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Authors(s)	Harris, Niall K.; O'Brien, Eugene J.; Wilson, Simon P.
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A MIXTURE MODEL FOR PREDICTING PATTERNS OF SPATIAL REPEATABILITY IN HEAVY VEHICLE FLEETS

Niall K. Harris (*corresponding author*)
School of Architecture, Landscape & Civil Engineering, University College Dublin
Dublin, Ireland
niall.harris@ucd.ie

Eugene J. OBrien
School of Architecture, Landscape & Civil Engineering, University College Dublin
Dublin, Ireland
eugene.obrien@ucd.ie

Simon P. Wilson
Department of Statistics, Trinity College Dublin
Dublin, Ireland
swilson@tcd.ie

Abstract

This paper presents a statistical heavy vehicle fleet model for predicting patterns of statistical spatial repeatability (SSR), i.e., the mean pattern of dynamic tyre force applied to a section of pavement by a large number of similar vehicles. Data from a Multiple Sensor Weigh-in-Motion (MS-WIM) system, collected for a sufficiently large number of vehicles, can be used to identify and measure SSR. A Bayesian analysis technique is used to infer the statistical distributions of fleet properties, given measured axle forces from MS-WIM data. The topic is introduced with the simple example of using the technique to predict distributions of axle weights, based on simulated MS-WIM measurements. The statistical Mixture model presented herein builds on previously presented models to add the necessary complexity and flexibility to represent the bimodal nature of truck fleets (e.g. the presence of both unladen and laden vehicles in the population). The model is numerically validated using simulated MS-WIM data to condition the Bayesian analysis.

INTRODUCTION

Statistical spatial repeatability (SSR) is an extension to the well known concept of spatial repeatability (O'Connor et al. 2000). SSR is the hypothesis that the mean of many patterns of dynamic tyre force applied to a pavement surface is repeatable for a fleet of trucks of a given type. This has significant implications for the response of flexible pavements, since the presence of areas of consistently high dynamic force at certain locations on a pavement will increase its' susceptibility to damage. Deterministic whole-life pavement performance models (WLPPMs), such as that described by Collop & Cebon (1995), account for the cumulative effects of dynamic heavy vehicle tyre forces on pavements by specifying the pattern of SSR as an input to the model. Clearly then, improving the general accuracy of predictions of SSR to match real-life fleet behaviour will better inform the subsequent predictions of the WLPPM.

It has been shown previously that information about the dynamic ride characteristics of a fleet of heavy vehicles may be statistically inferred from measurements of the dynamic tyre forces imparted by the fleet (Harris 2007, Wilson et al. 2006). Using a Bayesian statistical inference technique, the statistical distributions of unknown parameters are found for a dynamic heavy vehicle model, chosen to represent that fleet and thus predict SSR. The approach requires large numbers of dynamic force measurements, upon which the inference is conditioned. In practice, such measurements may be obtained from multiple sensor weigh-in-motion (MS-WIM) sites, which also provide relevant additional information regarding the vehicle fleet such as axle configurations, axle spacing and vehicle velocities. A description of the road profile at and on the approach to the MS-WIM site is also necessary. Once the statistical distributions of the dynamic model parameters have been found, patterns of SSR can then be predicted for any other road, assuming that the other road is traversed by a similar fleet). The model described uses relatively simple unimodal Gaussian distributions for a range of unknown quantities such as suspension stiffness, axle mass, damping, etc. However, it is likely that the statistical distributions of these parameters will be more complex due to factors such as the presence of multiple suspension types (e.g. air, steel, etc.) or due to non-Gaussian distributions of truck weights (see Fig. 1). This paper presents a fleet model which seeks to address these complexities by employing mixture distributions to model the unknowns. The model presented also accounts for correlation between vehicle mass and effective suspension stiffness which is observed by Fancher et al (1986).

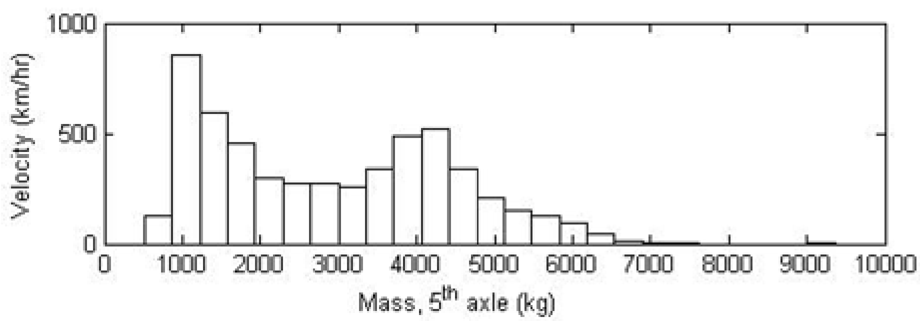
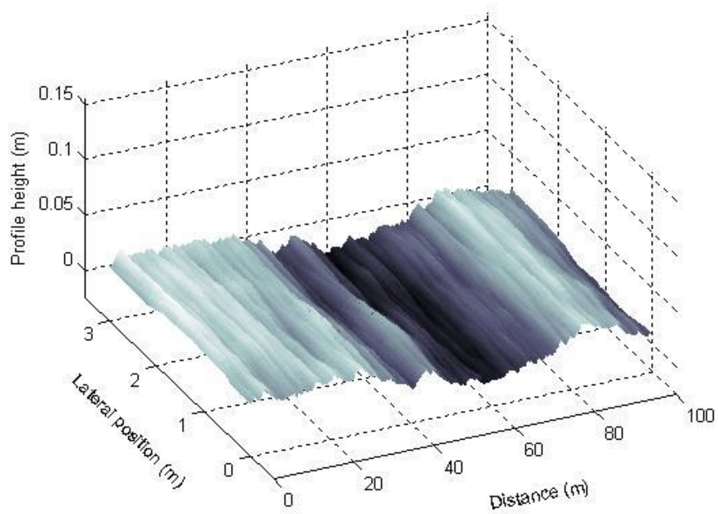


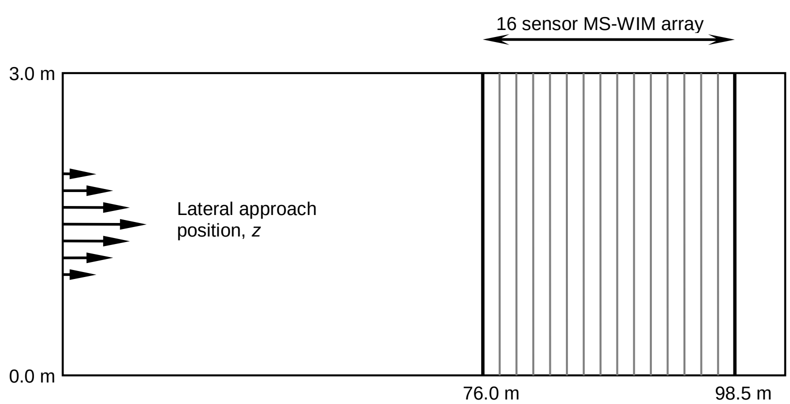
Figure 1 Evidence of non-Gaussian truck weight distributions (*from* Harris 2007)

Problem description

Consider the configuration shown in Fig. 2, representative of a road surface incorporating a 16 sensor MS-WIM installation. Measurements of heavy vehicle tyre forces are recorded at equidistant locations between longitudinal positions of 76 m and 98.5 m along an artificially generated three-dimensional road surface profile, AS1. The profile is generated to represent a pavement of good highway quality, with a roughness coefficient, $G_D(n_0)$, of 64 x 10^{-6} m^3/cycle. It is assumed that heavy vehicles traverse this section in the longitudinal direction with a statistically distributed lateral position, represented by the arrows. Use of a three-dimensional surface allows for the consideration of the effects of lateral approach, providing a varied but correlated excitation to the passing vehicles. It is assumed that the lateral position on the pavement is constant for each individual vehicle run and does not vary between sensors. Road surface AS1 is simulated using an inverse Fourier transform method, outlined by Cebon & Newland (1983). The length of generated profile also subjected to a three-dimensional moving average filter to simulate the envelopment of short wavelength disturbances by the tyre contact patch (Sayers and Karamihas 1996). A base wavelength of 0.25 m is used.



(a) 3-D road surface profile, AS1



(b) Geometry of sensor array

Figure 2 MS-WIM configuration

The goal of the analysis is to predict the pattern of SSR for a fleet of heavy vehicles by statistically inferring (based upon an original set of observed tyre force measurements, taken in this case from the MS-WIM sensor array) the characteristics of a dynamic fleet model. This analysis builds upon previously presented fleet models (Harris 2007, Wilson et al 2006) by allowing for the presence of both mixtures of populations and correlation between unknown parameters.

VEHICLE MODELLING

A linear quarter-car model, shown in Fig. 3, is used to simulate the force applied to a pavement as a heavy vehicle axle travels at a given speed. The model is used due to its suitability in representing the two fundamental modes of heavy vehicle axle ride – axle hop and body bounce – whilst remaining relatively simple to implement using numerical integration. As such, the quarter-car is also suitable for analyses such as the one presented herein, which require a large number of vehicle model evaluations to be performed.

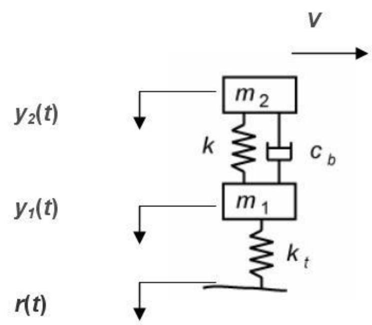


Figure 3 Quarter-car vehicle model

The quarter-car model travels at constant velocity, v, has two degrees-of-freedom, corresponding to the body bounce, $y_2(t)$, and axle hop, $y_1(t)$, vertical motions. The vehicle is excited by pavement profile, $r(t)$, which is determined by the vehicle lateral position/location, z. There are six parameters in the quarter-car pavement interaction model, sprung mass, m_2, unsprung mass, m_1, suspension spring stiffness, k, tyre stiffness, k_t and suspension damping, c_b. The equations of motion governing the quarter-car vehicle are:

$$m_2 \ddot{y}_2(t) + k\left(y_2(t) - y_1(t)\right) + c_b\left(\dot{y}_2(t) - \dot{y}_1(t)\right) = 0 \tag{1}$$

$$P - m_1 \ddot{y}_1(t) + k\left(y_2(t) - y_1(t)\right) + c_b\left(\dot{y}_2(t) - \dot{y}_1(t)\right) - F(t) = 0 \tag{2}$$

where P is the total static axle weight and $F(t)$ is the tyre force imparted to the pavement, given by:

$$F(t) = k_t\left(y_1(t) - r(t)\right) \geq 0 \tag{3}$$

Since the model is required to simulate the measurement of force at a given WIM sensor location, the simulated force of an arbitrary axle from the fleet (axle n) at one of the 16 sensor locations, m, is written as:

$$F_{nm} = F_n(t_m) \tag{4}$$

where $F_n(t)$ is the time history of impact force for the n^{th} truck, given by Eqn. (3), and t_m is the time the axle passes the m^{th} sensor location.

Fleet modelling approach

The model presented seeks to describe the behaviour of fleets of vehicles, with the dynamic parameters of the model conditioned by sets of measured data from MS-WIM installations. Such installations, such as the WIM-Hand installation in the Netherlands (van Loo 2001), allow for the segregation of measured data into distinct sub-populations which are easily analysed. For example, measured data may be segregated by vehicle class (e.g. 5 axle vehicles, 6 axle vehicles, etc.), axle number (e.g. steer axle, drive axle, etc.) and so on. The approach taken here is to analyse sets of MS-WIM data by vehicle type and axle number, in order to reduce the problem to one with less unknown parameters which can be well represented by the quarter-car model. By performing this analysis for each vehicle type, an informed model of heavy vehicle fleet behaviour may subsequently be built up.

BAYESIAN STATISTICAL INFERENCE

Given dynamic force data for a truck at different points on the road surface, it is possible, by effectively inverting numerically the quarter-car model, to infer the variables of the quarter-car that are most consistent with these data. Such fitting can be attempted through a wide variety of numerical methods such as, for example, minimising least squares errors using gradient descent or other optimization approaches. However, by instead conducting a similar analysis for a large number of similar vehicles simultaneously, it becomes possible to extract information about the overall distributions of unknown parameters for the whole vehicle fleet.

A Bayesian approach using hierarchical modelling (Gelman et al 2004) is adopted for this problem. Three stages are followed: probability model specification, statistical inference and prediction/model checking. A probability model is defined for the data (tyre forces from MS-WIM measurements) in terms of the quarter-car model variables (the unknown inputs), and subsequently the inferred probability distributions (known as the posterior distributions) are described. For the inference, the posterior distribution is simulated using Markov Chain Monte Carlo (MCMC) numerical methods. Finally, predictive distributions are plotted for quantities of interest and used to predict future patterns of SSR from the fleet (and to check that the fitted model is consistent with the data).

Assumptions

In order to reduce the complexity of the problem, a number of assumptions are made when specifying the fleet model. It is implicitly assumed that the availability of MS-WIM data will allow a high-quality estimate of the total mass, $m_1 + m_2$, for a given truck by averaging the sensor measurements or using a more elaborate algorithm (Dolcemascolo 1999). Further, moderate variation of the unsprung mass, m_1, will not induce significant change in the dynamic behaviour of the model and so it is assigned a constant value. Each estimate of m_2 is thus obtained by subtracting the constant m_1 value from the estimate of total mass. Further, the velocity of any given truck may also be determined from the MS-WIM measurements and is thus assumed known. This reduces the number of unknown parameters in the quarter car model to four: the suspension stiffness, k, tyre stiffness, k_t, suspension damping, c_b, and lateral approach position, z.

Stage I - Fleet probability model specification

For a fleet of heavy vehicles, the four unknown quarter-car parameters are assumed to independent, identically distributed (iid) random variables distributed according to the specified fleet probability model. The Bayesian hierarchical modelling approach (see Fig. 4) requires that for a fleet of N trucks, the measured MS-WIM forces (e.g. the data, $F^o{}_n$; $n = 1,2,\ldots,N$) for each n^{th} truck must be related to the corresponding unknown quarter-car parameters (the truck parameters, e.g. k_n, k_{tn}, c_{bn} and z_n). These in turn must be related to the so-called *hyperparameters*, which are means, variances, etc. that describe the overall distributions of the quarter-car parameters. This approach is necessary because it is not possible to relate the observed data (e.g. the MS-WIM measurements) directly to the hyperparameters, which are sought using statistical inference.

Lowest tier of hierarchy – Observed forces. Each instance of an observed axle force, $F^o{}_{nm}$, for truck n at sensor location m is assumed to be a Gaussian distributed random variable:

$$F^o{}_{nm} \sim \mathrm{N}(F_{nm}, \varepsilon^2 F_{nm}^2) \tag{5}$$

where F_{nm} is again the axle force according to the quarter-car model, which is a function of the quarter-car parameters, vehicle velocity and the road profile input. The relative variance is given by parameter ε^2 – this is to account for potential errors in measurement at the sensors, or discrepancies between the quarter-car model and its real life counterpart.

Middle and upper tier of hierarchy – Truck parameters and hyperparameters. The distributions of c_b and k_t are assumed to be univariate and independent of the other variables, consisting of bimodal mixtures of lognormal distributions. The distribution of lateral approach position, z, is assumed to be a simple Gaussian distribution. Finally, suspension stiffness, k, is most likely positively correlated to the sprung mass, m_2 (Fancher et al 1986), and as such, both parameters are assumed to follow a mixture distribution consisting of two bivariate lognormal modes. As it is assumed that m_2 is known, the distribution of k may subsequently be expressed as a univariate conditional distribution consisting of a mixture of two lognormals. Thus:

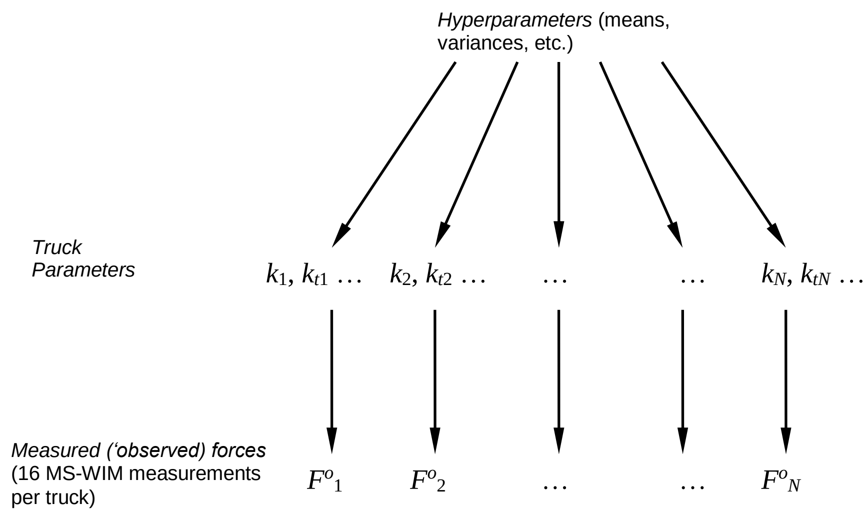


Figure 4 Hierarchical fleet modelling approach

$$
\begin{aligned}
&k_n \mid m_{2n} \sim \text{Mixture}(\mu_{(k|m_2)_1}, \mu_{(k|m_2)_2}, \sigma^2_{(k|m_2)_1}, \sigma^2_{(k|m_2)_2}, \lambda_k) \\
&k_{tn} \sim \text{Mixture}(\mu_{kt1}, \mu_{kt2}, \sigma_{kt1}{}^2, \sigma_{kt2}{}^2, \lambda_{kt}) \\
&c_{bn} \sim \text{Mixture}(\mu_{cb1}, \mu_{cb2}, \sigma_{cb1}{}^2, \sigma_{cb2}{}^2, \lambda_{cb}) \\
&z_n \sim \text{N}(\mu_z, \sigma_z{}^2)
\end{aligned}
\tag{6}
$$

for $n = 1,\ldots,N$.

where μ are mean hyperparameters, σ^2 are variances and λ values are scaling parameters denoting the relative proportions of the mixtures.

Stage II & III - Statistical inference and Model checking/prediction

Having specified completely the fleet probability model, the joint posterior distribution may be written using Bayes Law (Gelman et al 2004). Denoting the array of hyperparameters by ϕ, the matrix of truck parameters, **T** and the set of observed forces, $\mathbf{F^o}$, this is given by:

$$
p(\phi, \mathrm{T} \mid \mathbf{F^o}) \propto p(\mathbf{F^o} \mid \mathrm{T}, \varepsilon^2 \mathrm{F}^2)\, p(\mathrm{T} \mid \phi)\, p(\phi) \tag{7}
$$

This distribution represents the combined probability of every unknown quantity in the statistical inference problem, comprising of a very large number of unknowns. Each of the N trucks will have four unknown quarter-car parameters and three so-called indicator variables (which are binary variables denoting the mode that each mixture distributed quarter-car parameter comes from). This is in addition to the 17 unknown hyperparameters outlined in Eqn. (6), bringing the total number of unknowns to $7N$ +17.

Fortunately, this complex distribution can be simulated using Markov Chain Monte-Carlo (MCMC) methods (Gelman et al 2004, Tanner 1996). These MCMC techniques are extremely useful for simulating values from complex and high-dimensional distributions. Once the distribution represented by Eqn. (7) has been simulated, it is then possible to approximate the posterior probability distributions of parameters of interest (such as means, etc.) by using Monte-Carlo integration. Using the posterior distribution of Eqn (7), predictive posterior densities for the four quarter-car parameters are found. These probability distributions are then used to simulate new trucks in the heavy vehicle fleet, to predict patterns of SSR.

THEORETICAL TESTING

The Mixture fleet model is tested using a theoretical vehicle fleet. A fleet of vehicle axles is generated with random values for all the quarter-car variables (including those that the Bayesian fleet model assumes constant) and simulated MS-WIM measurements are recorded. Fig. 5 shows the simulated measurements for a fleet of 250 vehicles. To illustrate the bimodal nature of the data, the distribution of forces at sensors 2, 7 and 12 are also superimposed.

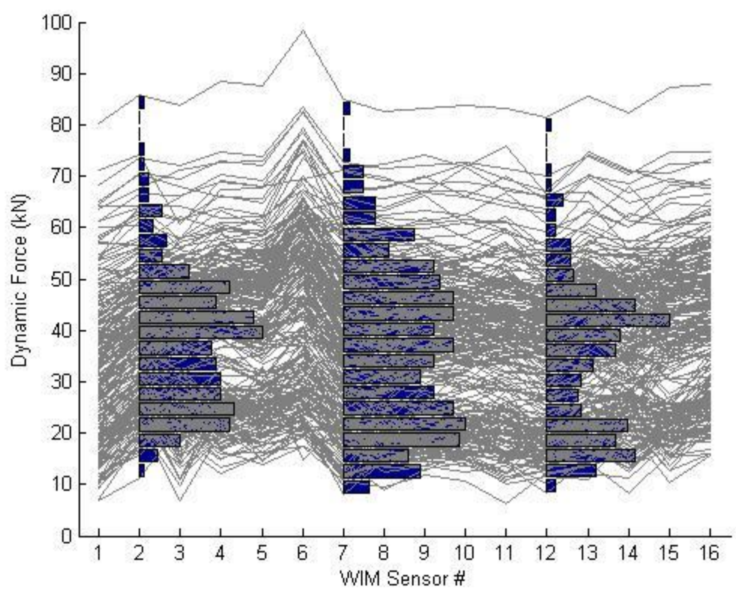


Figure 5 Dynamic force measurements from simulated vehicle fleet

Using the simulated MS-WIM measurements as an input (as well as the road surface profile and vehicle velocities), the Bayesian approach described is implemented and the inferred mixture fleet model is specified. Having specified the predictive densities for the quarter-car parameters, new vehicles may be simulated from the fleet and predictions made for the pattern of SSR. Fig. 6 shows a comparison between the pattern of SSR (the mean dynamic force at each sensor) and the 95% confidence limits for both the original data set and the inferred Mixture model. As can be seen, very good agreement is obtained for both the magnitudes and patterns of dynamic forces.

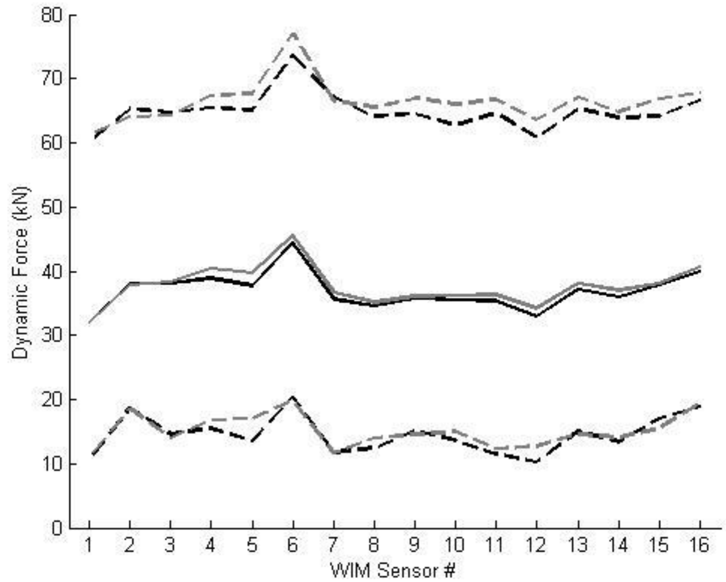


Figure 6 Mean (solid) and 95% limits (dashed) of dynamic force for Mixture model, predicted (▬) and 'observed' data (▬)

The fleet model is now considered to be well specified based upon the original data – it may now be used to predict patterns of SSR at other locations along the same or other pavements, assuming that the same fleet traverses it. This is illustrated in Fig. 7, which shows the pattern of SSR from the same vehicle fleet for a new pavement surface, AS2. The predictions from the Mixture model, determined from the data taken using surface AS1, are again very close to the 'true' values.

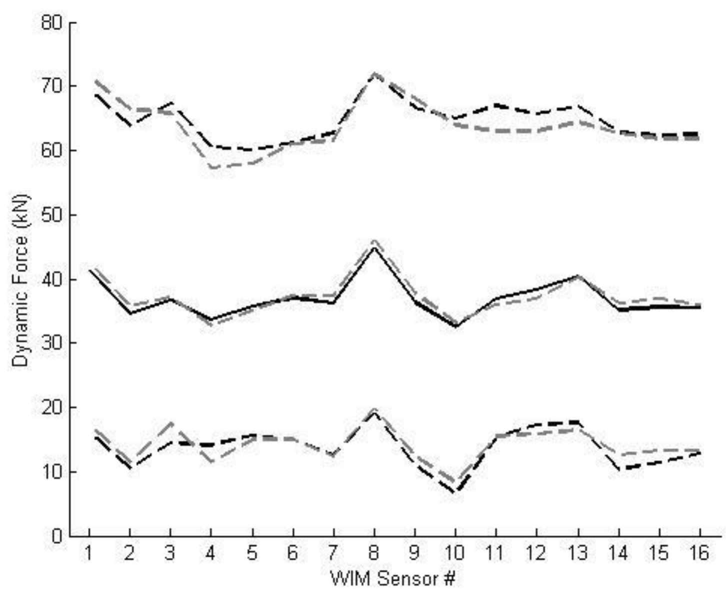


Figure 7 Mean (solid) and 95% limits (dashed) for impact forces for pavement surface AS2, predicted (—) and true (—)

CONCLUSIONS

A Mixture fleet model for predicting patterns of SSR, which describe the pattern of dynamic loading imparted by a fleet of vehicles to a section of pavement over time, is presented. A quarter-car vehicle model with randomly generated parameters is used to simulate multiple vehicles from a common fleet. The model employs both univariate and bivariate mixtures of two lognormal distributions for some of the unknown parameters. The statistical inference of the unknowns is presented within a Bayesian hierarchical framework and theoretically validated via a Bayesian analysis of a set of simulated MS-WIM forces. The Mixture model is shown to provide a good fit to the original data. The fleet model is also capable of predicting patterns of SSR for alternate pavement surfaces.

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