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Sensitivity of predicted bridge traffic load effects to the tails of truck weight distributions

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ABSTRACT

In the last two decades, simulations have been used to predict the characteristic traffic load effects on bridges using Weigh-In-Motion (WIM) data. The recorded Gross Vehicle Weight (GVW) is usually modelled by multimodal distributions. The parameters for these distributions are generally obtained by using different goodness-of-fit tests where the entire recorded data is considered. These parameters are then used as the basis for the simulations. In this work, the sensitivity of the predicted traffic load effects to these fittings is investigated. Generally, moment at mid-span for different return periods can be determined from simulations based on three different assumptions. The first approach is to use empirical distribution functions, i.e., direct simulations using the recorded GVW data. The second approach is to use parametric distribution functions to represent GVW from the recorded data. In the third approach, developed here, semi-parametric distribution functions are used to model the distributions of GVW. From these, load effects corresponding to different return periods are calculated and compared. The results are shown to be highly sensitive to the assumption adopted.

KEYWORDS Bridge, Traffic, Load Effect, Weigh-in-Motion, Simulation, Tail, Semi-Parametric.

INTRODUCTION

In order to design and assess highway bridges in an accurate manner, prediction of the design traffic load effects which may be expected during the intended lifetime, is required. It is stated by many that the design traffic load models, which are given in different codes, are very conservative. These loads are closely related to the level of traffic loads with a sufficiently low probability of acting on a bridge during its lifetime. As is discussed by [1], the increased cost of construction of a new bridge due to the use of an overestimated design load model is small and necessary to allow for uncertainty in future traffic and to simplify the design process. However, once a bridge is in service, the cost of an over-conservative evaluation can be much greater. This justifies the use of an approach which considers the actual traffic at the bridge site which is generally less than the notional traffic loading that will be specified for a road network or class. The measurement technique of Weigh-In-Motion (WIM) can be used to collect reliable unbiased traffic data without interrupting the traffic flow. A comprehensive probabilistic analysis of the WIM data provides a good knowledge of the actual maximum traffic load effects to which a bridge is subjected, provided there is no change in the underlying traffic weight profile. Ongoing monitoring of WIM data can ensure that this is the case. In the last two decades, many methods have been reported for the prediction of characteristic traffic load effects using WIM data [2-9]. A wide range of simulation and extrapolation techniques are reported for the prediction of the characteristic traffic load effects on bridges. It is usually assumed that data for Gross Vehicle Weight (GVW) are consistent with multimodal, in most cases Normal, distributions. The
parameters for these multimodal distributions are generally obtained by using goodness-of-fit tests where the entire data is considered. These parameters are then used as the basis for the simulations of truck crossing and meeting events on the bridge. In this work, the sensitivity of the predicted traffic load effects to these fittings is investigated. The investigation has been limited to the case of mid-span moment in simply supported bridges with spans ranging from 15 to 35m. The bridges are assumed to have two traffic lanes, one in each direction.

Generally, moment at mid-span is determined from simulations based on three different assumptions. The first approach is to use the histogram of the measured GVW data directly. The second approach is to use parametric distribution functions to represent the GVW histogram. In the third approach, semi-parametric distribution functions are used to model the GVW.

**WIM DATA**

The WIM data used in the present work was recorded on Highway A1 near Ressons in France. The highway is a dual carriageway with three traffic lanes in each direction. The data was collected on the two outer (slower) lanes in each direction. Only vehicles weighing at least 3.5 tonnes (only trucks) were registered. The measurement was performed continuously from a Monday at 18.00 to the following Saturday at 03.00 in September 1996. Figure 1 illustrates the GVW distributions, in each direction, determined using the WIM data. Clearly, the histograms have two main peaks representing two different truck populations. The first part of the histograms represents small trucks and unloaded large trucks. The second part of the histograms represent fully loaded large trucks.

**SIMULATIONS**

In order to determine the characteristic values of different traffic load effects, simulations are frequently performed using WIM data. Histograms such as those illustrated in Figure 1 are usually fitted with appropriate parametric probability distribution functions, which are then used for the simulations. The load effect data found from these simulations are fitted in turn to an Extreme Value distribution. Characteristic extreme values are found with return periods that go very much beyond the length of the recording periods of the available data. The key motivation for using the fitted distributions is to take into account other possible vehicle data, which are not obtained from the traffic records, in the particular period of data collection. There are usually
very few but important data points in the tails of the GVW histograms. Therefore, the parametric probability distribution functions which are usually obtained by fitting to the entire histograms for GVW might give a poor description of the histogram tails. The main goal of this work has been to investigate the sensitivity of the characteristic load effects to the recorded values that are at the tails of the GVW histograms. Earlier studies have shown that for short and medium span bridges, free-flowing traffic with two trucks present simultaneously on the bridge gives the critical loading events. The bridges considered here are in the 15 to 35m span range and have two traffic lanes with bi-directional traffic. Hence, it can be assumed that events where two trucks meet on the bridge give the critical loading scenarios.

**Truck Meeting Events**

In order to simulate the number of truck meeting events, an artificial traffic stream, which represents four weeks of traffic flow, is first simulated using the recorded data. The meeting events are defined as events involving the presence of at least one axle of a truck in each direction on the bridge simultaneously. The number of truck meeting events for bridges with spans of 15–35m is determined. As an example, the number of meeting events calculated from the simulated data for a 20m span is shown in Figure 2. As can be seen in the figure, the number of meetings between trucks with five axles driving in both directions are the dominant one, which is unsurprising given the frequency of 5-axle trucks and their length. From here on, only trucks with five axles are considered in this investigation. Clearly all trucks should be considered if the aim of this work had been other than to consider the sensitivity of results to the assumption adopted.

**MODEL FITTINGS TO GVW DISTRIBUTIONS**

**Non-parametric Fitting**

A “non-parametric” model that fits to observations is obtained by using an empirical distribution function. This function for a discrete random variable \( X = x_1, x_2, \ldots, x_n \) is discontinuous. It makes jumps at the points that are possible values, \( x_i \) and the size of the jumps is equal to \( 1/n \). In other words, this function is constant between consecutive (i.e. sorted into ascending order) \( x_i \)’s, and jumps by the same constant \( 1/n \) at each \( x_i \). The non-parametric cumulative distribution function used in this work is written as:

\[
\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq x)
\]

(1)

where:

\[ I(A) = \begin{cases} 
1 & \text{if } A \text{ is true,} \\
0 & \text{otherwise.}
\end{cases} \]

As can be seen in Figure 1, there are usually very few data points in the tail regions of histograms obtained from field data. Simulating data using the empirical distribution is the same thing as drawing new data from the original samples with the help of (1), which implies picking the values \( x_1, x_2, \ldots, x_n \) with a probability of \( 1/n \) in each case. This assumes, of course, that all possible outcomes are equally likely. Drawing a data repeatedly is the same thing as drawing data with replacement. This implies that no new data is generated in intervals for which no weight was
originally recorded when using the empirical distribution function for simulation. Obviously, this is not a problem in regions other than the tail where there is sufficient data.

**Parametric Fitting**
Multimodal distribution functions associated with different populations can be obtained; see for example [10]. For instance, the GVW distribution can be written as a sum of Normal distributions:

$$F(x) = \sum_{i=1}^{m} p_i \Phi \left( \frac{x - \mu_i}{\sigma_i} \right)$$

(2)

where $p_i$ is the proportion for mode $i$, $\mu_i$ is the mean value for mode $i$ and $\sigma_i$ is the standard deviation for mode $i$.

When using (2) to represent the GVW distribution, the parameters $p_i$, $\mu_i$ and $\sigma_i$ can be estimated using the $\chi^2$ test. Figure 3 illustrates distributions determined in this way for the measured 5-axle truck data. Since the $\chi^2$ test takes the entire data into consideration for the parameter estimations, it is very unlikely that the tail regions, where there are usually far fewer data points, will be well described by the fitted model. This poor fit in the tail region can be seen in Figure 3.

**Semi-parametric fitting**
Another way of fitting a distribution to a histogram of GVW is to use a semi-parametric distribution function. The idea is to model the histogram of the GVW with an empirical distribution function where there is sufficient data to give confidence in the frequencies and to model the tail with parametric distribution functions. This involves the use of all the data in the tail region to infer an overall tail trend which will be more representative than a direct replication of the histogram where data is very sparse. Unlike the parametric model, the function in the tail will not be affected by the frequencies elsewhere. In this work, the Normal distribution is judged to be appropriate to model the right tails of the GVW distributions for recorded 5-axle trucks. The parameters for this segment of distribution are estimated as follows.
The Normal probability plot shown in Figure 4 illustrates a comparison of the distributions of GVW of 5-axle trucks recorded in direction 1 during four different days. As can be seen in the figure, the distributions of the GVW in different days are comparable with the exception of the tail region. The region where the distributions differ starts at a truck weight of about 45 tonnes. A similar result from the comparison of GVW distributions is obtained for direction 2. Therefore, it is reasonable to assume that the modelling of the tail by the parametric distribution function should start at a GVW of 45 tonnes. This means that the GVW distribution is modelled using the empirical distribution function for truck weights less than 45 tonnes and the Normal distribution function is used to model the tail. This gives an acceptable confidence in the results. The parameters for the Normal distribution function, i.e., the mean $\mu_t$ and the standard deviation $\sigma_t$, are estimated by minimising:

$$
\chi = \sum_{i=1}^{\infty} \left[ \hat{F}(x_i) - \Phi \left( \frac{x_i - \mu_t}{\sigma_t} \right) \right]^2, \quad (x_i \geq 45 \text{ tonnes})
$$

(3)

The fitted semi-parametric models for GVW distributions are shown in Figure 5.

The solid curves in the tail region indicate the parametric parts of the models. A comparison of the magnified sections of the tails of the GVW distributions shown in Figure 3 and Figure 5 indicates the lack of a good fit in the tail region when using the parametric distribution function, whereas the semi-parametric distribution function always gives a good estimation of the tail of
the measured GVW. It should also be mentioned here that in many cases, distributions for measured GVW have more peaks than those investigated here. This leads to even poorer descriptions of the tails of the histograms by parametric distribution functions. For the semi-parametric distribution function, the appearance of the distributions of the GVW in the regions other than the tails are of less importance.

RESULTS

The mid-span moments are calculated for bridge spans of 15-35m. As an example, for a span of 25m, the distribution for the daily maximum moments obtained from the simulations done using the semi-parametric approach is shown in Figure 6, together with the fitted Generalised Extreme Value (GEV) and Gumbel probability density functions (see [11, 12]). As seen in the figure the shape parameter, $\xi$, for the functions is close to zero. For the results obtained from the direct and the parametric approaches, $\xi$ is 0.007 and 0.132 respectively. Similar results are obtained for the other bridge spans. This indicates that the characteristic loads should be calculated using the GEV model. However, to provide a more graphical illustration, the results from the three approaches are plotted on Gumbel probability paper in Figure 7. This illustrates the significant differences in the distributions of the daily maximum moments for the three simulation approaches.

![Figure 6](image6.png)

![Figure 7](image7.png)

As can be clearly seen from the Figure 7, the results from the simulations performed using the direct and semi-parametric models are closer to each other in all cases than those from the parametric model. If the semi-parametric model is accepted as the most appropriate, then the parametric model is clearly inaccurate. For the shorter spans, it is the heaviest individual axle, double bogie axles or triple bogie axles that give the critical loading case. As the span increases, the presence of the whole truck is more likely to cause the critical load effect. This is the reason why the differences in the distributions of the daily maximum moments obtained from the three simulation approaches increase with increasing bridge span, as seen in the figure.

Characteristic values for different return periods are calculated for each of the three approaches. An investigation performed using quantile-quantile plots has shown that the GEV model describes the distribution for the daily maximum moments reasonably well in all three approaches and for all studied bridge spans. Characteristic values for different return periods are calculated using the GEV model for bridge spans of 15 and 35m and illustrated in Figure 8.
can be seen in the figure, the results from the different approaches differ considerably. The values that correspond to a one-thousand-year return period are presented in Table 1. The differences between direct and parametric are very high (35%–45%) showing that the results obtained from the simulations performed using the parametric distribution function are particularly low. The difference between the semi-parametric and the parametric simulation approaches ranges from 15 to 24%. The differences between direct and semi-parametric are relatively low.

![Figure 8. Estimated return load verses return period.](image)

<table>
<thead>
<tr>
<th>$L$ [m]</th>
<th>Moment [kNm]</th>
<th>Difference [%]</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Direct</td>
<td>Semi.</td>
</tr>
<tr>
<td>15</td>
<td>2350</td>
<td>2157</td>
</tr>
<tr>
<td>20</td>
<td>4028</td>
<td>3384</td>
</tr>
<tr>
<td>25</td>
<td>5852</td>
<td>4743</td>
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<tr>
<td>30</td>
<td>7291</td>
<td>6315</td>
</tr>
<tr>
<td>35</td>
<td>9651</td>
<td>8214</td>
</tr>
</tbody>
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DS: Differences between direct and semi-parametric.  
DP: Differences between direct and parametric.  
SP: Differences between semi-parametric and parametric.

**CONCLUSIONS**

The objective of this work has been to investigate the sensitivity of the predicted traffic load effects to the chosen model for the distribution of measured GVW data. Three different simulation approaches have been used to determine traffic load effects corresponding to different return periods. The first approach uses the distribution of the measured GVW directly. The second and the third approaches use parametric and semi-parametric distribution functions to model the distributions for GVW. The histograms of GVW obtained from the direct and semi-parametric distribution functions are found to be comparable. The corresponding distribution obtained from the parametric distribution is quite different. This observation is similar for all studied bridge spans. This suggests that the parametric model describes the tails of the distributions for GVW poorly. The load effects that correspond to a one-thousand-year return period, calculated from the three different simulation approaches, corroborate this finding.
The authors believe that the semi-parametric distribution is the most appropriate for the GVW. The direct approach does not give any new information over the original data that is generally not representative and the parametric model describes the distribution of GVW poorly in the tail region. The semi-parametric distribution effectively generalises the trend in the tail region while reverting to a direct use of the histogram when there is sufficient data for a clear trend to be evident.

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