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<td>Williams, David; O Máirtín, Dónal; O'Keeffe, James</td>
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Expected prevalence of trading restriction in the absence of *Mycobacterium bovis* under the Bovine Tuberculosis Eradication scheme

D.H. Williams, D. O’Mairtin & J.J. O’Keeffe

**Introduction**

The Bovine Tuberculosis Eradication (BTE) scheme for the tuberculin testing cattle in the Irish Republic aims to test all cattle once a year. A trading restriction is enforced and 60-day follow-up testing carried out in cases of reactor disclosure, the restriction being lifted following two clear herd tests. The specificity of the test as administered in Ireland is believed to be in the range 0.998 to 0.999 (Monaghan *et al.*, 1994). For a herd free of *Mycobacterium bovis* infection, and specificity 0.9990, 0.9993 and 0.9996, the likelihood of being restricted for some period during a year due to the disclosure of reactors can be calculated as a function of herd size. For a population with known herd size distribution it is then possible to derive the expected prevalence of herd restriction. O’Keeffe (1993) calculated the annual prevalence of herd restriction in the absence of *M. bovis* following tests on clear herds. Here we take into account the contribution of follow-up reactor re-tests. In order to apply the results to a population of herds where the herd size distribution is known, a comparison is made with the observed data from the East Offaly Badger Research Project (EOP), 1988–1995 (Ó Máirtín *et al.*, 1998).

**Definitions**

Under the BTE scheme all herds are tested at regular intervals and pass from restricted to non-restricted status on the basis of the tuberculin test results. We consider the eight states presented in Table 1, each state representing a possible status of a herd during a 60-day period. In practice herd tests will not take place exactly according to a strict timetable; the interval between two annual tests may be more or less than 12 months, and on disclosure of a reactor the interval to the reactor re-test may not be exactly 60 days. Consequently what is modelled here is an idealisation of normal practice.

**Table 1. States representing testing and herd restriction status prior to any scheduled test.**

<table>
<thead>
<tr>
<th>State</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$C_0$</td>
<td>the herd is unrestricted and subject to its annual test</td>
</tr>
<tr>
<td>$C_i$, $i = 1,...,5$</td>
<td>the herd is unrestricted in the $i$-th period following the annual test</td>
</tr>
<tr>
<td>$R_0$</td>
<td>the herd is restricted and a reactor was found in the previous period</td>
</tr>
<tr>
<td>$R_1$</td>
<td>the herd is restricted and no reactors were found at the previous period</td>
</tr>
</tbody>
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ERAD/TEAGASC 50 Tuberculosis Investigation Unit, UCD
Transitions
Under the protocol of the testing scheme the following are possible transitions from one state to another at the end of a 60-day period:

- \( C_0 \rightarrow R_0 \) if a reactor is found
- \( C_0 \rightarrow C_1 \) if no reactors are found
- \( C_i \rightarrow C_{i+1} \) for \( i = 1, \ldots, 4 \)
- \( R_0 \rightarrow R_0 \) if a reactor is found
- \( R_0 \rightarrow R_1 \) if no reactors are found
- \( R_1 \rightarrow R_0 \) if a reactor is found
- \( R_1 \rightarrow C_3 \) if no reactors are found

\( R_1 \rightarrow C_3 \) if no reactors are found since a herd then passes to \( C_0 \) after 180 days. This corresponds to the six-month check test.

Transition matrix

\[
M = \begin{bmatrix}
R_0 & R_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 \\
R_0 & p & q & 0 & 0 & 0 & 0 & 0 \\
R_1 & p & 0 & 0 & 0 & q & 0 & 0 \\
C_0 & p & 0 & 0 & q & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
C_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
C_5 & 0 & 0 & 1 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]

Steady-state probabilities
The long-term probabilities, \( p_i, i = 1, \ldots, 8 \) of a herd being in state \( i \) are solutions to the equations \( \Sigma_i p_i = 1 \) and \( M^T P = P \) where \( P = (p_1, p_2, \ldots, p_8)^T \). The solutions are:

\[
p_i = x_i / \Sigma j x_j \text{ where } x_1 = p/q^3, \quad x_2 = p/q^2, \quad x_3 = x_6 = x_7 = x_8 = 1/q, \quad x_4 = x_5 = 1.
\]

After the testing regime has been in operation for a sufficiently long time the proportion of herds in each of the eight states would approximate these long-term probabilities.

Transition probabilities
A herd enters restriction on the disclosure of one or more reactors. Assuming independence of animals in disclosing a reactor, a herd of \( n \) animals will enter restriction with probability

\[
p = 1 - \text{spec}^n
\]

where spec is the specificity of the test.

For example, with \( n = 50 \) and spec = 0.999, \( p = 0.049 \), so that the probability of no reactors at a herd test is \( q = 1 - p = 0.951 \), in this case.

The transition matrix \( M \) is an \( 8 \times 8 \) matrix where the \((i, j)\) entry is the probability of a herd (of a given size) moving from state \( i \) to state \( j \). Under the protocol of the BTE scheme \( M \) is given by:

Restriction prevalence
To calculate expected restriction prevalence for herds of a given size, note that a herd is restricted at a period during a year if, and only if, it passed through one of the sequences of states listed in Table 2, where \( c = 1 / \Sigma j x_j \).

Multiplying each sequence of the probability by the long-term probability of the first state in the sequence and summing gives the expected restriction prevalence:

\[
\text{prevalence} = cp \{ 2 + 4/q + 1/q^2 + 1/q^3 \}.
\]
East Offaly Project
In the Project Area of the EOP there has been a reduction in the level of *M. bovis* in the cattle population following removal of badgers during the period (Ó Máirtín et al., 1998). It is informative to compare the prevalence of trade restriction in this area with the levels expected in the absence of infection. For a population of herds these are calculated using the formula in the last section for each herd size in the population and then averaging. The resulting values should be lower that the observed levels of restriction prevalence. In this way information can be obtained about values of test specificity consistent with the data.

Both observed and expected prevalences are shown in Figure 1 for a range of values of specificity.

The expected values calculated for the East Offaly Project Area increase over the years. This was due to increasing herd size as a result of a trend towards intensification (Sheehy et al., 1991). Table 3 shows mean herd sizes based on test results. The observed prevalence decreased from 1989 through 1995, to a level that is approximately the expected prevalence in the absence of disease for specificity 0.9996.

**Table 2. Sequences of states involving part or all of a trade restriction.**

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Probability</th>
</tr>
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<tbody>
<tr>
<td>R₀ → ...</td>
<td><em>p₁</em> = <em>c</em> / <em>q</em>³</td>
</tr>
<tr>
<td>R₁ → ...</td>
<td><em>p₂</em> = <em>c</em> / <em>q</em>²</td>
</tr>
<tr>
<td>C₀ → R₀</td>
<td><em>p₃</em> <em>p</em> = <em>c</em> / <em>q</em></td>
</tr>
<tr>
<td>C₁ → C₂ → C₃ → C₄ → C₅ → C₀ → R₀</td>
<td><em>p₄</em> <em>p</em> = <em>c</em></td>
</tr>
<tr>
<td>C₂ → C₃ → C₄ → C₅ → C₀ → R₀</td>
<td><em>p₅</em> <em>p</em> = <em>c</em></td>
</tr>
<tr>
<td>C₃ → C₄ → C₅ → C₀ → R₀</td>
<td><em>p₆</em> <em>p</em> = <em>c</em> / <em>q</em></td>
</tr>
<tr>
<td>C₄ → C₅ → C₀ → R₀</td>
<td><em>p₇</em> <em>p</em> = <em>c</em> / <em>q</em></td>
</tr>
<tr>
<td>C₅ → C₀ → R₀</td>
<td><em>p₈</em> <em>p</em> = <em>c</em> / <em>q</em></td>
</tr>
</tbody>
</table>

**Table 3. Mean herd sizes (1988 to 1995) of herds tested in the Project Area of the East Offaly Project.**

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Mean herd size</td>
<td>48.9</td>
<td>51.2</td>
<td>53.8</td>
<td>55.2</td>
<td>55.2</td>
<td>56.2</td>
<td>59.1</td>
<td>60.9</td>
</tr>
</tbody>
</table>
Figure 1. Expected prevalence of trading herd restriction in the absence of *M. bovis* infection for tuberculin test specificity 0.9990, 0.9993, 0.9996 and as observed in the Project Area of the EOP 1988 to 1995.

Trading restriction prevalence

Conclusions
1. The expected prevalence of trade restriction in the absence of *M. bovis* under the BTE scheme for tuberculosis testing cattle herds can be calculated in a way that takes into account the effects of reactor re-testing, etc.
2. The calculation emphasises the importance of the herd size distribution in assessing observed prevalence of trade restriction.
3. Prevalence in 1995 in EOP Project Area would suggest a near absence of disease and/or test specificity of at least 0.9996.

References

