially been predicted to be available at these times are in reality unavailable, and thus are scheduled to charge either earlier or later.
Fig. 2.13 shows results for the end-of-feeder customer. This customer’s initial base load forecast, realised base load, and the corresponding EV charging profiles are given. The resulting forecast and realised voltages are also displayed in Fig. 2.14. Although there are substantial deviations between the realised base load and the initial forecast, those times at which this EV is scheduled to charge are ones of relatively low underlying demand which, generally, do not experience a substantial variation from day to day. The significant change to this EV’s charging pattern, observed in Fig. 2.13, is not a consequence of base load changes, but predominantly the result of a reduced BSOC, as well as electricity price variations discussed in the previous subsection. Initially, it was predicted that this customer would have 11 kWh remaining in their EV battery upon returning home at 17:30. In reality they had only 4.6 kWh, which gives an additional energy requirement of 6.4 kWh. The rolling method has the capability to alter this EV’s charging profile to cope with the increased energy needs of the customer. The increase in EV load is reflected in the customer’s voltage in Fig. 2.14, with a more prolonged period of depression (01:00-04:30) than initially anticipated (04:00-06:00). However, the voltage is maintained at a safe operating level at all times.
Figure 2.13: Individual customer’s initial forecast base load and EV load, and realised base load and EV load

Figure 2.14: Individual customer’s initial forecast voltage and realised voltage

2.4.4 Results Summary

Table 2.3 compares two performance metrics for aggregate EV charging for the uncontrolled case, the controlled case without rolling optimisation, and the controlled case with rolling optimisation.

The cost metric is calculated by summing the product of the realised electricity price and the EV load for each time step, and dividing the result by the sum of EV load to give a normalised cost. The uncontrolled cost of €0.0822/kWh is almost double that of both the controlled cases, as the peak plug-in time for EVs coincides with the realised electricity price peak.
Table 2.3: Cost and energy comparison

<table>
<thead>
<tr>
<th></th>
<th>Normalised Cost (€/kWh)</th>
<th>% Required Energy Delivered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled(^1)</td>
<td>0.0822</td>
<td>100</td>
</tr>
<tr>
<td>Controlled no Rolling</td>
<td>0.0466</td>
<td>80.02</td>
</tr>
<tr>
<td>Controlled Rolling</td>
<td>0.0416</td>
<td>100</td>
</tr>
</tbody>
</table>

The energy metric is defined as the percentage of the realised energy required by EVs that was ultimately delivered. Controlled charging without rolling optimisation only delivers 80.02% of the required energy. The reason for the deficit is the assumed perfect forecasts for the EV availability and the BSOC, which leads to charge allocated to times that EVs are not actually available, and insufficient energy provided to those vehicles with increased requirements. Each customer inputs the time by which they desire their battery to be full, and therefore it is assumed that if a customer’s battery is fully charged by that time, then the customer is satisfied. The results show that the controlled rolling optimisation provides the least cost solution while delivering 100% of the energy required by customers, thus all customers are deemed to be satisfied. If the penetration level of EVs were to become significantly high, it may be the case that the total energy required by EVs could not be delivered, even with the controlled rolling method, however the method will deliver the maximum possible load, within the constraints of the particular network.

The computation times for the 12-hour load flow and optimisations, for the presented results, ranged between 48.7 seconds and 324.8 seconds on an Intel Xeon 3.1 GHz machine with 4 GB RAM. The computation time for the optimisation depends on the EV penetration level, the initial conditions provided and whether or not the constraints are binding.

### 2.5 Conclusions

The work in this chapter utilises a three-phase unbalanced load flow method which accurately models the unbalances that are inherent to distribution...
networks, ensuring that individual phases are not overloaded. It performs multi-period optimisations to minimise the cost of EV charging, which is an advantage to both customers and DSOs alike. Finally, incorporating the updated forecasts using the rolling optimisation manages the uncertainties associated with user behaviour. The combination of these attributes provides a novel approach to the problem of controlling EV charging. Examining the three phase voltages for an individual bus demonstrates the importance of the unbalanced load flow in maintaining acceptable operating levels. Balanced modelling of distribution networks does not give an accurate representation of the system state, and such modelling is likely to inadvertently push systems past their limits.

Controlled charging of EVs has been shown here to prevent issues associated with uncontrolled EV charging. Scheduling EV charging with respect to electricity price, for example, can lead to fewer network limitation breaches, as well as reducing the cost of charging for both the system operator and the customer, by scheduling EVs to charge at low demand hours. The use of electricity price in the objective also takes advantage of the multi-period capability of the method, allowing the optimal charging times to be identified across a specified time horizon.

The results presented indicate that fully delivering the energy needed by customers, in an economical manner, requires consideration of forecast errors. Growing penetration levels of variable energy resources, such as wind and solar, mean that electricity prices are not as predictable as they perhaps once were, and individual customer demand is difficult to predict, therefore taking account of such changes is essential. The reduction of the effect of forecast errors is accomplished through the incorporation of updated forecasts using a rolling method which not only provides a more optimal solution in terms of cost, but also ensures that, assuming correct plug-out times are provided, customers do not find themselves undercharged upon plugging out.

The method presented in this chapter can reassure DSOs that networks are being represented and controlled in such a way that the risk of voltage violation, or network overloading, is mitigated. DSOs would also have comfort in the
knowledge that an entire network would not have to be upgraded purely because an increased peak load is causing difficulties at a certain time of day. There are many potential applications for the method described in this work, such as control of additional distributed energy resources e.g. CHP units, heat pumps or micro-generation. The method could also play a valuable role in demand side management schemes, which aim to control other types of load such as water and space heating/cooling or refrigeration.

Although the load flow and optimisation formulation discussed in this chapter proves effective, it is somewhat limited in its capabilities, e.g. the sensitivity matrix provides an approximation of what voltage will be. A more streamlined and flexible approach for determining optimal solutions for three-phase systems is desirable. Such a solution is proposed in Chapter 3.
CHAPTER 3

Three-Phase Unbalanced Optimal Power Flow

3.1 Introduction

Traditionally, distribution networks were designed to be a passive part of the power system, utilised purely for power delivery to the customer, and a part of the power system that needed little to no control (Willis, 2004). As discussed in Chapter 1, the introduction of new DER to residential distribution networks has the potential to cause issues on the networks that may not have been anticipated when they were originally designed. Preventing the issues associated with DER has led to the proposal of control schemes, which have the capability to manage the different forms of DER in a way that can mitigate potential issues. Such control could also provide a degree of flexibility to system operators and enhance the utilisation of distribution networks.

Planning and operational tools for transmission systems are well established, with OPF formulations at the core of most of these tools (Momoh and Zhu, 2001; Yoshida et al., 2000; Padhy, 2004; Saravanan et al., 2005; Alsac and Stott, 1974). Unbalanced distribution network approaches to OPF have also been proposed (Cao et al., 2013; Bruno et al., 2011; Carpinelli et al., 2006; Paudyal
and Dahal, 2011; de Araujo et al., 2013). In the previous chapter, a load flow and optimisation methodology was discussed, which formed the basis for the work in this chapter. The load flow and optimisation method provides advantages in terms of ease implementation from a DSO perspective, as it can be executed in any programming software. However, the formulation presented here introduces further benefits in terms of accuracy, flexibility, and simplicity.

In this chapter, a multi-period three-phase unbalanced optimal power flow formulation is presented which is based on the three-phase load flow method developed in Penido et al. (2008), that has also been employed in Chapter 2. The TOPF formulation is capable of modelling both radial and meshed networks, and can model systems containing a mix of single-phase, two-phase and three-phase lines. There is also explicit representation of a neutral wire, which plays an integral part in unbalanced systems. The formulation is implemented as a single NLP. It does not require the use of any external load flow programs, and can be solved using various widely available non-linear solvers. The combination of these attributes results in a novel, efficient, and effective TOPF method.

The methodology is outlined in Section 3.2, a detailed description of the test cases is given in Section 3.3, Section 3.4 outlines the main results of the test cases, and conclusions are presented in Section 3.5.

3.2 Methodology

3.2.1 Unbalanced Power Flow

As in Chapter 2, the power flow method at the core of this work is the four conductor current injection method (FCIM) proposed by the authors in Penido et al. (2008). This Newton-Raphson based method performs a three-phase power flow, in rectangular coordinates, with additional representation of the neutral conductor. It can be used for balanced or unbalanced power flow, and on both radial and meshed feeders. At each iteration of the power flow method, the known currents injected at each bus by loads, generators, etc. are used to calculate the
current mismatches. The Jacobian matrix is formed using the elements of the nodal admittance matrix, and, combined with the current mismatches, is used to compute the voltage mismatch for each bus. A simplified version of the mismatch equation is presented in (3.1), where $\Delta I$ is the current mismatch matrix, $\Delta V$ is the voltage mismatch matrix and $J$ is the Jacobian matrix.

$$\Delta I = J \times \Delta V$$  (3.1)

### 3.2.2 Three-Phase Unbalanced Optimal Power Flow

The current mismatch equations given in Penido et al. (2008) are used as the basis for the TOPF formulation presented here, by applying a constraint which forces these mismatches to equal zero. This constraint, along with others that will be described in the following subsections, allows the optimisation problem to solve iteratively for voltages and other control variables.

#### Equality Constraints

**Current Mismatches:** The following equations (3.2)-(3.5) represent the calculation of the real and imaginary current mismatches at bus $k$ phase $d$ and bus $k$ neutral ($n$) phase. The two components of these equations are the current injection contribution from shunt elements, e.g. loads and generators, and the current injection contribution from series elements, e.g. lines and transformers.

As previously stated, the mismatch equations are constrained to equal zero as shown in (3.6).

$$\Delta I_{Re_k}^d = \frac{P_k^d(V_{Re_k}^d - V_{Re_n}^d) + Q_k^d(V_{Im_k}^d - V_{Im_n}^d)}{(V_{Re_k}^d - V_{Re_n}^d)^2 + (V_{Im_k}^d - V_{Im_n}^d)^2}$$

$$- \sum_{t \in \Omega_p} (G_{kk}^{dt}V_{Re_t}^t - B_{kk}^{dt}V_{Im_t}^t)$$

$$- \sum_{i \in \Omega_{ki}, i \neq k} \sum_{t \in \Omega_p} (G_{ki}^{dt}V_{Re_t}^t - B_{ki}^{dt}V_{Im_t}^t)$$  (3.2)
\[
\Delta I^d_{Im_k} = \frac{P^d_k(V^d_{Im_k} - V^n_{Im_k}) - Q^d_k(V^d_{Re_k} - V^n_{Re_k})}{(V^d_{Re_k} - V^n_{Re_k})^2 + (V^d_{Im_k} - V^n_{Im_k})^2} \\
- \sum_{t \in \Omega_p} (B_{kk}^{dt}v^t_{Re_k} + G_{kk}^{dt}v^t_{Im_k}) \\
- \sum_{i \in \Omega_{ki}} \sum_{t \in \Omega_p} (B_{ki}^{dt}v^t_{Re_i} + G_{ki}^{dt}v^t_{Im_i}) 
\]

(3.3)

\[
\Delta I^n_{Re_k} = - (I^a_{Re_k,lg} + I^b_{Re_k,lg} + I^c_{Re_k,lg}) \\
- \sum_{t \in \Omega_p} (G^{nt}_{kk}v^t_{Re_k} - B^{nt}_{kk}v^t_{Im_k}) \\
- \sum_{i \in \Omega_{ki}} \sum_{t \in \Omega_p} (G^{nt}_{ki}v^t_{Re_i} - B^{nt}_{ki}v^t_{Im_i}) 
\]

(3.4)

\[
\Delta I^n_{Im_k} = - (I^a_{Im_k,lg} + I^b_{Im_k,lg} + I^c_{Im_k,lg}) \\
- \sum_{t \in \Omega_p} (B^{nt}_{kk}v^t_{Re_k} + G^{nt}_{kk}v^t_{Im_k}) \\
- \sum_{i \in \Omega_{ki}} \sum_{t \in \Omega_p} (B^{nt}_{ki}v^t_{Re_i} + G^{nt}_{ki}v^t_{Im_i}) 
\]

(3.5)

\[
\Delta I^d_{Im_k} = \Delta I^d_{Re_k} = \Delta I^n_{Re_k} = \Delta I^n_{Im_k} = 0 
\]

(3.6)

**Slack Bus and Voltage Controlled Bus Voltages:** The voltage magnitude at the slack bus, and at any other voltage controlled bus are held at the specified values \(V^d_{\text{slack}}\) and \(V^d_{\text{spec}}\) respectively, as shown in (3.7) and (3.8).

\[
V^d_k = V^d_{\text{slack}} \quad k \in \Omega_{k_s} 
\]

(3.7)

\[
|V^d_k| = V^d_{\text{spec}} \quad k \in \Omega_{k_v} 
\]

(3.8)

**Inequality Constraints**

The inequality constraints define the limits of the specific network and therefore may change according to each network’s specifications.
Lines and Cables: The current flowing between bus $k$ phase $d$ and bus $i$ phase $d$, i.e. the line or cable current, $I_{ki}^d$, should be less than or equal to the rated current of that specific line $I_{ki}^{rated}$.

$$
|I_{ki}^d| \leq I_{ki}^{rated} \quad \text{for} \quad i \in \Omega_k \quad (3.9)
$$

Transformers and Voltage Regulators: The total apparent power flowing through phase $d$ of any transformer (T) or voltage regulator (R), $S_{ki}^d$, cannot exceed the given apparent power rating of $S_{ki}^{d, rated}$.

$$
|S_{ki}^d| \leq S_{ki}^{d, rated} \quad \text{for} \quad k, i \in \Omega_{kT} \quad (3.10)
$$

$$
|S_{ki}^d| \leq S_{ki}^{d, rated} \quad \text{for} \quad k, i \in \Omega_{kR} \quad (3.11)
$$

Voltage Limits: The voltage at all buses is limited between the lower and upper bounds $V_{min}$ and $V_{max}$ respectively.

$$
V_{min} \leq |V_k^d| \leq V_{max} \quad (3.12)
$$

Additional constraints can easily be introduced to account for specific equipment limitations, etc.

3.2.3 Objective Function

The optimisation allows for minimisation (or maximisation) of an objective function $f$. The objective function can be varied to accommodate the needs of the particular network. The multi-period capability is achieved by introducing an extra time dimension to the relevant parameters and variables, and allows for the introduction of time dependent variables and objective functions. Specific objective functions will be described in further detail in Section 3.4.
3.2.4 Component Models

Loads

Loads are modelled using the composite ZIP representation. This means loads are represented by a composition of three different types of load; constant impedance load \((Z)\), constant current load \((I)\), and constant power load \((P)\). The proportion of each type of load utilised is determined by the voltage dependence of the load (Collin et al., 2014). The active and reactive power demand, \(P_{D_k}^d\) and \(Q_{D_k}^d\) respectively, which are used to calculate the mismatch equations in (3.2)-(3.5), are given in terms of their ZIP components in (3.13) and (3.14). The active and reactive power injection at bus \(k\) phase \(d\), given by \(P_{k}^d\) and \(Q_{k}^d\) respectively, is defined as the generation, \(P_{G_k}^d\) and \(Q_{G_k}^d\), minus the demand, \(P_{D_k}^d\) and \(Q_{D_k}^d\), as shown in (3.15) and (3.16).

\[
\begin{align*}
P_{D_k}^d &= P_{P_k}^d + P_{I_k}^d \left| V_k^d - V_n^k \right| + P_{Z_k}^d \left| V_k^d - V_n^k \right|^2 \\
Q_{D_k}^d &= Q_{P_k}^d + Q_{I_k}^d \left| V_k^d - V_n^k \right| + Q_{Z_k}^d \left| V_k^d - V_n^k \right|^2 \\
P_k^d &= P_{G_k}^d - P_{D_k}^d \\
Q_k^d &= Q_{G_k}^d - Q_{D_k}^d
\end{align*}
\]

(3.13) \hspace{1cm} (3.14) \hspace{1cm} (3.15) \hspace{1cm} (3.16)

Voltage Regulators

A step voltage regulator, as shown in the example in Fig. 3.1 (a), is essentially an autotransformer with a load tap changing mechanism on the series winding (Kersting, 2002). The function of the voltage regulator is to change its tap settings to step the voltage at a secondary bus either up or down, depending on the desired voltage at some bus. In Fig. 3.1 (a), the voltage at Bus 2 is determined by multiplying the voltage at Bus 1 by the turns ratio.

Figure 3.1 (b) gives an overview of how voltage regulators are modelled in this formulation. A new dummy bus, Bus 1’, is introduced between Bus 1 and Bus 2, such that the segment between Bus 1 and Bus 1’ is treated as an ordinary line
The segment between \textit{Bus 2} and \textit{Bus 1}' is a new regulator segment. The voltage regulation is implemented by introducing current injection variables at \textit{Bus 2} and \textit{Bus 1}'. The constraints given in (3.18)-(3.20) apply to the regulator segment, and are used to guide the current injection variables to the appropriate solution. Equation (3.18) relates the complex voltage at \textit{Bus 2} to the complex voltage at \textit{Bus 1}' by means of the turns ratio \( r_{Bus 1',Bus 2}^d \). The turns ratio, and therefore the tap settings, are modelled as continuous variables for the purposes of this work. Equations (3.19) and (3.20) define the complex current flows from bus \textit{Bus 1}' phase \( d \) to \textit{Bus 2} phase \( d \), and vice versa, as minus the sum of the phase \( d \) current flows to the other connecting buses.

\[
S_{Bus 1',Bus 1}^d = S_{Bus 2,Bus 1'}^d 
\tag{3.17}
\]

\[
V_{Bus 2}^d = r_{Bus 1',Bus 2}^d \times V_{Bus 1'}^d 
\tag{3.18}
\]

\[
I_{Bus 1',Bus 2}^d = - \sum_{i \in \Omega_{Bus 1'}_{1'}} I_{Bus 1',i}^d 
\tag{3.19}
\]

\[
I_{Bus 2,Bus 1'}^d = - \sum_{i \in \Omega_{Bus 2}_{2,i}} I_{Bus 2,i}^d 
\tag{3.20}
\]
3.2.5 Implementation

The various elements described above are combined to formulate a non-linear optimisation. The network configuration, loads and generation, and various ratings and limits are provided as parameters. Voltages, currents, powers and any control variables are implemented as variables and constraints. Many of the variables and constraints, as well as certain objective functions, are non-linear. For example, the current mismatch constraints given in (3.2)-(3.5) above, which result in the non-linear formulation of the problem. The formulation has been implemented using the AIMMS optimisation modelling environment (AIMMS, 2015), and is solved using the NLP solver CONOPT (CONOPT, 2000). There are various non-linear solvers available which would also be suitable for solving this problem, such as KNITRO (KNITRO, 2015) and IPOPT (IPOPT, 2015).

3.3 Test Cases

Two test feeders were chosen for the purposes of this work, the IEEE 123-node test feeder, and a practical suburban feeder.

3.3.1 IEEE 123-node test feeder

The IEEE 123-node test feeder, given in Fig. 3.2, is one of the distribution test feeders provided by the IEEE Distribution System Analysis Subcommittee. In reality, the feeder consists of more than 123 buses, 129 when dummy buses are added for voltage regulators. There are 4 step voltage regulators, 3 shunt capacitor banks, numerous switches, multiple voltage levels (4.16 kV nominal) and a combination of overhead lines and underground cables present on the feeder.

This feeder was selected as an appropriate test for validating the formulation by performing the TOPF with a dummy objective, i.e. performing an ordinary load flow, and comparing results with the provided solution. The configuration of the feeder was also altered to demonstrate the capability of the method to provide solutions for meshed feeders. Data for this and other distribution test
feeders are available at the IEEE distribution test feeders website IEEE (2013). Further information on the feeders can be found in Kersting (2001).

### 3.3.2 Practical test feeder

The practical test feeder is the same feeder utilised for the work in Chapter 2 Section 2.3.1. A detailed diagram of the feeder is given in Fig. 2.3. In order to demonstrate the multi-period TOPF, grid-connected PV modules have been added to a number of the houses on the feeder. Each PV system also has a battery installed for storage purposes. A 24-hour time period, in time steps of 30 minutes, was chosen as a test case for the formulation. The 24-hour period that was chosen was a low demand “sunny” day in Summer so as to observe the maximum impact of the PV systems. Demand and PV data were chosen accordingly and will be described in further detail below.

---

**Figure 3.2:** Single line diagram of IEEE 123-node test network
Demand profiles

Demand profiles for the practical feeder are generated in the same way as the initial demand profiles described in Chapter 2, Section 2.3.2.

Photovoltaic modelling

At nominal low voltage (230 V), the definition of microgeneration according to ESB Networks (2009), gives a maximum power rating of 5.75 kWp. However, after assessing the average roof area available on the houses in the above feeder, a system with a power rating of 2 kWp was chosen. The National Renewable Energy Laboratory (NREL) have developed an online tool, PVWatts (NREL, 2015), which uses the PV system parameters, along with historical meteorological data, to calculate hourly active power outputs. A sample 24-hour period was chosen from the hourly data obtained using the PVWatts tool. As demand data is in 30-minute intervals each hourly data point is repeated for the following half-hourly time step.

PV inverters, which are used primarily to convert the DC power produced by PV modules to AC power, are also capable of both producing and absorbing reactive power (Carvalho et al., 2008; Vasquez et al., 2009). The reactive power capability chart for a PV inverter is given in Fig. 3.3. Although, at present, it is common for inverters to operate within a specified power factor range (ABB, 2015; Sungrow, 2015) and therefore have limited reactive power capabilities, in this work it is assumed that the full reactive power capability is available, as may be the case in the future. If the inverter has a kVA rating, $S_{\text{rated}}$, equal to the rating of the PV module, $P_{\text{rated}}$, then the reactive power capability is given by the black dashed line. It is clear that, in this case, if the PV module is producing maximum active power, the inverter would not be able to inject or absorb any reactive power. However, if the inverter has a kVA rating $S_{\text{rated}}$, which is slightly higher than the rating of the PV module, the reactive capability is given by the red dotted line, and the inverter would still be capable of providing or absorbing some reactive power, even if the PV module was producing maximum active
power, $P_{rated}$. It can be seen that if the PV module is not producing any active power the inverter can still provide or absorb reactive power, however, for the purposes of this work, it is assumed that when the PV module is not in use, i.e. during the night, reactive power support is unavailable. It is also assumed that PV inverters have a kVA rating equal to the rating of the PV module, i.e. 2 kVA.

![Diagram of reactive power capability of PV inverter](image)

**Figure 3.3:** Reactive power capability of PV inverter

This chart provides inequality constraints for the PV inverter reactive power limits, which are given in 3.21.

$$-\sqrt{(S_{inv_h}^{d})^2 - ((P_{PV_h}^{d})^2) \leq (Q_{inv_h}^{d})_h \leq \sqrt{(S_{inv_h}^{d})^2 - ((P_{PV_h}^{d})^2)} (3.21)}$$

**Battery modelling**

Each PV system is equipped with a 5 kWh battery for storage purposes, based on IEEE Standards Association (2007). The rate of charge/discharge of batteries at each time step $h$, $(P_{storage}^{d})_h$, must be between an upper and lower bound, as shown in (3.22). The upper bound is the energy stored in the battery at time step $h$, $(E_{storage}^{d})_h$, which is defined in (3.23). The lower bound is minus the available capacity of the battery at time step $h$. A negative value of $(P_{storage}^{d})_h$ indicates that the battery is charging, while a positive value indicates it is discharging. All batteries are assumed to be empty at the beginning of the 24-hour period.

$$- (E_{storage}^{max} - (E_{storage}^{d})_h) \leq (P_{storage}^{d})_h \leq (E_{storage}^{d})_h \quad (3.22)$$
\[
(E^d_{\text{stor},k})_h = \sum_{g=1}^{h-1} (P^d_{\text{stor},k})_g
\]  

(3.23)

3.4 Results and Discussion

The following subsections present the results for the IEEE 123-node test feeder which is used primarily for validation purposes, and the practical feeder which demonstrates the TOPF using grid connected PV systems. Results from overall feeder and individual customer perspectives are analysed and discussed.

3.4.1 IEEE 123-node test feeder

Radial configuration results (validation)

The data for the IEEE 123-node test feeder, obtained from IEEE (2013), was used as the input to the formulation and a dummy objective was introduced so that an ordinary load flow could be performed. Table 3.1 gives the average and maximum percentage difference of the voltage magnitudes between the solution of the TOPF and the solution provided by the IEEE Distribution System Analysis Subcommittee, for phases \(a\), \(b\), and \(c\). It can be seen that the percentage difference never exceeds 0.1% with the maximum difference on phase \(c\) as low as 0.056%.

**Table 3.1:** Percentage difference of voltage magnitudes for TOPF vs IEEE solutions

<table>
<thead>
<tr>
<th></th>
<th>Average difference (%)</th>
<th>Maximum difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase a</td>
<td>0.0236</td>
<td>0.0965</td>
</tr>
<tr>
<td>Phase b</td>
<td>0.0209</td>
<td>0.0808</td>
</tr>
<tr>
<td>Phase c</td>
<td>0.0158</td>
<td>0.0560</td>
</tr>
</tbody>
</table>

The tap settings for the four regulators were all equal to the provided solution. The solution time was 3.32 seconds on an Intel Xeon 3.1 GHz machine with 16 GB of RAM.
Meshed configuration results

A load flow was also performed on the 123-node feeder with two of the normally open switches being closed. The switch locations can be seen in Fig. 3.2, where the first three-phase switch is located between buses 151 and 300, and the second single-phase switch is located between buses 54 and 94. The single-phase switch is on phase $a$. Closing these switches transforms the configuration of the feeder from radial to meshed.

Solutions for the voltage magnitudes for phases $a$, $b$, and $c$ are given in Fig. 3.4, and are compared to the earlier solutions for the radial configuration. The bus names given refer to the names shown in Fig. 3.2. It can be seen that meshing the network has led to increased voltage magnitudes for phases $b$ and $c$. Phase $a$ also experiences increases in some areas of the feeder, however, the closing of the single-phase switch between buses 54 and 94 has resulted in lower voltages around those buses.

Overall, the tap settings for the meshed feeder are similar to the radial configuration settings, however, there is a variation in the settings for phase $a$ of the regulator located between buses 160 and 67. This can be attributed to the closing of the single-phase switch between buses 54 and 94, and the resulting decrease in voltage magnitudes for surrounding buses.

The solution time for the meshed configuration was 3.71 seconds on the same machine given above. This feeder was also tested with various demand profiles, feeder configurations and objective functions and consistently showed good convergence properties and solution times.

3.4.2 Practical feeder TOPF

The practical feeder was used as the test feeder for the TOPF by adding grid connected PV modules to the roofs of ten of the south-facing houses on the feeder, giving a PV penetration level of 13.5%. So that the impact of unbalanced generation could be observed, a higher number of PV systems have been allocated to phase $a$. In order to demonstrate the flexibility of the method, two objectives
Minimum loss objective

In this case, the objective function is to minimise the sum of the active power losses on the feeder over all phases, branches and time steps, as seen in (3.24), subject to the earlier defined constraints. The multi-period capability is expressed by the summation over all time steps \( h \) in (3.24). The active power output of the PV systems and the energy stored are the control variables.
\[ f = \min \sum_{h \in \Omega_h} \sum_{k \in \Omega_k} \sum_{i \in \Omega_i, i \neq k} \sum_{d \in \Omega_d} (P_{L_ki})_h \] (3.24)

The maximum output for each of these 2 kWp PV systems over the 24-hour period is given in Fig. 3.5. It can be seen that the PV system produces power between the hours of 05:00 and 20:00 with maximum power output between 12:00 and 13:00. In this case, although PV output is modelled as a variable, all of the PV systems produced maximum available output at all times.

![Figure 3.5: Maximum output for each PV system over the 24-hour period](image)

Figure 3.6 shows the aggregate storage and PV outputs, as well as the aggregate feeder demand over the 24-hour period. Positive storage output means that power is being injected into the feeder, while negative output means that power is being absorbed. It is clear that the storage output follows the feeder demand while also taking advantage of the PV output. During the times that the PV systems are producing a large amount of power, the storage systems are absorbing significant power. This stored energy is later injected into the feeder, when the demand is high and PV output is low.

The aggregate stored energy over the 24-hour period is given in Fig. 3.7, where the dotted lines represent the aggregate maximum storage capacity (50 kWh). This figure compares the energy stored when a multi-period TOPF is performed,
and when 48 single period TOPFs are performed. The lack of knowledge regarding future feeder requirements in the single period scenario results in little to no energy being stored, and PV output being curtailed. In contrast, the multi-period case allows for a degree of planning, and stores a significant amount of energy for use later in the day, when it is of most benefit to the feeder.

The total feeder losses are 18% lower in the multi-period scenario than the single period case. The solution time for the TOPF for 48 time periods using the
practical test feeder and the minimum loss objective was 70.12 seconds, using the same machine given above.

**Minimum unbalance objective**

The objective in this scenario is to minimise the voltage unbalance on the feeder over all phases, buses, and time steps as shown in (3.25), where $V_{unb_k}^d$ is the voltage unbalance at bus $k$ phase $d$. The voltage unbalance for bus $k$ phase $d$, given in (3.26), is equal to the deviation of the voltage magnitude at bus $k$ phase $d$, from the average voltage at bus $k$, divided by the average voltage at bus $k$. The control variable in this case is the PV inverter reactive power output. The PV active power is not treated as a variable in this scenario, therefore, all PV modules are assumed to produce maximum available active power output at all times.

$$f = \min \sum_{h \in \Omega_h} \sum_{k \in \Omega_k} \sum_{d \in \Omega_d} (V_{unb_k}^d)_h$$ \hspace{1cm} (3.25)

$$V_{unb_k}^d = \frac{|V_k^d| - |\overline{V}_{abc}^k|}{|\overline{V}_{abc}^k|}$$ \hspace{1cm} (3.26)

Figure 3.8 shows the reactive power output for an individual PV system located at bus a4 in Fig. 2.3. The reactive power limits are dictated by the active power output. Comparing to Fig. 3.5, it can be seen that the limits are somewhat restricted as the active power output increases, while the limits are close to the inverter kVA rating as the active power output approaches zero. This inverter’s reactive power output is at the limits for a large proportion of the time, as is the case with the majority of the inverters, showing that the availability of reactive power is fully employed in order to reduce voltage unbalances.

Figure 3.9 gives the voltage magnitudes at the three-phase bus a4, for phases a, b, and c for a case with no PV, one with all PV systems providing maximum available active power output, and the minimum unbalance objective case, respectively. Focusing on those times that the PV systems are capable of providing output, namely 05:00-20:00, it is clear that although having no PV on
the feeder leads to varying voltage magnitudes among the phases, allowing the PV modules to provide the maximum possible output, without any reactive support, only exacerbates the voltage unbalance problem, with total voltage unbalance being 6% higher than the case with no PV. In particular, the additional PV systems on phase $a$ lead to larger deviations in voltage between phase $a$, and phases $b$ and $c$. In contrast, the introduction of the minimum unbalance objective results in similar voltage profiles for all three phases. The addition of reactive power support aids greatly in the reduction of voltage deviations, particularly at those times of day when the active power output is relatively low, e.g. early morning and early evening.

The sum of voltage unbalances in this case is 26% less than the maximum active power output scenario. The solution time for the TOPF for this case was 86.41 seconds, using the same machine given above.

The objectives presented here are purely sample objectives for the purposes of demonstrating the capabilities of the method. Various objectives can be incorporated, to accommodate the needs of each individual feeder.
This chapter introduces a multi-period, three-phase, unbalanced, optimal power flow formulation. The formulation allows for the integration of all of the different elements that are inherent to unbalanced distribution networks, such as capacitors, ZIP loads and voltage regulators.

The formulation was tested using two feeders, the IEEE 123-node feeder, and a practical test feeder. The solutions produced by the TOPF for the IEEE 123-node feeder were almost identical to those of the reference solution. The results from the practical test feeder show that the addition of storage to a PV system, paired with the multi-period TOPF, can significantly reduce losses, while inverter reactive power control can improve voltage unbalance.

**Figure 3.9:** Voltage magnitudes at bus a4, for phases a, b, and c, for the scenario with no PV, with maximum PV output, and with the minimum unbalance objective

### 3.5 Conclusions

This chapter introduces a multi-period, three-phase, unbalanced, optimal power flow formulation. The formulation allows for the integration of all of the different elements that are inherent to unbalanced distribution networks, such as capacitors, ZIP loads and voltage regulators.

The formulation was tested using two feeders, the IEEE 123-node feeder, and a practical test feeder. The solutions produced by the TOPF for the IEEE 123-node feeder were almost identical to those of the reference solution. The results from the practical test feeder show that the addition of storage to a PV system, paired with the multi-period TOPF, can significantly reduce losses, while inverter reactive power control can improve voltage unbalance.
The new computational formulation provides a unique solution to the TOPF problem due to the combination of the following features. The TOPF is capable of finding optimal solutions for both radial and meshed networks, while the modelling of a neutral wire gives accurate representations of unbalanced networks. The multi-period capability allows time dependent variables and objectives to be incorporated. The formulation has shown good convergence properties and solution times. Finally, it is formulated as a single optimisation program, and can be solved using any of the widely available non-linear solvers.

There are many potential applications for the formulation described in this chapter, such as control of additional DER, e.g. CHP units, heat pumps or micro-wind, as well as control of the reactive power associated with certain types of DER. The formulation could also play a valuable role in demand side management schemes, which aim to control other types of load, such as water heaters or refrigerators. An operational planning application to determine volt-var curves for PV inverters has been established and will be discussed in further detail in Chapter 4.
4.1 Introduction

The inverters that are used to grid connect PV have capabilities outside of just converting DC power to AC. They are also capable of curtailing the active power output as well as injecting and absorbing reactive power (Hassaine et al., 2014). A volt-var curve is a function that can be provided to a PV inverter which specifies what the reactive power output should be based purely on the voltage at that node. Most curves are in piecewise linear form, with full capacitive operation at low voltages, full inductive operation at high voltages, and a sloping region in between. Although standards in Europe (European Committee for Standardisation, 2013) have allowed reactive power from distributed generators to support voltage for quite some time, standards in the US have only recently embraced this advanced capability (IEEE Standards Association, 2015). Accordingly, a considerable amount of research has focused on how best to utilise this potentially advantageous resource (Demirok et al., 2011; Farivar et al., 2012; Jahangiri and Aliprantis, 2014; Su et al., 2014; Weckx et al., 2014).
Many PV inverter studies focus on providing optimal settings in the context of active network management i.e. an online approach. This chapter examines an offline solution, in the same vein as Cuffe et al. (2014) which determines the controllable reactive power available for DG resources. The work expands on studies undertaken in EPRI, detailed in O’Connell et al. (2015) and summarised in Appendix B, as part of a demonstration project which aims to utilise the capability of distributed PV inverters. The formulation in Appendix B produces volt-var curves for a utility-scale PV inverter using a combination of MATLAB optimisation and EPRI’s OpenDSS distribution modelling software. A number of objectives, such as mitigation of voltage variation and loss reduction, are examined. The work in this chapter models a low voltage distribution feeder with the addition of a number of PV systems. The TOPF formulation from Chapter 3 is utilised to determine optimal reactive power settings for various load and PV active power scenarios. As the reactive power output given by a volt-var curve depends solely on the voltage at that bus, the aim of the scenarios is purely to produce a wide range of voltages at each PV bus, so that the reactive power settings determined by the TOPF span a range of voltages and result in a practical volt-var curve. The objective examined aims to minimise voltage deviations from predefined set-points. This objective can mitigate the voltage deviations caused by substantial PV penetrations, as well as potentially reducing voltage unbalance. The results are utilised to generate individual volt-var curves for each PV inverter as well as a universal curve for all inverters. The resulting curves are then analysed for a particular set of conditions in order to compare the results and validate the formulation. The curves can subsequently be applied to the actual PV inverters to perform a decentralised type of voltage control.

The chapter is organised as follows. The methodology is outlined in Section 4.2, a detailed description of the test case is given in Section 4.3, Section 4.4 outlines the main results and conclusions are presented in Section 4.5.
4.2 Methodology

4.2.1 Three-Phase Optimal Power Flow

The TOPF utilised is as described in Chapter 3, Section 3.2. All ratings and limits for voltage, current, and power also apply to this work. Loads are modelled using the composite ZIP representation, and PV inverters use the same capability chart as shown in Fig. 3.3.

The objective for the purposes of this work is to minimise the sum of the voltage deviations at PV buses from their optimal voltage set-points, over all phases, PV buses, and scenarios, as seen in (4.1), subject to the TOPF constraints. The definition in (4.1) shows that the deviation is squared. For the purposes of optimisation, the smooth square function results in more efficient convergence than the non-smooth absolute function (Drud, 1996).

\[
\min f \\
f = \sum_{m \in \Omega_m} \sum_{k \in \Omega_k_{PV}} \sum_{d \in \Omega_d} \left( \left| V_{k_m}^d \right| - V_{opt_k}^d \right)^2
\]  

(4.1)

The optimal voltage set-points are determined by performing a separate multi-scenario TOPF without the addition of PV but including a variable reactive power source at each bus in order to control the voltage. The aim is to calculate the optimal voltage set-point for each PV bus, \( V_{opt_k}^d \), that will minimise the total feeder voltage unbalance. Minimisation of voltage unbalance is beneficial for distribution networks as significant levels of voltage unbalance can cause problems for three-phase loads, in particular three-phase motors, which may experience significant degradation due to overheating. Minimising the voltage deviation from these set-points should therefore have an additional benefit in the form of unbalance reduction.
4.2.2 Formulation

A number of residential PV systems are added to a test network. Voltage set-points are determined for each PV bus as described in the previous section. The purpose of the work is to find the optimal volt-var curve for each PV system with the objective of minimising voltage deviations. This is achieved by performing a multi-scenario TOPF simulation. Scenarios are developed which represent an appropriate number of combinations of load and active power output of PV systems. The aim of the scenarios is not to try and incorporate all of the demand and PV possibilities that may occur on the feeder, but rather to achieve a significant range in voltages at each PV bus. The concept of volt-var curves implies that the optimal reactive power setting of a particular PV inverter is based solely on the voltage at that PV bus, and therefore the specific external circumstances that lead to the occurrence of that voltage, are irrelevant.

For each scenario, a load flow is performed without the addition of any reactive power support from the PV inverters. The resulting voltages should represent the range of voltages that each PV node is likely to experience. These are the voltages that will be utilised to create the volt-var curves. A multi-scenario TOPF is then performed with the principal decision variable being the reactive power setting of each PV inverter. A scatter plot of the reactive power setting against the voltages for the non-reactive power support solution is then plotted for each inverter. Volt-var curves are then fitted to the scatter plot data. A flowchart describing the formulation is given in Fig. 4.1.

This formulation could be utilised by system operators in an operational planning capacity for a feeder with significant PV penetration levels. Similarly, it could be employed when a new PV system is installed on a particular feeder to determine the volt-var curve for the new PV system. This would not require modifying the volt-var curves for any existing PV systems. The method is formulated as a non-linear optimisation program. The formulation has been implemented using AIMMS (AIMMS, 2015) optimisation modelling environment,
and is solved using the non-linear programming solver CONOPT (CONOPT, 2000).

4.3 Test Case

4.3.1 Test Network

The network utilised in this work is the section of actual LV network introduced in Chapter 2. Grid connected PV systems, which are described in further detail in one of the following subsections, are allocated to 10 of the houses on the test feeder, these are represented as a dot in Fig. 4.2. The name, location, and phase of each PV system is given in Table 4.1. It can be seen that there are a larger number of PV systems allocated to phase $a$, so as to observe unbalance effects. Although, at present, it is common for inverters to operate within a specified power factor range (ABB, 2015; Sungrow, 2015) and therefore have limited reactive power.
capabilities, in this work it is assumed that the full reactive power capability is available, as may be the case in the future.

Table 4.1: Name, location, and phase of PV systems

<table>
<thead>
<tr>
<th>PV Bus Name</th>
<th>Three-Phase Bus</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV 1</td>
<td>a2</td>
<td>a</td>
</tr>
<tr>
<td>PV 2</td>
<td>a2</td>
<td>a</td>
</tr>
<tr>
<td>PV 3</td>
<td>a3</td>
<td>a</td>
</tr>
<tr>
<td>PV 4</td>
<td>a6</td>
<td>a</td>
</tr>
<tr>
<td>PV 5</td>
<td>a9</td>
<td>a</td>
</tr>
<tr>
<td>PV 6</td>
<td>a2</td>
<td>b</td>
</tr>
<tr>
<td>PV 7</td>
<td>a8</td>
<td>b</td>
</tr>
<tr>
<td>PV 8</td>
<td>a4</td>
<td>c</td>
</tr>
<tr>
<td>PV 9</td>
<td>a7</td>
<td>c</td>
</tr>
<tr>
<td>PV 10</td>
<td>a9</td>
<td>c</td>
</tr>
</tbody>
</table>

Figure 4.2: Diagram of 85 node test network with PV location given by (●)

4.3.2 Demand Scenarios

Daily demand profiles were generated using the residential load modelling tool described in McKenna and Keane (In Press) at a 1-min time resolution. The demand profiles consist of apparent power, power factor, Z load component, I load component and P load component values for each customer and time step. The maximum aggregate active power demand for the feeder was located and the demand for each customer at that time was utilised as the maximum
demand scenario. These values were then scaled to generate 15 demand scenarios between the minimum aggregate demand and the maximum aggregate demand. The reason that values are scaled is that the objective of the scenarios is purely to produce a wide range of voltages at each PV bus. As such, scaled values of demand between the minimum and maximum values are sufficient to achieve this goal. The 15 demand scenarios are given in Fig. 4.3 where each block in a column shows a single customers demand.

![Figure 4.3: Aggregate demand for each scenario](image)

### 4.3.3 PV Active Power Scenarios

At nominal low voltage (230 V), the definition of microgeneration according to ESB Networks (2009), gives a maximum power rating of 5.75 kWp. However, after assessing the average roof area available on the houses in the test feeder, a system with a power rating of 2 kWp was chosen. PV active power output is varied between 0 kW and 2 kW in increments of 0.2 kW to give 11 PV active power scenarios. Combining the demand and PV active power scenarios gives a total of 165 scenarios. As previously stated, the goal of the scenarios is not to encompass every possible combination of load and PV active power, but to produce a wide range of voltages at each PV bus.
4.4 Results and Discussion

4.4.1 Volt-Var Curves

The optimal voltage set-points for each PV bus are given in Table 4.2. These are the set-points determined by the first TOPF in Fig. 4.1 which aims to minimise voltage unbalance. The voltage set-points vary between 0.993 pu and 1.002 pu.

<table>
<thead>
<tr>
<th>PV Bus</th>
<th>Phase</th>
<th>Set-Point (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV 1</td>
<td>a</td>
<td>1.000</td>
</tr>
<tr>
<td>PV 2</td>
<td>a</td>
<td>1.000</td>
</tr>
<tr>
<td>PV 3</td>
<td>a</td>
<td>1.002</td>
</tr>
<tr>
<td>PV 4</td>
<td>a</td>
<td>0.997</td>
</tr>
<tr>
<td>PV 5</td>
<td>a</td>
<td>0.993</td>
</tr>
<tr>
<td>PV 6</td>
<td>b</td>
<td>1.002</td>
</tr>
<tr>
<td>PV 7</td>
<td>b</td>
<td>0.993</td>
</tr>
<tr>
<td>PV 8</td>
<td>c</td>
<td>1.000</td>
</tr>
<tr>
<td>PV 9</td>
<td>c</td>
<td>0.999</td>
</tr>
<tr>
<td>PV 10</td>
<td>c</td>
<td>0.995</td>
</tr>
</tbody>
</table>

The resulting volt-var scatter plot and curve for each PV system is given in Fig. 4.4. The plots show the reactive power as a proportion of the available reactive power of the inverter, against the voltage at that bus with no reactive power support. A positive value means that reactive power is being injected to the feeder (capacitive) while a negative value means it is being absorbed (inductive). The marker represents the phase that each bus is on. The phase a buses are represented by a circle, phase b by a triangle, and phase c by a diamond. The curves are given in piecewise linear form and are represented in red in Fig. 4.4. The slopes of each volt-var curve are given in Table 4.3, where the negative value indicates a decreasing slope. Generally, all of the inverters have similarly shaped volt-var curves. The shape indicates that reactive power should be injected into the feeder when the voltage at that bus is below its voltage set-point in order to increase the voltage. Conversely, when the voltage at any PV bus is above its set-point the curves indicate that reactive power should be absorbed for the purposes of reducing the feeder voltage towards the set-point. The reactive power around the set-point voltage is close to zero. Although the shapes of the curves
are comparable, the slopes are somewhat varied. It seems that the closer to the feeder head the PV system is, the steeper the slope of the volt-var curve. This is reflected by the larger slope values in Table 4.3. PV systems closer to the end of the feeder tend have a more gradual slope, given by the smaller slope values in Table 4.3. This indicates that, at the start of the feeder, affecting voltage requires large amounts of reactive power to be injected/absorbed, whereas towards the end of the feeder, less reactive power is needed to change voltage levels. The inverter with the largest slope is PV 3 and the smallest is PV 5, both located on phase $a$. PV 3 is located close to the feeder head while PV 5 is located at the end. The average slope value is $-364 \text{ pu}^{-1}$. 

Figure 4.4: Individual volt-var scatter plots and curves for 10 PV systems
Table 4.3: Slopes of individual volt-var curves

<table>
<thead>
<tr>
<th>PV Bus</th>
<th>Phase</th>
<th>Slope (pu⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV 1</td>
<td>a</td>
<td>-294</td>
</tr>
<tr>
<td>PV 2</td>
<td>a</td>
<td>-286</td>
</tr>
<tr>
<td>PV 3</td>
<td>a</td>
<td>-625</td>
</tr>
<tr>
<td>PV 4</td>
<td>a</td>
<td>-270</td>
</tr>
<tr>
<td>PV 5</td>
<td>a</td>
<td>-196</td>
</tr>
<tr>
<td>PV 6</td>
<td>b</td>
<td>-606</td>
</tr>
<tr>
<td>PV 7</td>
<td>b</td>
<td>-312</td>
</tr>
<tr>
<td>PV 8</td>
<td>c</td>
<td>-357</td>
</tr>
<tr>
<td>PV 9</td>
<td>c</td>
<td>-435</td>
</tr>
<tr>
<td>PV 10</td>
<td>c</td>
<td>-345</td>
</tr>
</tbody>
</table>

Fig. 4.5 shows the universal volt-var scatter plot if the system operator wants to provide all inverters with the same curve. The reactive power, in this case, is plotted against the average voltage of all the PV buses. The curve has similar characteristics to the individual curves with operation being in the capacitive region for lower voltages and in the inductive region for higher voltages. The set-point for the universal curve is calculated as 0.998 pu. The slope is relatively gradual in comparison to the individual curves, with a value of -192 pu⁻¹, which is less than the average of the individual slopes (-364 pu⁻¹). Referring back to the observations made about the individual curves, the slope of the universal curve indicates that the inverters towards the end of the feeder have more influence on the universal curve than those at the start. Although implementing this universal curve may be more straightforward, it could result in less than optimal operation.

In order to verify the assumption that the resulting reactive power settings that define the volt-var curves are based only on the PV bus voltage, and not the particular circumstances that caused the voltage, a comparison is performed. Using the same approach as previously described, a volt-var curve is determined for PV 9, with it being the only PV system on the test feeder. The scatter plot for this case is shown in Fig. 4.6 along with the original PV 9 scatter plot and volt-var curve that were determined with 10 PV systems on the feeder. The new scatter plot is given in blue. It is clear that the only difference is that the range of voltages experienced at that bus is smaller due to having less PV systems on the feeder. The original volt-var curve however is still applicable. This shows that
adding a new PV system will not require new volt-var curves to be determined for existing inverters but just for the inverter of the new system being added.

Figure 4.5: Universal volt-var scatter plot

Figure 4.6: Comparison of PV 9 volt-var curves with 1 PV system and 10 PV systems on the feeder

4.4.2 Validation

In order to examine and validate the volt-var curves from the previous section in further detail, a 24-hour period is investigated as a test case. Both the individual and the universal curves are applied to the inverters on the test feeder, and
the results are analysed against a case without reactive power support. Fig. 4.7 shows the active power output of the PV systems over the 24-hour period. The PV systems produce power between 05:00 and 19:30 with the maximum power being generated at 12:00 and 12:30. The maximum active power output is 1.42 kW which is less than the inverter rating of 2 kVA, therefore the inverters can produce reactive power throughout the day.

![Active Power Output of PV Systems](image)

**Figure 4.7:** Active power output of PV systems over 24-hour period

The total reactive output for all 10 PV inverters is given in Fig. 4.8 for the cases using the individual curves and the universal curve. It is clear that in both cases reactive power is primarily being injected to the feeder which would indicate that voltages generally need to be increased in order to reach their set-points. It should be noted that individual inverters may be absorbing reactive power at certain times, but on an aggregate feeder scale, capacitive operation is dominant. The individual curves result in larger amounts of reactive power being injected in comparison to the the case when the universal curve is employed.

The reactive power output of the individual inverter for PV system PV 7 is shown in Fig. 4.9 for the individual and universal curves. The reactive power limits, as defined by the active power output, are shown by the solid black lines. The resulting voltages at that bus are presented in Fig. 4.10 along with the voltages for the case with no reactive support. The individual and universal curve
result in substantially different reactive power outputs for this inverter. At certain times, the individual curve results in inductive operation while the universal curve results in capacitive operation. The reason for this can be seen when the curve for PV 7 in Fig. 4.4 is compared to the universal curve in Fig. 4.5. The individual curve has a voltage set-point of 0.993 pu while the universal one is slightly higher at 0.998 pu. The individual curve also has a much steeper slope than the universal curve which leads to reactive power outputs that are closer to the reactive limits than the case with the universal curve. The voltages in Fig. 4.10 reflect the reactive power output, with capacitive operation resulting in voltages above the case with no reactive support and vice versa for inductive operation. It is also evident that during later hours both curves are providing the maximum amount of reactive power possible in order to increase the low voltages caused by high demand.

Fig. 4.11 shows the resulting voltages at 17:30 for all buses on phases a, b, and c for the cases with no reactive power support, and with the individual volt-var curves employed. The red markers show the locations of the PV systems. It is clear that implementing the volt-var curves drives the voltage closer to the set-points on all phases. Phase a sees the greatest benefit as it has the most PV systems connected, with a total voltage change of 0.22% over the test day. Phase
Chapter 4. Optimal Volt-Var Curves for PV Inverters

Figure 4.9: Reactive power output of a single PV inverter over 24-hour period

c also has a considerable voltage change of 0.21%, while phase b, which has the least PV systems connected, experiences only a minor voltage decrease of 0.09%.

Figure 4.11: Phase a, b, and c voltages at 17:30 for the case without reactive power support and the case where individual volt-var curves are applied
Figure 4.10: Voltage for an individual customer for the cases with no reactive power support, and using individual and universal curves

Over the test day, the additional reactive power flowing in the lines results in increased losses. Total daily active power losses increase from 17.07 kWh in the base case, to 17.80 kWh and 18.12 kWh with the universal and individual curves respectively. However, the overall impact on voltage is beneficial. Table 4.4 compares values for the sum of voltage deviations from the set-point values, and the sum of voltage unbalance for the cases with no reactive power support, and with the addition of the universal and individual curves. The voltage unbalance for bus $k$ phase $d$ is equal to the deviation of the voltage magnitude at bus $k$ phase $d$, from the average voltage at bus $k$, divided by the average voltage at bus $k$. Given that the objective when generating the volt-var curves is to minimise the voltage deviations from their respective set-points, it is assumed that a reduction in voltage unbalance should also occur. The data in the table shows that while both voltage deviations and unbalance are reduced with the use of the universal curve, the results are improved further with the application of individual curves. These results demonstrate that both individual and universal curves can provide benefits to the feeder, which allows the system operator to decide which is more valuable to them, optimality, or ease of implementation.
### Table 4.4: Comparison of metrics for various reactive power cases

<table>
<thead>
<tr>
<th>No Reactive Support</th>
<th>Voltage Deviation (pu)</th>
<th>Voltage Unbalance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal Curve</td>
<td>3.48</td>
<td>7.11E-03</td>
</tr>
<tr>
<td>Individual Curves</td>
<td>2.33</td>
<td>6.48E-03</td>
</tr>
</tbody>
</table>

#### 4.5 Conclusions

The work presented in this paper determines optimal volt-var curves for distributed PV inverters. The TOPF method accurately models three-phase networks and their associated components, as well as providing optimal solutions for distribution system control variables. Using the TOPF formulation, a number of PV systems are added to a test network and a range of load and active power output scenarios are assessed. A reactive power setting is determined for each PV inverter for each scenario and the resulting reactive power settings are then plotted. An individual volt-var curve is determined for each PV system. These curves can subsequently be incorporated by the system operator to improve the day to day operation of their feeders.

Reduction of voltage unbalance/deviations is employed as an overall objective. Distribution feeders can face significant levels of voltage unbalance due to the dominance of single-phase loads, i.e. loads are not split evenly among the phases. The addition of single-phase PV systems to distribution feeders may result in even higher levels of unbalance. Three-phase loads, in particular motors, can experience shortened lifetimes due to overheating caused by voltage unbalance. Therefore, it is imperative to ensure that distribution feeders do not incur considerable levels of voltage unbalance.

The results show individual volt-var curves for the 10 PV systems on the test feeder, as well as a universal curve for all inverters. Implementation of these curves shows favourable results, with both the universal and the individual curves resulting in voltages closer to the desired set-point values. As well as reducing voltage deviations, the curves have been shown to reduce feeder
voltage unbalance, with the individual curves providing improved results over
the universal curve.

The formulation discussed here could be utilised by system operators offline
in an operational planning capacity to determine volt-var curves for distributed
PV inverters. Since the external factors that produce the voltage at each PV
bus are assumed unimportant in relation to the volt-var curves, once the curves
have been implemented there is no requirement for them to be updated, unless
the configuration of the feeder is considerably altered. Furthermore, if a new
PV system is introduced to the feeder, analysis would be required for only that
system, the existing volt-var curves would still be applicable. Depending on the
requirements of the specific feeder, alternative objectives could also be examined
utilising this formulation.
Robust Operation Using Information Gap Decision Theory

5.1 Introduction

Concerns over the future of distribution networks have led distribution system operators to implement field trials and impact studies for distributed resources, such as electric vehicles, on their LV networks (Green E-Motion Project, 2015; Richardson et al., 2013). Some of the studies described in Ochoa and Mancarella (2012) focus on utilising existing distribution equipment to improve the planning and operation of LV networks, and move towards a more active approach. Although gaining further control and observability of LV networks is desirable, it is not a trivial task. As discussed in Chapter 2, distribution networks have a large degree of uncertainty associated with them. The TOPF developed in Chapter 3 is a powerful tool, however, in order to provide robust solutions for distribution systems, uncertainties must be accounted for.

Various approaches are available for dealing with the uncertainties in unbalanced distribution networks (Caramia et al., 2007; Das, 2006; Viswanadha Raju and Bijwe, 2008; Cao et al., 2013). IGDT (Ben-Haim, 2006) is a decision making method that has recently been incorporated into power systems research.
to combat the problems caused by uncertainties. The vast majority of this research has been concerned with electricity markets and energy procurement. In Zare et al. (2011) the optimal electricity procurement strategy for a large customer is determined considering an electricity pool, bilateral contracts and on-site generation. Pool price uncertainty is modelled using IGDT. The authors in Mohammadi-Ivatloo et al. (2013) introduce an IGDT-based method for generation self-scheduling considering uncertain electricity prices. The work in Soroudi and Ehsan (2013) produces an energy procurement strategy for a distribution network which has distributed generation present, while also taking account of network constraints. IGDT is used to examine uncertainties in both the electricity pool price and the demand level.

IGDT is a decision making method that, unlike stochastic or robust optimisation, does not require knowledge of probability distributions, membership functions or detailed uncertainty sets. It is capable of modelling severe uncertainty without significantly increasing the complexity of the underlying problem. The simplicity and reliability of the method make it ideal for incorporation into unbalanced distribution network analysis, as it can account for the considerable uncertainty that distribution networks face, without increasing the computational burden of an already intricate formulation.

The work presented in this chapter utilises the IGDT method in conjunction with the TOPF developed in Chapter 3. For the purposes of this work, the method will be used as an operational tool to provide network operators with robust day ahead decisions for a particular LV network. The focus of this work is to deal with the uncertainty of apparent power demand in the network. The TOPF and IGDT method assesses the level to which demand can be varied from its predicted value, for a given acceptable variation in the optimal predicted losses, while ensuring that the network is operating within its safe technical limits.

The chapter is organised as follows. The methodology is outlined in Section 5.2, a detailed description of the test case is given in Section 5.3, Section 5.4 outlines the main results and conclusions are presented in Section 5.5.
5.2 Methodology

5.2.1 Information Gap Decision Theory

IGDT is a method for decision making under severe uncertainty, i.e. the decision maker has limited information as to what the actual value of an uncertain parameter will be. An information gap refers to the discrepancy between what is known and what is possible. In the context of power system uncertainty, the information gap describes the disparity between the operating point that the decision maker deems most likely, and the actual operating point, which is unknown. IGDT uses nested sets, with each set defining a particular information gap level, and each element representing a possible operating point.

The required input for IGDT is an initial prediction of what the values of the uncertain parameter will be. Decision makers can then assess either a robustness function or an opportuneness function. The robustness function indicates the maximum allowable deviation from the predicted value, in an undesired direction, while maintaining an acceptable deterioration in the objective value. The opportuneness function indicates the minimum required deviation from the predicted value, in a desired direction, which will result in an anticipated improvement in the objective value. The uncertain parameter belongs to an uncertainty set $U$ which is defined in (5.1)-(5.3). In contrast to robust optimisation or interval based mathematical modelling, the uncertainty set used in IGDT is not precisely known. In fact, it is also subject to uncertainty. The uncertain parameter is represented by $\Lambda$. The only information available regarding $\Lambda$ is its predicted value $\bar{\Lambda}$. The $\alpha$ in (5.1)-(5.3) is called the radius of uncertainty. It is also an uncertain value. It describes the deviation of the actual uncertain parameter values from the predicted values. A comprehensive description of the IGDT method can be found in Ben-Haim (2006) and Ben-Haim (2004), which include theoretical definitions of both the robustness and opportuneness algorithms, various model descriptions, and detailed worked examples.
\[ U(\alpha, \bar{\Lambda}) = \left\{ \Lambda : \frac{\Lambda - \bar{\Lambda}}{\Lambda} \leq \alpha \right\} \] (5.1)

\[ 0 \leq \alpha \] (5.2)

\[ \Lambda \in U(\alpha, \bar{\Lambda}) \] (5.3)

### 5.2.2 Network Modelling in the TOPF

The TOPF methodology from Chapter 3 is also employed in this chapter. The main TOPF equations from Chapter 3, i.e. (3.2)-(3.12), which solve the power flow and impose network limits, therefore apply to this work, along with the constraints defined in the following sections.

**Tap Changers and Voltage Regulators**

A detailed discussion of how tap changers are modelled in this formulation is given in Chapter 3 and will not be repeated here. However, the key voltage equation is shown in (5.4), where \( r_{ik}^d \) refers to the turns ratio at phase \( d \) between buses \( i \) and \( k \).

\[ V_k^d = r_{ik}^d \times V_i^d \] (5.4)

In order to reduce complexity, the turns ratio, and therefore the tap settings, are modelled as a continuous variable in the main TOPF iteration. To ensure that tap settings are modelled accurately, the resulting tap settings are then rounded to the nearest integer and modelled as a parameter. A second TOPF iteration is then performed to confirm that the new integer tap settings provide a valid solution and do not result in any constraint breaches. If a constraint is breached then the bound is tightened on the violated constraint, and the procedure restarts. Although the rounded tap settings may not be optimal, they provide a more realistic representation of actual tap settings. This should eliminate the need to introduce integer modelling to the formulation.
Switches

In order to find optimal switch settings, a configuration dimension has been introduced to the TOPF, where $\Omega_f$ is the set of configurations. The objective is then minimised or maximised over all configurations, time steps, buses and phases. The optimal configuration for each time step is subsequently identified by comparing the configurations at each time step, and determining which gives the optimal objective value. This determines whether switches are open or closed.

5.2.3 Formulation

The IGDT based TOPF is used as a day ahead operational tool for the purposes of this work. The method can provide the system operator with robust and optimal set-points for network control variables. These set-points will allow the network to be operated in such a way that deviations in the uncertain parameter will not cause any technical limits to be breached, and will not cause the objective to vary by more than a specified value.

The uncertain variable in this work is the apparent power demand $\bar{S}_{D_k}$, where $\bar{S}_{D_k}$ is the predicted value for the demand. As IGDT is a method for decision making under severe uncertainty, it is assumed that the system operator has limited information as to what the actual value of the demand will be, and wants to make their operating decisions as robust as possible. The primary objective is to minimise the active power energy losses. The following steps describe the proposed solution procedure:

Step 1: Calculate the predicted optimal active power losses $\bar{P}_L$. A deterministic TOPF, using the predicted values for the demand, $S_{D_k}^d$, is performed to obtain a predicted value for the losses. This is the amount of losses that the network operator will face if there is no demand uncertainty. The set of equations in (5.5) should be solved to obtain $\bar{P}_L$. The total active power losses are calculated by summing the losses, $(P_{L_k}^d f)_{h}$, over all phases, connected buses, time steps, and configurations.
\[ \bar{P}_L = \min \sum_{f \in \Omega_f} \sum_{h \in \Omega_h} \sum_{k \in \Omega_k} \sum_{d \in \Omega_d} \left( P_{L_{ki}}^d \right)_h \]

\[ \alpha = 0 \]

\[ \left| S_{Dk}^d \right| = \left| \bar{S}_{Dk}^d \right| \]

Subject to TOPF constraints

Step 2: Repeat with tap settings set as an integer parameter.

Step 3: Set the tolerable limit of the losses, \( P_{L_c} \), when demand uncertainty exists in the model. The limit \( P_{L_c} \) is set as a percentage of the predicted losses using a new parameter \( \beta \), known as the tolerable loss variation. This means that, depending on whether robustness or opportuneness is being assessed, it is acceptable for losses to be either \( 1 + \beta \) times larger (robustness) or \( 1 - \beta \) times smaller (opportuneness) than the predicted value \( \bar{P}_L \), as in (5.6).

\[ P_{L_c} = (1 \pm \beta) \bar{P}_L \] (5.6)

Step 4: Depending on whether robustness or opportuneness is being analysed, perform a new iteration of the TOPF either maximising or minimising the radius of uncertainty \( \alpha \). This is achieved by solving the set of equations in (5.7), using \( \max \alpha \) and \( 1 + \alpha \) for robustness, or \( \min \alpha \) and \( 1 - \alpha \) for opportuneness.

\[ \max / \min \alpha \]

\[ f \leq P_{L_c} \]

\[ \left| S_{Dk}^d \right| = (1 \pm \alpha) \left| \bar{S}_{Dk}^d \right| \]

Subject to TOPF constraints

Step 5: Repeat with tap settings set as an integer parameter.
Step 6: If desired, repeat steps 2-5 using different values for the tolerable loss variation $\beta$. This allows the network operator to assess how the radius of uncertainty $\alpha$ is affected by changes in $\beta$, and therefore make more informed decisions about the appropriate operating point.

Step 7: Finish.

Using only an initial demand prediction, the results from steps 1-7 will provide the system operator with a range of set-points for the switch and tap settings that will ensure that the losses do not exceed the specified optimal value. Furthermore, the accuracy of the three-phase modelling provided by the TOPF will ensure that the network is operated in a safe manner at all times.

It is worth noting that it is not necessary to repeat steps 2-5. If the system operator knows the specific increase in losses that they are satisfied to tolerate, then only one value of $\beta$ needs to be assessed, and steps 2-5 need only be performed once. The $\alpha$ value obtained in step 4 is optimal due to the integration of the TOPF. It is possible to compute a unique value of $\alpha$ for each customer load, by adding a new dimension to the $\alpha$ variable, however, doing so would not provide any additional benefit. The smallest $\alpha$ would be the limiting value for the feeder, and that is what is captured by the formulation presented here.

The method is formulated as a NLP. The formulation has been implemented using AIMMS (AIMMS, 2015) optimisation modelling environment, and is solved using the non-linear programming solver CONOPT (CONOPT, 2000).

5.3 Test Case

5.3.1 Test Network

The network utilised in this work is a section of actual LV network provided by the Irish DSO, ESBN (ESB Networks, 2015). It represents a typical suburban network in Ireland. A detailed diagram of the network is given in Fig. 5.1. The network consists of two radial feeders, feeder $A$ and feeder $B$ shown by the dashed boxes in Fig. 5.1, that can be connected through a switch located at the end of
each feeder. In this work, when the switch is open the network will be referred to as *radial*, and when the switch is closed it will be referred to as *meshed*. The 400 kVA distribution transformers at the head of each feeder are assumed to have tap changing capabilities, with 32 tap settings each, 16 positive and 16 negative. *Feeder A* serves a total of 63 buses, 9 three-phase buses (including the MV and LV buses), and 54 single-phase customer buses. *Feeder B* serves a total of 60 buses, 8 three-phase buses, and 52 single-phase customer buses, giving a total of 123 buses for the network as a whole. Each customer load and single-phase cable is modelled individually on its corresponding phase, and is indicated by an arrow in Fig. 5.1. The three phase buses are named in Fig. 5.1 as $a1-a7$ and $b1-b6$.

![Feeder A and Feeder B diagrams](image)

**Figure 5.1:** 123 bus practical test network

### 5.3.2 Demand Profiles

Demand profiles were generated using the residential load modelling tool described in McKenna and Keane (In Press). The demand profiles were generated for a high load Winter day giving the worst case scenario for demand increases. The demand profiles are for a 24 hour period in 30 minute time steps, giving 48 time steps, and consist of apparent power, power factor, $Z$ load component, $I$ load component and $P$ load component values for each customer and time step. These demand profiles are used as the predicted values for apparent power.
demand. The magnitude of the apparent power is considered as uncertain as it has the most potential to cause issues, from a feeder perspective, if forecasts are incorrect.

5.3.3 Simulation Cases

The TOPF and IGDT method, as described in Section 5.2, is used to determine optimal and robust tap and switch settings for the test network, for a 24-hour period in 30-min time steps, with the objective of minimising active power losses. The uncertain variable is the apparent power demand, and there are two simulation cases. The first case represents LV network demand at present, and assumes that all customers have similar demand uncertainty. The second case assesses a future scenario where certain customers are involved in demand response and therefore have a more significant level of demand uncertainty than other customers. Both cases are discussed further in the following subsections.

**Full Uncertainty Case**

The full uncertainty case assumes that all of the apparent power demand is uncertain. The robustness problem is formulated as in Section 5.2.3 but with a larger number of IGDT iterations, using increasing values of the tolerable loss variation $\beta$. This allows the system operator to choose the operating point which has the required level of robustness against demand increases.

**Demand Response Uncertainty Case**

The demand response uncertainty case assumes that a number of customers on the test network are participating in a demand response scheme. Twenty five customers have been chosen at random points along the test network shown in Fig. 5.1. These customers all have flexible demand that can be controlled by a third party operator, when necessary, to alleviate system-wide issues. These customers are therefore more likely to have a higher level of uncertainty associated with their demand, so for this case it is assumed that only the demand
response customers’ apparent power demand is uncertain. Both the robustness and opportuneness functions are assessed in this case.

5.4 Results and Discussion

The following subsections present the results for the two test cases. These results primarily present what a system operator would be concerned with, e.g. the control variable set-points.

5.4.1 Full Uncertainty Case

This case assumes that all of the demand on the test network is uncertain. Fig. 5.2 shows the resulting radius of demand uncertainty $\alpha$ values for the corresponding tolerable loss variation $\beta$ values. Using the case where $\beta=2$ as an example, we can see that the corresponding value of $\alpha=0.74$. This means that if energy losses are permitted to increase to 3 times their predicted value, using the resulting tap and switch settings, demand can be up to 1.74 times its predicted value, and no network limits will be breached. It is clear that there is a somewhat proportional relationship between allowable active power loss energy increases and apparent power demand increases. However, the demand cannot increase as much as the losses can. In fact, the $\alpha$ values are less than half of the $\beta$ values, and this ratio decreases as $\beta$ increases further.

Fig. 5.3 depicts the status of the switch that connects feeders A and B, for each time step and each value of $\beta$. A dark box indicates that the switch is closed, while a light box represents the switch being open. The switch is predominantly in the closed position showing that the meshed configuration is optimal in terms of energy losses for the majority of time steps. There is a window of time during the morning, however, when the optimal switch position is open. The optimal configuration does not change as $\beta$ increases, as all of the load on the network increases uniformly.
The resulting tap settings for the tap changers on feeders A and B are given in Fig. 5.4. The results given are for the phase \( a \) tap settings when \( \beta = 0 \) and \( \beta = 2 \). Phases \( b \) and \( c \) and other \( \beta \) values were also calculated but are not given here. The difference in the tap settings between \( \beta = 0 \) and \( \beta = 2 \) is relatively small, with the tap settings for \( \beta = 2 \) set slightly higher than \( \beta = 0 \), due to voltage drops experienced as a result of the increase in demand. Comparing feeders A and B
it is clear that the tap settings are quite similar throughout the day. The only times that the tap settings on the two feeders are significantly different are when the switch is open, as shown in Fig. 5.3.

The optimal switch settings seen in Fig. 5.3 are reflected in the loss comparison presented in Fig. 5.5. This figure shows the percentage difference in the total energy losses between the fully radial network configuration and the optimal configuration (from Fig. 5.3), given in light grey, and the fully meshed configuration and the optimal configuration, given in black. It is clear that both fully radial and fully meshed operation produce higher losses than the optimal case, however fully radial operation gives losses approximately 1.6% higher than optimal for all values of $\beta$. This accounts for the largely meshed operation observed in Fig. 5.3.

Fig. 5.6 presents the aggregate apparent power load for each feeder, given by the solid line, as well as the actual imported apparent power at the source node of each feeder, given by the dashed line. The values shown here are for the meshed
configuration when $\beta = 0$. Generally, feeder $A$ imports more power than its load requires, which is to be expected due to losses. However, feeder $B$ imports less power than its load requires. In fact, even during times when the feeder $B$ load is larger than the feeder $A$ load, for example from 10:30-12:00 or 15:00-16:30, feeder $B$ still imports less than its load requirement. This indicates that, generally, feeder $B$ experiences higher losses than feeder $A$, which again explains why meshed is more often the optimal operation choice rather than radial for this network.

Using these results, the system operator can decide which tap and switch settings to use. If the system operator decides to operate the feeders using the settings for $\beta=0$, they are taking the biggest risk in terms of safe feeder operation, but will have the lowest loss value if the actual demand is equal to the predicted value. Conversely, if the system operator implements the settings for $\beta=2$, they are reducing their risk level but will incur higher losses if the actual demand is equal to the predicted value.

5.4.2 Demand Response Uncertainty Case

The demand response uncertainty case targets those 25 customers who are participating in a wider demand response scheme and assesses the uncertainty
associated with their apparent power demand. Due to the nature of demand response, i.e. load can be either increased or decreased depending on the needs of the system, both the robustness and opportuneness functions are assessed in this case. The resulting radius of demand uncertainty, $\alpha$, values are given for the corresponding tolerable loss variation, $\beta$, values in Fig. 5.7. The $\beta$ values are still in reference to the total network losses in this case. Values to the left of the dashed line represent the opportuneness function while values to the right represent the robustness function, with the zero value being the base case. There is more scope for robustness, as further increases in $\beta$ in the direction of opportuneness would require these customers to generate power, i.e. their load values would need to be negative, which would be possible if they had grid-connected micro-generation, however that is not considered in this work. It should also be noted that the $\alpha$ values are much higher in this case than in the full uncertainty case, as there are fewer customers with uncertain load, therefore their load can be

\textbf{Figure 5.6:} Aggregate apparent power demand for each feeder vs the apparent power imported at the source node of each feeder in the meshed configuration.
increased significantly more. In the figures that follow Fig. 5.7, $\beta$ values for the opportuneness function will be presented as negative values.

![Graph](image)

**Figure 5.7:** Radius of demand uncertainty $\alpha$ vs tolerable loss variation $\beta$

Fig. 5.8 shows the total demand for the demand response customers. The predicted demand is given by the solid line, the dashed line shows the demand for $\beta = -0.3$ and the dotted line shows the demand for $\beta = 2$. It is evident that for the losses to be reduced to 0.7 times their base value, i.e. $\beta = -0.3$, it is necessary for the demand response customers’ total demand to be close to zero. As seen in Fig. 5.7, allowing the losses to increase threefold, i.e. $\beta = 2$, means that the aggregate demand for these customers can be almost 4 times larger than the initial predicted value, i.e. $\alpha \approx 3$. It should be noted that it would be unlikely for 25 customers to have demand this high, however it illustrates the capabilities of the formulation.

The switch status for the demand response case is presented in Fig. 5.9. Unlike the fully uncertain case, there is more variation in the switch settings in this case due to the fact that only the demand response customers’ loads are considered uncertain. The difference between the load on *feeder A* and *feeder B* determines whether *meshed* or *radial* operation is optimal. *Radial* operation will be optimal when this difference is small, whereas *meshed* will be optimal when it is large. At certain time steps, although the $\beta$ values mean that the overall load is lower than in the base case, the difference between the *feeder A* and *feeder B* load is more
Chapter 5. Robust Operation Using Information Gap Decision Theory

Figure 5.8: Aggregate load for demand response customers for varying levels of tolerable loss variation $\beta$

significant, leading to *meshed* operation. This also occurs in reverse at higher $\beta$ values.

![Diagram](image.png)

**Figure 5.9:** Switch status for each time step for increasing values of the tolerable loss variation $\beta$
The resulting phase $a$ tap settings for the demand response case are given in Fig. 5.10. There is much more variation in the tap settings between $\beta=0$ and $\beta=2$ than previously seen in Fig. 5.4 for the full uncertainty case. Most of these variations are increases due to the increased level of demand. There is also a significant variation in the tap settings on feeders A and B. Both of these differences can be attributed to the switch settings shown in Fig. 5.9. There is generally more radial operation in this case in comparison to the previous case, and the switch settings change as $\beta$ increases, which does not occur in the fully uncertain case.

![Figure 5.10: Tap settings for phase $a$ of the tap changers on feeders A and B for $\beta=-0.3$, $\beta=0$ and $\beta=2$](image)

Fig. 5.11 shows the percentage difference between the radial and meshed losses and the optimal losses. In a similar fashion to the fully uncertain case, the percentage difference between the radial and optimal losses is much higher than that for the meshed and optimal. Deviation from the base case increases the difference between the radial and optimal losses regardless of what direction that deviation moves. The difference in the meshed and optimal losses increases
as $\beta$ increases, which is to be expected as the overall load increases. These results correlate to the switch settings seen in Fig. 5.9.

An alternative random selection of 25 customers was chosen for the demand response scheme and the formulation was performed again for the purposes of illustration. The resulting radius of demand uncertainty, $\alpha$, values are compared to the $\alpha$ values for the original customer selection in Table 5.1. The original values are given by $\alpha_1$, while the new values are called $\alpha_2$. It is clear that although the customers participating in the demand response scheme have changed, the $\alpha$ values do not vary significantly.

**Table 5.1:** Comparison of $\alpha$ values for two different sets of demand response customers

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Opportuneness</th>
<th>Base</th>
<th>Robustness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.82, -0.58, -0.37, -0.18</td>
<td>0.5</td>
<td>1.71, 2.36, 2.92</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.82, -0.59, -0.38, -0.18</td>
<td>0.99</td>
<td>1.77, 2.43, 3.02</td>
</tr>
</tbody>
</table>

It is worth noting that, for all of the above cases, the difference between the total losses when the tap settings are modelled as a continuous variable, and when they are modelled as an integer parameter, is zero. This is due to the fact that the difference between the continuous and integer tap settings is small and thus
has no effect on the feeder. This indicates that, for LV networks similar to the test network used in this work, and regulators with a large number of tap settings (32 in this case), continuous modelling of tap settings should be sufficient.

The results presented show that changes in demand, be it increases or decreases, can significantly vary the optimal operating point of a network. However, the switch and tap settings provided by the TOPF and IGDT method allow system operators to be sure that their network is robust against these changes. This means that when the set-points provided by the formulation are implemented, demand variations will not result in voltage, current, or power limit breaches, and will not increase the minimised losses more than the value specified by the tolerable loss variation.

5.4.3 Convergence

Convergence of the formulation was analysed by solving the IGDT TOPF problem using the CONOPT solver, for the full load uncertainty case, for 100 different predicted load scenarios, using a $\beta$ value of 1. The results in Table 5.2 show the convergence, as well as the iterations and computation time, for each of the 100 scenarios. The results show that all 100 scenarios converged, with a mean computation time of 325 seconds.

Table 5.3 shows the convergence status, iterations, time and $\alpha$ value for the full uncertainty case, using one predicted load value, for $\beta=1$, using three different NLP solvers that are available with AIMMS. The problem converged for all three solvers with similar computation times.

The convergence results show that, regardless of the predicted value of the uncertain parameter, the formulation does not present a significant computational burden, and can be solved in reasonable time using a range of different non-linear solvers.
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5.4.4 Validation

In order to validate the results, the full uncertainty case from Section 5.4.1 is considered at $\beta=2$. The results from Fig. 5.2 indicate that by implementing the resulting tap and switch settings, the demand can be up to 1.74 times the predicted value without increasing the losses by more than 3 times their predicted value. Three times the predicted value of losses gives $P_{LC} = 29.03 \text{ kWh}$. 500 load flow simulations were performed with the tap and switch settings held at the values given in Fig. 5.3 and Fig. 5.4, but with the demand varying between 1 and 1.74 times the predicted value in a normally distributed fashion. The total

Table 5.2: Convergence data for different predicted load scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Converged</th>
<th>Time (s)</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Y</td>
<td>317 31</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>342 42</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
<td>344 51</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Y</td>
<td>330 38</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Y</td>
<td>324 33</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Y</td>
<td>317 26</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Y</td>
<td>331 37</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Y</td>
<td>322 30</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Y</td>
<td>326 32</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Y</td>
<td>335 38</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Y</td>
<td>334 40</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Y</td>
<td>341 40</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Y</td>
<td>327 33</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Y</td>
<td>422 71</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Y</td>
<td>328 34</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Y</td>
<td>337 35</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Y</td>
<td>334 38</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Y</td>
<td>342 38</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Y</td>
<td>345 43</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Y</td>
<td>321 27</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Convergence data for different solvers

<table>
<thead>
<tr>
<th>Solver</th>
<th>Converged</th>
<th>Time (s)</th>
<th>Iterations</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONOPT 3.14V</td>
<td>Y</td>
<td>259.32</td>
<td>46</td>
<td>0.419</td>
</tr>
<tr>
<td>IPOPT 3.11</td>
<td>Y</td>
<td>302.189</td>
<td>25</td>
<td>0.419</td>
</tr>
<tr>
<td>KNITRO 9.1</td>
<td>Y</td>
<td>337.867</td>
<td>49</td>
<td>0.420</td>
</tr>
</tbody>
</table>
daily losses from each simulation were recorded and are shown in Fig. 5.12 as a histogram. The black dashed line shown in Fig. 5.12 indicates $P_{LC}$, the maximum allowable loss value of 29.03 kWh. It is clear that by implementing the IGDT results for the taps and switch, the losses for all 500 simulations are kept below the desired value of 29.03 kWh.

![Figure 5.12: Histogram of losses for 500 load flow simulations using robustness settings](image)

### 5.5 Conclusions

The work discussed here describes a new methodology to combat the problem of uncertainty on LV distribution networks. The TOPF method accurately models three-phase networks and their associated components, as well as providing optimal solutions for distribution system control variables. The IGDT method is capable of assessing robustness and opportuneness functions for an acceptable improvement or deterioration in the objective function. This method can allow network operators to make LV network decisions that will be robust against potential deviations, without adding a high level of complexity to the TOPF problem.

The formulation was tested using an actual suburban LV network to determine optimal and robust tap and switch settings. The results provide a range of operating points, with varying levels of robustness, for a network operator to
choose from. In particular, for the demand response case, where only a number of customers’ loads are modelled as uncertain, the switch and tap settings vary significantly with the level of robustness/opportuneness. The method is shown to provide control decisions that will keep losses as low as possible, while ensuring that significant deviations from the predicted load will not cause any unnecessary issues.

The method discussed here could be utilised for planning and operational applications for distribution networks. In particular, it would be highly beneficial for the management of DER by accounting for the uncertainty associated with these resources, e.g. solar forecast uncertainty for PV. The formulation described in this chapter could be used to provide optimal and robust operating points for these new technologies.
Conclusion

The issue of climate change has incited major changes to be implemented in the area of power systems. Renewable resources and related technologies are becoming more prevalent throughout the power system, and have more recently begun to emerge on distribution systems in the form of DER. Distributed resources could provide significant benefits to distribution networks, however, works discussed in Chapter 1 show that, if left unmanaged, DER could prove unfavourable for distribution assets and power quality. It is, therefore, crucial to be prepared with suitable solutions for the management and control of DER. The work in this thesis provides a number of solutions for the coordination of DER that not only model distribution networks in the most detailed manner, but also account for the uncertainties that are inherent to these networks and technologies.

The work in Chapter 2 describes a rolling load flow and optimisation methodology that is used to control the charging schedule for a number of EVs on a distribution feeder. Three-phase load flow simulations were performed and Jacobian matrices were extracted and inverted to be incorporated as network sensitivity matrices in an optimisation. The optimisation aimed to minimise the
cost of EV charging over a 24-hour period while ensuring that network limitations were not breached. The results showed that scheduling EVs to charge at low cost times provided a significant reduction in cost over an uncontrolled case, as well as eliminating voltage violations that were experienced with the uncontrolled case. A non-rolling case was also examined, and although it too resulted in reduced costs, it only delivered 80% of the energy required by EVs while the rolling methodology resulted in 100% of energy being delivered.

Expanding on the load flow and optimisation methodology in Chapter 2, an advanced TOPF formulation was developed in Chapter 3. The TOPF integrated the main current mismatch equations from the power flow method used in Chapter 2 as constraints in a single optimisation program environment. The formulation models the various elements that exist on distribution systems such as capacitors, voltage regulators and ZIP loads. It is also capable of modelling radial and meshed feeders. The TOPF was validated using the IEEE 123-node feeder in both its radial and meshed configurations. Percentage differences were negligible. A practical feeder was used as a test case for minimum loss and minimum unbalance objectives using PV. Results showed the potential benefits, such as loss and unbalance minimisation, that could be achieved with the use of the TOPF formulation.

The TOPF from Chapter 3 was also employed for the work discussed in Chapter 4. This chapter examined an operational planning approach to PV inverters. Optimal volt-var curves were determined by running a multi-scenario TOPF which had an objective of minimising voltage deviations from predetermined set-points. The resulting reactive power settings were plotted against the corresponding voltages and volt-var curves were fitted to these plots. Individual volt-var curves were determined for each PV system on a test feeder, as well as a universal curve that was uniform for all inverters. Validation of the resulting curves was performed by implementing them for a test day and analysing the results. Voltages deviations and unbalance were both shown to be reduced.

The importance of uncertainty has been emphasised in Chapter 2, however, the TOPF developed in Chapter 3 only provides deterministic solutions. In
Chapter 5 the IGDT uncertainty technique was integrated to the TOPF formulation to provide robust operational decisions, in a day ahead time frame. The IGDT method allows robustness and opportuneness functions to be assessed to determine how much the uncertain parameter can change for a tolerable change in the objective value. The work in Chapter 5 examined how much apparent power could vary by, for a range of tolerable active power loss increases and decreases. The method determined the optimal tap and switch settings for each tolerable loss limit. The results show that the method can provide decisions which will ensure the network is operated safely and that losses are kept below the desired value.

The work developed in this thesis has provided solutions that fulfil the necessary criteria for distribution network management. Distribution systems are modelled accurately using three-phase representations. The solutions have the flexibility to incorporate various forms of DER as well as existing distribution system components. Optimisation approaches ensure that results provided by the developed formulations yield the best possible outcomes. The inherent uncertainty associated with distribution networks and DER is considered using robust techniques. The combination of the aforementioned attributes means that these solutions provide a set of core building blocks, necessary for future distribution management systems.

6.1 Future Work

While the work undertaken during this thesis provides significant contributions in the form of distribution management solutions, the availability of such solutions naturally gives rise to ideas for new applications, providing scope for further research.

The work in this thesis has primarily focused on a typical day ahead time resolution of 30 minutes. Many forms of DER operate at much higher time resolutions such as 1 second or 1 minute. For example, the active power output of a PV system can reduce substantially with a passing cloud. In order to capture
these effects, highly granular models need to be examined. This could be achieved using the solutions developed in this thesis along with the 1 minute demand data that can be generated using the method in McKenna and Keane (In Press).

Integrating DER to distribution systems is not a trivial task and is made more difficult by the fact that there are various different types of DER that may need to be assessed. Concerning DER planning, DSOs are generally interested in the penetration level at which that resource will begin to cause adverse effects, or in other words, the hosting capacity of feeders for a particular resource. This knowledge allows DSOs to decide the appropriate course of action, e.g. time of use tariffs, active DER control, or network reinforcement, and be aware of when this course of action will require implementation. The characteristics of distribution systems can vary greatly from feeder to feeder, so hosting capacities usually need to be assessed on a feeder by feeder basis, for a given form, or multiple forms, of DER. Determining the hosting capacity of a particular feeder for a particular resource can be achieved using stochastic techniques as in Rylander and Smith (2012), however optimisation methods may provide a simpler solution. The TOPF method discussed in Chapter 3 could be employed to determine generic hosting capacities for feeders. This would not be a technology specific study but could assess general ranges of hosting capacities for additional generation and/or load. This could provide DSOs with awareness of the limitations of their feeders, regardless of what the technology may be.

The IDGT TOPF from Chapter 5 has the potential to provide solutions for DER in an operational capacity. The IGDT formulation could be used to account for the uncertainties associated with PV, EVs, heat pumps, etc. and make robust decisions regarding their operation. Multi-objective optimisation could also be examined with the IGDT TOPF as in Soroudi and Ehsan (2013) for the purposes of assessing multiple uncertain parameters. Furthermore, the rolling approach from Chapter 2 could be combined with the TOPF to produce a real-time operational solution for uncertain distributed resources.

As discussed in Chapter 1, CVR is the theory that reducing distribution feeder voltages will result in a reduction in energy and peak demand. Although
many CVR trials have been carried out (Kennedy and Fletcher, 1991; Lefebvre et al., 2008; Diskin et al., 2012), the lack of detailed end use data makes it difficult to determine whether the observed energy savings can be attributed to reduced voltage levels. The success of CVR depends entirely on the voltage dependency of the load. One of the main flaws with the CVR concept is the underlying assumption that the majority of the load is voltage dependent and thus if voltage is reduced, load will also reduce. Although this may have been the case in previous times, particularly with the dominance of incandescent lighting, there has been a substantial shift in the types of appliances that are being utilised at present. A number of these appliances, primarily electronic devices such as computers and televisions, operate closer to a constant power type of load, which draws more current at lower voltages and consequently increases demand (Bokhari et al., 2014). The demand profiles generated using the comprehensive load modelling methodology described in McKenna and Keane (In Press), include the ZIP representation of the load at each time step. There is therefore scope to use the TOPF from Chapter 3 to determine the optimal tap settings over the course of a day, with the objective of minimising load, or a combination of load and losses.

The distribution management methodologies described in this thesis are all centralised approaches. Centralised control assumes complete knowledge of the system and its conditions. Although centralised schemes provide the most accuracy and detail, there is a significant level of feeder knowledge required. Furthermore, substantial communication infrastructure is needed between the system operator and the resources being controlled. Both knowledge and communication requirements could be reduced using a local or decentralised control approach. Local control assumes knowledge of local conditions only and decisions are made using only the local information. Although local control is less demanding in terms of communications, there would likely be a trade off in terms of optimality and accuracy. A local control scheme could be developed in the future, and compared against the centralised methods described in this thesis to assess which would be more appropriate in a distribution context.
In recent years, technologies known as real-time digital simulators (RTDS Technologies, 2015; Opal RT Technologies, 2014), have been developed which allow for electric power systems to be modelled and simulated in real time. The real time simulation allows not only for testing and validation of power system control, but also for examination of hardware devices in the loop. The hardware in the loop capability refers to replacing a simulated model of a device with the actual physical component. This interface allows for testing of both how devices would react to control signals, as well as how the simulated power system would react to the output of the device. A real time simulator could be employed to validate the control strategies developed in this thesis, as well as test hardware such as PV systems, inverters, and batteries. Examination of hardware devices could also be used to validate or improve the models of these distributed resources.

It is evident that there is a great deal of opportunity for further research in the area of distribution systems and DER. The ideas and solutions developed in this thesis can form the basis for potential future work. Applying the developed management schemes could ensure that, regardless of the technologies that emerge, the safe and reliable operation of distribution networks can be maintained in the future.


References


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Rylander, M. and Smith, J. (2012). Stochastic analysis to determine feeder hosting capacity for distributed solar PV.


United States Environmental Protection Agency (August 2015). Carbon pollution emission guidelines for existing stationary sources: Electric utility generating units.


During the course of my PhD, I had the opportunity to undertake a three month internship with the Irish DSO, ESB Networks, as a member of their Smart Networks group. The internship took place between June and August 2012 and focused on the analysis of an ongoing CVR field trial. The primary circuit under examination was a 20 kV rural circuit, consisting of two sections that were fed from different substations, and separated by a normally open point. The CVR trial was in place between September 2011 and September 2012, and operated on a day on day off schedule, i.e. reduced voltage one day and nominal voltage the following day. The voltage was controlled by tap changers at the substations with voltages typically being lowered by 1%-2% on the previous nominal voltage day. There were numerous monitoring devices in place to collect data, from substation level down to customer connection points. The CVR factor is defined as the percentage energy decrease per 1% reduction in voltage, as shown in (A.1).

\[ CVR_f = \frac{\% \text{Energy Decrease}}{\% \text{Voltage Decrease}} \]  

(A.1)

CVR factors were calculated on an hourly, daily, weekly and monthly basis to assess the impact of CVR, and ultimately determine whether it is a scheme...
that should be deployed on a wider scale. Sample results are given in Fig. A.1 and Fig. A.2. Fig. A.1 shows the resulting average hourly CVR factors for the two sections of the rural feeder. Positive values indicate energy reductions, while negative values denote energy increases. It is evident that, generally, CVR factors are above zero for both circuit sections, which means that, overall, energy is being reduced. Section B has relatively similar CVR factors at all times of the day, with all values being positive and close to 1. Section A has more variation, with high CVR factors during the night/early morning, and low factors during the day/early evening. This trend may be due to a higher level of resistive loads, such as lighting, cooking, and heating, being used during the night which are most sensitive to voltage changes.

![Figure A.1: Average Hourly CVR Factors for Trial Circuit](image)

The average daily CVR factors for the two circuit sections are given in Fig. A.2. The two circuit sections have similar values for all days except Thursday and Friday which have significantly different values. The variation highlights the fact that the impact of CVR is heavily dependent on the feeder that it is being implemented on, and the overall load composition of that feeder. Although overall the results from this trial were positive, with an average CVR factor of 1.25, it is difficult to ascertain whether the energy savings observed are actually a result of reduced voltage or just a reduction in customers’ consumption. Furthermore,
changes to the load mix in the future, due to the prevalence of electronic devices and the decline of incandescent lighting, could mean that CVR may not be as effective in years to come.

![Figure A.2: Average Daily CVR Factors for Trial Circuit](image-url)
At the beginning of 2015, I travelled to Knoxville, Tennessee to commence a three month internship with the Electric Power Research Institute. I was a member of the Power System Studies team and carried out research on a demonstration project which examined how the capabilities of PV smart inverters could be employed in distribution systems. The work consisted of developing a methodology for determining optimal volt-var curves for a utility scale PV inverter. The method utilises optimisation methods along with distribution feeder models to produce optimal volt-var curves for specific objectives. Various combinations of load and solar levels were examined, with both the load and solar level being increased incrementally between their respective minimum and peak values. An optimisation was then be performed for each combination of load and solar which determines the optimal reactive power setting for that particular operating point. Within the optimisation, the OpenDSS platform provides the three-phase power flow results, which determine the solution for the selected objective and constraints, for the particular reactive power setting. Optimal reactive power solutions and corresponding voltages are subsequently used to produce optimal volt-var curves.
The feeder under examination was an actual feeder that was provided for the study by a US utility. The feeder has one large scale PV system which has a maximum active power output of 1 MW, with an inverter which is rated slightly higher at 1.036 MW. This means that the inverter is capable of providing reactive power support even when the active power output is at its maximum. The objectives that were examined were aiming to minimise: voltage variation at the PV connection bus from 1.02 pu, active power losses, and active power consumption (load and losses). The results for each of these objectives are shown in Fig. B.1, Fig. B.2, and Fig. B.3 respectively. Each point on these figures represents the optimal reactive power setting for a given solar and load level. The reactive power is given as a proportion of the available reactive power which is based on the inverter size and the current active power output. The colour of each curve represents the solar level, which is the per unit output of the 1 MW PV system.

The results for the minimum voltage variation objective are given in Fig. B.1. The general trend shows capacitive operation for voltages up until the set-point of 1.02 pu, after which inductive operation begins. It is also evident that the full reactive capability is only required at the highest solar level, due to the inverter reactive power being limited by the active power output of the PV system at certain voltages.

Figure B.1: Volt-var curves for each solar level for the minimum voltage variation objective
The results for the minimum loss objective are presented in Fig. B.2. It is evident that for minimising losses the optimal reactive power settings are primarily capacitive, becoming somewhat inductive at higher voltages. This is as expected, as increasing the voltages will decrease the current, which will in turn decrease the line losses on the feeder, as line losses are proportional the square of the current. The optimal reactive power settings become less capacitive as the feeder load decreases (given by voltage increases in Fig. B.2), indicating that the transformer losses are a more significant proportion of total losses in this region.

Figure B.2: Volt-var curves for each solar level for the minimum loss objective

Figure B.3 shows the optimal reactive power settings for the minimum consumption objective. For high solar levels, full inductive operation is employed across the range of voltages. This is due to the voltage dependency of the load. The active power load on this feeder has a conservation voltage reduction (CVR) factor of 0.8, which means that for a 1% variation in voltage, the active power load will vary by 0.8%. Therefore, the optimal reactive power setting for minimising consumption will reduce voltages as much as possible. The lower the solar level and the higher the load (given by low voltages in Fig. B.3), the less reactive power absorbed, due to the voltage constraint.

The results show that the shape and main operating region of the optimal volt-var curve depends greatly on what the objective is. Depending on the needs of
Figure B.3: Volt-var curves for each solar level for the minimum consumption objective

the system, and availability of communications, smart inverters could be provided with a different volt-var curve for each solar level, or one approximation curve, which shows the average trend from all solar levels. The work undertaken in EPRI provided significant insight and was the primary influence for the work described in Chapter 4.