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Bayesian updating of bridge safety model

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ABSTRACT: This paper investigates the sensitivities of and correlation between the different parameters influencing the load on a bridge and its resistance to that load. The actual safety, i.e. the probability of failure, is calculated by combining the load and resistance models. The usefulness of updating the developed bridge safety model using damage indicators from a Structural Health Monitoring system is also examined.

KEY WORDS: Reliability; Safety; Bridge; Probabilistic; Bayesian Updating; Damage Indicator.

1 INTRODUCTION

1.1 Motivation

Probabilistic assessment of bridges has been the subject of various studies in recent decades. It has been widely accepted that evaluating an existing bridge according to the standards and codes used for new structures can lead to unnecessary demolition or repair, and thus to high economic cost and an increase in the associated environmental impact.

In several studies in the literature, the authors developed a complete or partial bridge safety model based on probabilistic assessment of load and/or resistance. The main focus of these studies however varies widely. Rocha et al. [1] focus on the safety assessment of short span railway bridges for identifying critical train speeds and the associated probability of failure. Hajializadeh et al. [2] primarily study the effect of spatial correlation of both load and resistance on the probability of failure of concrete road bridges. The principal aim in Zhou et al. [3] is to quantify the effect of foreseen traffic growth on the time-dependent reliability of a reinforced concrete (RC) bridge, accounting for structural deterioration caused by chloride induced corrosion. Marsh & Frangopol [4] also concentrate on the deterioration model of an RC bridge deck but improve the model by incorporating corrosion rate sensor data.

These works can also be distinguished through the simplifications, idealizations and the applied levels of these. In the present work both load and resistance of a one-directional (single lane) bridge are modelled in a probabilistic context and the bridge safety is found by combining them and calculating the probability of load exceeding capacity. The structural model of the bridge is greatly simplified and loads other than dead and traffic loads are ignored, since the emphasis is placed on realistic traffic loading, the probabilistic assessments and the global methodology.

1.2 Bayesian updating

Bayesian updating is a powerful technique for combining a probabilistic model with a limited volume of information from measurement to achieve a more reliable model. It has already been used for several different problems in the field of structural engineering, such as for estimating bridge characteristic load effects [5], for prediction of the effects of corrosion on RC beam bearing capacity [6] or for updating either fatigue reliability of steel structures [7] or degradation of RC structures [8] using non-destructive test data. In this paper, a simple deflection-based damage indicator (DI) is presented and used as the measurement data for updating the bridge safety model and thus for obtaining a better measure of the actual condition of the studied RC bridge. It is acknowledged that more realistic damage indicators exist – deflection is used here simply to demonstrate the concept. Bridge safety model

1.3 Resistance model

The resistance of the bridge is defined as the bending moment capacity of an RC deck cross-section. This resistance is calculated using both deterministic and random variables for the geometric, concrete and steel reinforcement properties. The incorporated random variables and their descriptions have been taken from the literature [9], [10], [11], [12].

The bridge is represented by a simplified 1D beam model. Spatial variability is not accounted for along the length of the beam as no deterioration process is considered. The bridge has short span and the damage is assumed to be at a random location, as would be the case for damage due to impact.

1.4 Dead load model

As for the resistance model, the dead load model incorporates random variables that can be defined according to findings reported in the literature (e.g. after Akgül[13]). It accounts for the weight of concrete and the surface asphalt layer. Other bridge equipment is not taken into account as the corresponding load and its variation are relatively small. It can also be noted here that other loads, such as wind, thermal or seismic, are not considered in the current study.

1.5 Traffic load model

The traffic load model for this study is based on Weigh-in-Motion (WIM) data. The raw WIM data was cleaned and

filtered for quality assurance purposes. There are different techniques for cleaning WIM data [14][15][16] and the final result depends on individual subjective decisions. For the purpose of this study it is decided to focus on trucks that can travel without special permit on roads, i.e., standard vehicles.

Using this population of trucks, the bending moment can be calculated at each segment of the bridge induced by each vehicle at each instant in time. However, it would not be efficient and useful to calculate and store all these load effects as the database may involve hundreds of thousands or even millions of heavy vehicles. Therefore, it is decided to calculate daily maximum values in accordance with the Block Maximum approach as its efficiency is well known [17], [18], [3]. A drawback of this method is that only one load effect per day/block is considered, regardless of the heaviness of the actual day's traffic. In this way, an inaccuracy is introduced through possibly ignoring some extreme trucks in the database. It is also to be noted that this study considers short span bridges. Hence the daily maximum load effects can be obtained from 1-truck loading events based on the assumption that these events are independent and identically distributed (iid).

There are several ways to approximate extreme data through the use of inferential statistics. For maximum bridge load effects, some of the most used tail fitting (and other) methods are studied in detail in Hajializadeh's work [19]. For bending moments induced by passing of the trucks of the database, the focus, in terms of safety, needs to be placed on the tail of this distribution, i.e., based on extreme value theory. This tail however can be defined in several ways. Some authors suggest the top $2\sqrt{n}$ data [20], where n is the number of blocks, some the top 30% [14] while Hajializadeh [19] uses the top 10% of the data as a compromise between the previous two definitions. In the current case, it is decided to study both the top $2\sqrt{n}$ and 30% of data of the daily maximum bending moments for tail fitting. For fitting this block maximum data, the Generalized Extreme Value (GEV) distribution is chosen after Castillo [20]. GEV is a family of three distributions: Gumbel (type I), Fréchet (type II) and (negative) Weibull (type III), and is written as [21] :

$$G(x;\mu,\sigma,\xi) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \tag{1}$$

where μ is the location parameter, σ is the scale parameter and ξ is the shape parameter. Depending on the individual cases, any of the above three distributions can be confidently fit to the collection of extreme data, though some authors [20] argue that Fréchet is unsuitable for bridges given the natural limit on the space available and the capacity of axles.

1.6 Probability of failure

In the present study, the failure of the bridge is defined as the failure of any of its segments (one or more at a time). Hence the probability of failure of the bridge, assuming that the segments' failures are independent of each other, can be expressed as:

$$Pf = \sum_{i=1}^{k} Pf_i \tag{2}$$

where k is the number of segments and:

$$Pf_i = P(C_i - Ed_i - Et_i \le 0) \tag{3}$$

where C_i is the moment capacity, Ed_i is the moment induced by dead load and Et_i is the moment induced by traffic in the ith segment of the bridge. It has to be noted however that the occurrence of failure of one segment might not be independent from the failure of other segments. Therefore, using the formulas presented in equations 2 and 3 may lead to overestimation of the probability of failure considering that in the case of simultaneous failure of various segments it would erroneously report various failure events.

There are several possible ways to obtain the probability of failure expressed in equations 2 and 3. However due to the complexity of civil engineering structures, analytical solutions can rarely be derived. Hence, Monte Carlo simulation is widely preferred and its suitability has already been proven in several studies [22], [19], [4] for computing probabilities of structural failure. Moreover, it can easily address the problem of overestimation of probability of failure as shown in the following formulas:

$$Pf_{MC} = \frac{\sum_{j=1}^{N_{sim}} I_j}{N_{sim}} \tag{4}$$

where N_{sim} is the total number of simulations, j is the actual simulation and I_j can be expressed as:

$$I_j = \begin{cases} 1, \text{ if } G_j \le 0 \\ 0, \text{ otherwise} \end{cases} \tag{5}$$

where the limit state function, G, of the jth simulation is written as:

$$G_j = \min\left[C_{ij} - Ed_{ij} - Et_{ij}\right] \tag{6}$$

where i=1…k and k is the number of segments in the bridge.

1.7 Bayesian updating

Until this point it has been assumed that the bridge is generally healthy, i.e. neither local damage nor a time-dependent deterioration process is assumed. However, the aim of the methodology is to evaluate ageing bridges or bridges subject to impact where the occurrence of damage cannot be ignored. In most of the studies to date, damage of reinforced concrete bridges is accounted for through a corrosion process that decreases the area of the reinforcement and results in a time-dependent resistance model as presented by Enright & Frangopol [23]. Additionally, in many recent studies the spatial variability of resistance is also accounted for [24], [4], [22].

In the current paper, it is proposed to apply Bayesian updating based on a damage indicator (DI). There are various

damage indicators that can be used in the proposed model. The important point is to find a DI which is

a. Relatively easy to measure;
b. Gives relevant information about possible damage extent;
c. And damage location in the bridge.

2 EXAMPLE APPLICATION

2.1 The bridge model

The simplified bridge model which is used in the example application is presented in Figure 1.

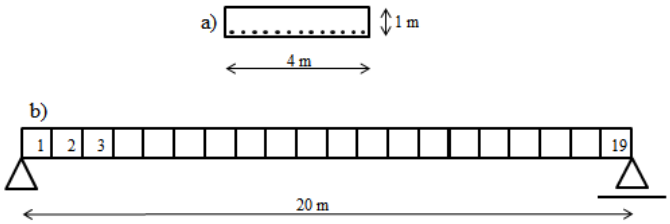

Figure 1. Example RC solid slab bridge's a) cross-section and b) longitudinal view.

2.2 Initial Weigh-in-motion (WIM) data

An extensive database of Weigh in Motion (WIM) data was available, courtesy of the Federal Highway Administration's Long-Term Pavement Performance (LTPP) program [15]. For the purpose of this study one year (2011) of data from 19 sites in the USA are reviewed and after some preliminary investigation, a typical site is chosen: Illinois, I57. The total number of heavy vehicles in this database is 832,307 for 253 days (all working days in 2011) with an average daily truck traffic in the monitored lane of 3281. The main data of the trucks recorded can be seen in Table 1.

Table 1.The mean values and Confidence Intervals (CI) of the main parameters of WIM data.

	Unit	Mean	$CI_{95\%-}$	$CI_{95\%+}$
Speed	[km/h]	105.95	94.94	117.47
GVW	[tons]	22.93	3.81	35.88
Length	[m]	19.84	6.1	24.10
No. of axles	[-]	4.57	2	5
Wheel base	[m]	16.10	3.60	20.86
Gap behind	[sec]	37.00	1.41	106.6

The database had already been cleaned according to federal and state regulations [15] to give only the trucks travelling without a permit, assuming that the road/bridge owner is able to monitor and even to restrict the flow of permit trucks if needed. There are however some surprisingly heavy, long vehicles or vehicles having a surprisingly high number of axles. These may happen for one of two reasons. Firstly, there had been no cleaning regarding the Gross Vehicle Weight (GVW) due to the inability to distinguish between permit and illegally over-loaded tucks. This can result in some suspiciously heavy vehicles passing through the filter that may be outliers or permit trucks. Secondly, there are so-called 'grandfather rights' and 'routine permit' trucks [25], which can have more specific axle-configurations and more than 8 axles in total but which however can travel regularly on the roads of the state.

The distribution of GVW for the trucks included in this study is shown in Figure 2. There are two major modes visible: one around 4 tons, which is caused mainly by 3 and 4 axle trucks and one around 35 tons, caused by fully loaded 5 and 6 axle trucks. In general, excluding the lightest vehicles, the histogram of the overall GVW strongly resembles the shape of the GVW histogram of the 5-axle trucks only (Figure 2). This is due to the fact that more than 75% of the total number of vehicles in the database are 5-axle trucks.

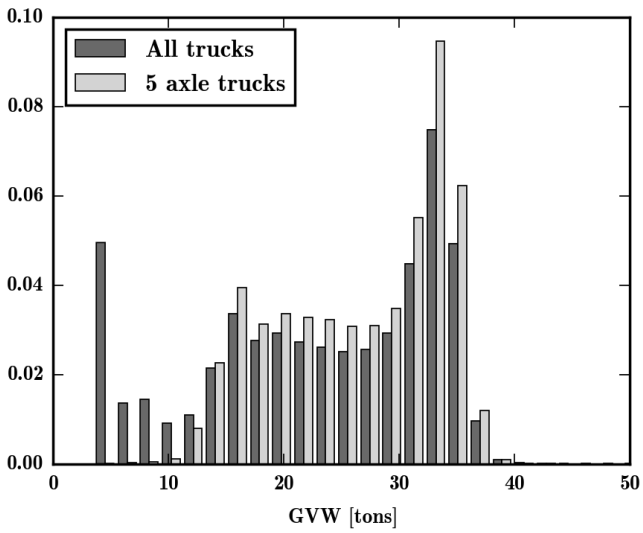

Figure 2. GVW histogram for all the trucks and for only 5-axle trucks at Illinois, I57 site.

2.3 Data filtering

There are several ways of further cleaning/filtering WIM data, depending on the purpose of the study in question. The most common aspects and recommendations can be found in the study of Enright and OBrien [26]. However, in the present work it is agreed to apply as few additional filters as possible in order not to exclude any truck that is realistic and can infer significant load effect in the bridge. After studying the population of trucks in detail, only one severe issue arose and led to further filtering of the database.

The 'gap behind' data show how far two consecutive trucks travel from each other. 0.2 sec can be set as a lower limit for a reasonable gap. Therefore, trucks traveling closer than 0.2 sec (depending on the speed this can correspond to about 4-5 metres) are investigated further. These trucks can be divided into two groups: those with very similar speed and those with different speed. The trucks which travel very closely at around the same speed are assumed to actually be the same truck and are therefore merged into one vehicle. It is assumed that an error occurred in the measurements and the original truck happened to be split into two parts. On the other hand, trucks traveling very close to each other but having different speeds are simply excluded (filtered out) from further study.

2.4 *Importance sampling of trucks*

The primary aim of using WIM data is to calculate realistic bending moments in the bridge induced by the traffic load. To achieve this the bending moment induced during the vehicles' passing is calculated using the bridge's influence line. The Block Maximum method is applied to provide a time reference and to reduce the quantity of data.

To obtain the maximum bending moment per day for each segment (using the WIM data), Monte Carlo simulation could be used directly but it would reduce the computational advantage of the Block Maximum method. Therefore, it is decided to use importance sampling to simulate only the trucks that are expected to have a critical effect on the bridge, i.e. that have a high chance to cause the daily maximum load effect. It is clear that the maximum bending moment per day will be triggered by trucks with either

1. high axle loads and/or GVW, or
2. small inter-axle spacings and/or wheelbase.

2.5 *Daily maximum load effect*

For each day of the year the trucks assumed to be the most critical are sampled from the population and the maximum bending moments they induce in each segment of the bridge during their passing are calculated. Afterwards the daily maximum values are chosen from the resulting data for each segment. The parameters used to condition the importance sampling (for identifying critical trucks) are as follows:

1. Trucks with the greatest GVW (top 20 trucks per day);
2. Trucks with the highest maximum individual axle loads (top 20 trucks per day);
3. Trucks with the highest tandem axle loads (top 20 trucks per day);
4. Trucks with the highest tridem axle loads (top 10 trucks per day);
5. Trucks having the highest density of weights, i.e. the highest GVW/wheelbase ratio (top 20 trucks per day).

This importance sampling process results in a maximum of 90 trucks (usually around 60 due to overlapping among the conditions) per day. The contribution of the sampling conditions regarding each segment of the bridge can be seen in Figure 3. The absolute daily maximum load effects obtained through importance sampling are validated against the maximum daily values of the original WIM data of one week. Each graph shows from segments 0 to 18 (x axis) how many days out of the 253 (y axis) the condition is successful in sampling the truck causing the absolute maximum load effect. There is a clearly visible trend differentiating the segments closer to the middle of the bridge from those closer to the edges. Therefore, it can be seen that a single criterion for the importance sampling cannot be used if the aim is to study all segments of the bridge. In addition, the maximum effectiveness, reached at the 6th segment by the 5th condition (load density), still does not exceed 85% (which would be 215 out of 253), which indicates once more the need of a multi-criteria system.

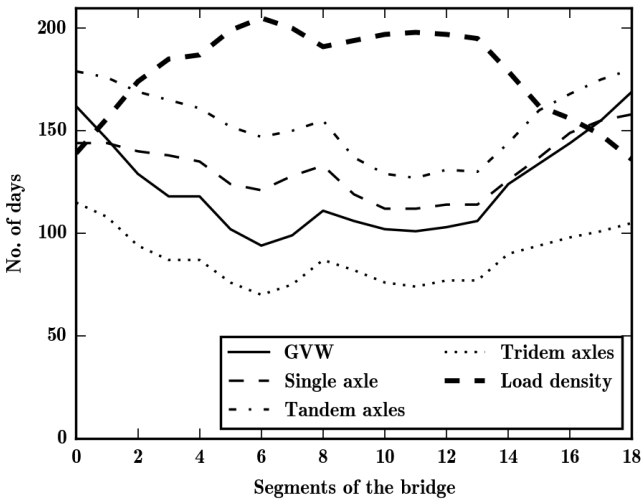

Figure 3. The contributions of the different importance sampling conditions to the daily maximum bending moment.

2.6 *Final load effect distribution*

In the original database, there are more than 800 000 trucks. They are however still collected only for the time duration of one year, leading to 253 daily maximum values of load effect. To overcome this limitation, it is necessary to introduce inferential statistics. The ultimate goal is to find the distribution of the most critical load effects the bridge may experience in the return period which may lead to its failure.

Using a Gumbel probability paper plot it is possible to obtain an idea about which GEV distribution the data follows most closely (see Figure 4). Applying the maximum likelihood method, both Gumbel and Weibull distributions are fitted for tail assumptions ($2\sqrt{n}$ and 30%). In most cases it is difficult to decide between the different fits by engineering judgement. Therefore, the Kolmogorov-Smirnov test is applied and it provides two main conclusions. Both the Weibull and Gumbel distributions fit well to both types of tail data (all p-values, i.e. risks of incorrectly rejecting the proposed distributions, are much higher than the chosen significance level of 0.05). However, as a decision is unavoidable, it is advised to make it segment-based (see some resulting p-values in Table 2) to obtain the most realistic tail distribution for each segment. After some investigations, it is decided to use $2\sqrt{n}$ number of data (which gives higher p-values, i.e. closer fits, for the critical segments close to the midspan of the bridge) but to allow for both Weibull and Gumbel fits depending on the segment in question. For each segment, the GEV distribution fit with the higher p-value is chosen according to the Kolmogorov-Smirnov test results that can be found in Table 2.

Table 2. P-values of the Kolmogorov-Smirnov tests

Segment	Tail of $2\sqrt{n}$		Tail of 30%	
	Gumbel	Weibull	Gumbel	Weibull
6	0.644	0.614	*0.667*	0.432
7	*0.785*	0.717	0.654	0.648
8	*0.899*	0.760	0.651	0.755
9	0.903	*0.946*	0.706	0.776
10	*0.963*	0.856	0.410	0.609

As an example the Gumbel probability paper plot of daily maximum bending moment in the mid-segment of the bridge and its different tail fittings are shown in Figure 4.

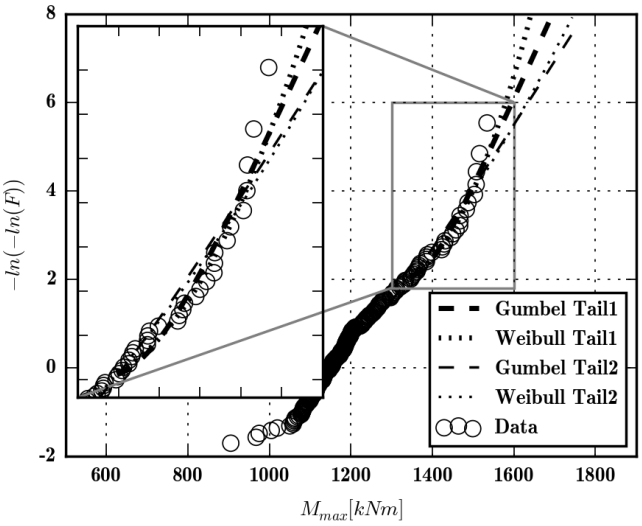

Figure 4. Gumbel probability paper plot of the daily maximum bending moment at the mid-segment of the bridge.

2.7 Resistance and dead load models

In this example, the bridge is assumed to be a simply supported RC solid slab bridge, whose geometry is presented in Figure 1. To build the resistance and dead load models, several parameters are taken stochastically, using the descriptors found in the literature for defining their distributions (Table 3). All variables are considered to be normally distributed.

Table 3. Descriptors of the stochastic parameters.

Parameter	Nominal value	Unit	Mean value	Standard deviation
Slab depth	1000	[mm]	+ 0.8	3.6
Concrete cover	50	[mm]	+ 6	11.5
Concrete compressive strength	45	[N/mm^2]	+ 7.4	6
Steel yield strength	400	[N/mm^2]	+36	21.3
Unit weight of concrete	23.68	[kN/m^3]	x 1.05	x 0.11
Unit weight of asphalt	22.73	[kN/m^3]	x 1.0	x 0.25
Thickness of asphalt	80	[mm]	+ 0	40

Using Monte Carlo simulation, resistance is calculated given both deterministic and stochastic parameters. For the resulting data normal distribution is fitted and this distribution is identical for all the segments, since spatial variability of resistance is ignored at this point. The dead load is calculated similarly; however, the induced bending moments are already spatially variable and so an empirical distribution of moments due to dead load is obtained for each segment separately. In the next step, a normal distribution is fitted to each of them.

2.8 Probability of failure

As described above, three distributions are obtained for each segment of the bridge. A normal distribution for the bending moment capacity, a normal distribution also for the dead load and finally a GEV distribution for the traffic load. The capacity distribution is taken here to be identical for all the segments, while the load distributions vary between segments. The probability of failure is obtained by Monte Carlo simulation according to the equations 4-6.

For the case of a healthy bridge, (without assuming any damage) it can be easily understood that for a simply supported bridge, the mid-segment represents the critical part, while the segments close to the supports can expect significantly lower probability of failure. However, if accounting for damage this can change and it cannot be foreseen which segments will be the most critical. It is therefore necessary to consider all segments of the bridge when calculating the probability of failure.

It can also be observed that the greatest daily maximum bending moment does not always appear in the exact middle segment, but somewhere close to it. This slightly unusual result can be explained in two ways. At first, although a single moving force would always induce the greatest bending moment in the mid-segment (in the case of the simply supported bridge under study), a moving truck load can behave differently thanks to the widely varying axle configurations (see Figure 5). It is also worth noting that the daily maximum bending moment at each segment is not necessarily caused by the same single truck. It is indeed more common to observe 3-4 different trucks per day that are responsible for the all the daily maximum bending moments on the bridge.

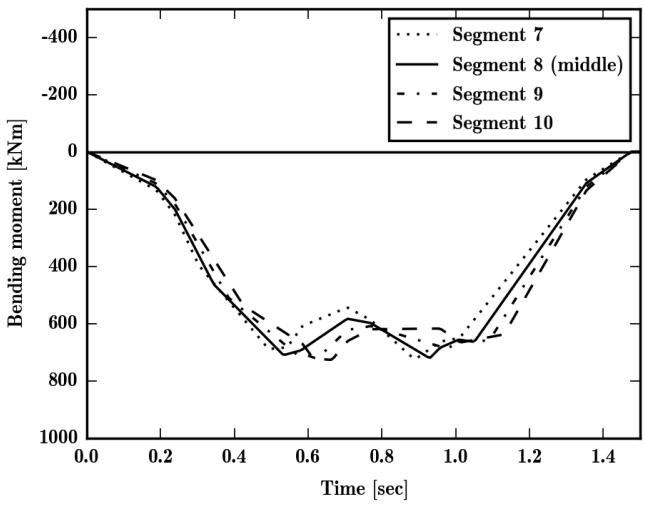

Figure 5. Truck induced bending moments in 4 different consecutive segments of the bridge due to a 5-axle truck passing

The probability of failure is calculated through equations 4-6 using Monte Carlo simulation with a sample size of 10^7 and accounting for full spatial correlation for both the dead load and the traffic load along the bridge. This is on the basis that if one segment of the bridge experiences an extremely great induced bending moment, then it is very probable that all the others will do as well. However, the probability of failure is calculated for each segment separately. The calculated general probability of failure of the studied bridge is 1.40×10^{-5} with a 95% confidence interval of [1.17×10^{-5}; 1.63×10^{-5}]. These values correspond to a bridge in a healthy state and this number should serve as a reference value when implementing damage in the model.

3 FUTURE WORK

Bayesian updating is briefly introduced in this paper; however, the methodology is not yet implemented. The bridge safety model, that is explained and shown in this work, will be the subject of further studies that involve data from deflection-based damage indicators. The ultimate goal is to achieve a realistic general bridge safety model that is able to incorporate some new information on the damage state of the bridge that can be obtained easily, regularly and cost-effectively.

4 CONCLUSIONS

A general bridge safety model is presented in this paper. Three main parts of the model can be distinguished: moment capacity, moment induced by the dead load and moment induced by the traffic load. The moment capacity and dead load models involve several stochastic variables based on the literature and are kept as simple as possible. On the other hand, more attention is paid to the traffic load model and the calculation of the traffic-induced bending moment in the bridge. The traffic load model incorporates one year of WIM data obtained from a single site in Illinois, USA. The methodology particularly emphasizes how to build a realistic traffic load model and discusses several issues that may arise. The Block Maximum method and tail fitting are used to obtain realistic extreme bending moments in an efficient way. Throughout the work, Monte Carlo simulation is applied and importance sampling is used to keep this sometimes computationally expensive simulation method at a sustainable level. The main achievement of the present work is the relatively simple but reliable healthy bridge safety model that can be later used in a Bayesian updating framework (see in Section 3).

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REFERENCES

[1] Rocha, J.M., Henriques, A.A. and Calçada, R. (2014), 'Probabilistic safety assessment of a short span high-speed railway bridge', *Engineering Structures* 71: 99-111.

[2] Hajializadeh, D., OBrien, E.J. and Stewart, M.G. (2015), 'The sensitivity of bridge safety to spatial correlation of load and resistance', *Structures* 5:22-34.

[3] Zhou, X.Y., Schmidt, F., Toutlemonde, F. and Jacob, B. (2014), 'Applying Weigh-in-motion traffic data to reliability based assessment of bridge structures', ICOSSAR2013, *11th International Conference on Structural Safety & Reliability*, Jun 2013, New York.

[4] Marsh, P.S. and Frangopol, D.M. (2007), 'Reinforced concrete bridge deck reliability model incorporating temporal and spatial variations of probabilistic corrosion rate sensor data', *Reliability Engineering & System Safety* 93:394-409.

[5] Enright, B., Leahy, C. and OBrien, E.J. (2014), 'A Bayesian approach for estimating characteristic bridge traffic load effects', *Civil Engineering Research in Ireland*, Belfast, UK, 28-29 August.

[6] Ma, Y., Zhang, J., Wang, L. and Liu, Y. (2013), 'Probabilistic prediction with Bayesian updating for strength degradation of RC bridge beams', *Structural Safety* 44:102-109.

[7] Zhang, R. and Mahadevan, S. (2000), 'Model uncertainty and Bayesian updating in reliability-based inspection', *Structural Safety* 22: 145-160.

[8] Décatoire, R. (2015), *Optimisation of inspection plans for structures submitted to a stochastic degradation context: application on RC structures*, PhD Thesis. Université de Nantes, Faculté des Sciences et des Techniques.

[9] Mirza, S.A. and MacGregor, J.G. (1979), 'Variations in dimensions of reinforced concrete members', *J Struct Div*. 105(4): 751-766.

[10] McGee, R.W. (2001), *On the service life modelling of Tasmanian concrete bridge*, PhD Thesis, University of Tasmania, Hobart, Australia.

[11] Attard, M.M. and Stewart, M.G. (1998), 'A two parameter stress block for high strength concrete', *ACI Structural Journ*al. 95(3): 305-317.

[12] Seo, D., Shin, S. and Han, B. (2010), 'Reliability-based structural safety evaluation of reinforced concrete members', *Journal of Asian architecture and building engineering* 9(2): 471-478.

[13] Akgül, F. (2002), *Lifetime system reliability prediction for multiple structure types in a bridge network*, PhD thesis, University of Colorado, Boulder, Colorado.

[14] Enright B. (2010), *Simulation of traffic loading on highway bridges*, PhD Thesis. College of Engineering, Mathematical and Physical Sciences, University College Dublin, Ireland.

[15] Leahy, C. (2013), *Using weigh-in-motion data to predict extreme traffic loading on bridges*, PhD thesis, College of Engineering and Architecture, University College Dublin, Ireland.

[16] Sivakumar, B., Ghosn M. and Moses F., *Protocols for collecting and using traffic data in bridge design*, NHCRP report 683, National Academy of Sciences, Washington, 2011.

[17] Caprani, C.C., OBrien, E.J. and McLachlan, G.J. (2007), Characteristic traffic load effects from a mixture of loading events on short to medium span bridge, *Structural Safety* 30: 394-404.

[18] OBrien, E.J., Bordallo-Ruiz, A. and Enright, B. (2014), 'Lifetime maximum load effects on short-span bridges subject to growing traffic volume's, *Structural Safety* 50: 113-122.

[19] Hajializadeh, D. (2013), *Combined probabilistic modelling of load and resistance in bridges*, PhD thesis, College of Engineering, Mathematical and Physical Sciences, University College Dublin, Ireland.

[20] Castillo, E., *Extreme value theory in engineering*, Academic Press, New York, 1988.

[21] Coles, S., *An introduction to statistical modeling of extreme values*, Springer-Verlag, London, 2001.

[22] Darmawan, M.S. and Stewart, M.G. (2007), 'Spatial time-dependent reliability analysis of corroding pretensioned prestressed concrete bridges', *Structural Safety* 29: 16-31.

[23] Enright, M.P. and Frangopol, D.M. (1998), 'Probabilistic analysis of resistance degradation of reinforced concrete bridge beams under corrosion', *Engineering Structures* 20(11): 960-971.

[24] Stewart, M.G. and Al-Harthy, A. (2008), 'Pitting corrosion and structural reliability of corroding RC structures: Experimental data and probabilistic analysis', *Structural Safety* (93): 373-382.

[25] USDOT (2000), *Comprehensive truck size and weight study. Washington DC: US Department of Transportation*, FHWA-PL-00-029.

[26] Enright, B. and OBrien, E.J., *Cleaning Weigh-in-Motion data: Techniques and recommendations*, University College Dublin, 2011, available from: iswim.free.fr/doc/wim_data_cleaning_ie.pdf