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CROSS ENTROPY WEIGHT MINIMIZATION OF A COMPRESSION STRUT

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ABSTRACT: In this study, a population-based optimization algorithm is used to minimize the weight of a compressive strut. A geometrically nonlinear analysis is carried out to get an accurate measure of the structure’s true capacity, allowing for individual member and overall structure (and sub-structure) buckling. To overcome the computational challenge of nonlinear analysis, the study uses a simple definition of the onset of instability and hence the number of iterations is cut to a minimum.

INTRODUCTION

Optimization techniques have been developing in parallel with the growth in computing power through the years. Additive manufacturing now means that structural complexity is no longer a constraint and very complex structures can be manufactured with minimum weight. In recent years, with access to increased computing power, the trend in optimization has been towards Evolutionary Algorithms (EA). While they are computationally demanding, EA’s are very robust and perform well for complex problems with nonlinear, stochastic, temporal or chaotic components (Kicinger et al., 2005).

The earliest EA algorithm is the Genetic Algorithm (GA) (Goldberg, 1989). Inspired by the GA, many other EA approaches and variations have been developed. The Structured GA (Dasgupta and McGregor, 1991) retains redundant genetic material in its genotype and uses a gene activation mechanism which can reactivate dormant parts of the genotype after several generations. The Messy GA (Goldberg et al., 1989) allows for variable length genotype strings – crossover between parents can happen between segments of string that have different locations and lengths in each parent. Mühlenbein et al. (1991) apply what they term Parallel GA to problems with continuous variables and later Mühlenbein and Schlierkamp-Voosen (1993) apply Breeder
GA to such problems. Cross Entropy (Rubinstein and Kroese, 2004) is a simple but powerful EA algorithm that has been applied to a number of diverse Engineering problems (e.g. Belay et al. 2008, Dowling et al. 2012).

Additive Manufacturing (AM) has made it possible to manufacture highly complex structural forms and opens up the potential for much more efficient structures in terms of the strength to weight ratio. In this paper, the internal topology and member sizes in a compressive strut are optimized using a Cross Entropy algorithm. Most previous studies use allowable-stress approaches or simple Euler Buckling definitions of failure. When structures are optimized, buckling dominates the behaviour of the members and it is well known that Euler Buckling is a poor representation of true behaviour. In this study, a geometrically nonlinear analysis is carried out to get an accurate measure of the structure’s true capacity, allowing for individual member and overall structure (and sub-structure) buckling. The nonlinear analysis used is approximate – it is based on the premise that the structure will be stable provided the onset of instability is prevented. This algorithm uses a pre-defined ‘ground structure’, i.e., all possible nodal coordinates and member orientations are pre-defined. However, members are allowed to have near-zero area. Provided the ground structure is sufficiently refined, the solution is a very close approximation to a fully optimized topology.

A case study of a simple optimized truss is used to illustrate the process. As well as a large vertical force, the loading includes a small horizontal force to promote buckling. It is optimized using the simplified geometrically nonlinear analysis to minimize weight subject to constraints on strength and buckling. The optimizations result in dramatic reductions in total weight, relative to a solid alternative. The numerical modelling will, in the near future, be complemented with an experimental programme where samples are manufactured and tested.

### OPTIMIZATION ALGORITHM

Firstly, an initial estimation of the areas for all members of the strut is inputted into the algorithm. Standard deviations, chosen as a percentage of the means of the initial areas, are also specified (typically 10% of the mean was found to be adequate). A population of solutions, consisting of the areas, is then generated through Monte Carlo simulation, using the initial vector mean and standard deviation.

![Figure 1. The Cross-Entropy algorithm](image)

Each solution has a fitness (objective function) which is determined by the total volume and the degree to which it satisfies stability constraints at each node. Once the fitness for each solution
in the population has been determined, the solutions are assembled in ascending order of fitness. The low performing 90% of solutions are then discarded. The top performing 10%, known as the elite set, form the basis for the next generation. This is done by calculating the vector mean and standard deviation of the areas of the elite set and using Monte Carlo simulation to generate the next generation. To ensure continuous improvement between generations, the top performing solution from each generation is carried through to the next. In each successive generation, the vector standard deviation tends to get smaller so the process is restarted a number of times to prevent premature convergence. This Cross Entropy algorithm is illustrated in Figure 1.

**FITNESS EVALUATION**

The goal of the optimization process is to find the minimum possible weight of the strut, under a compressive load, while preventing excessive stress in any member and preventing instability at any node. Assuming the same density for all members, minimizing weight amounts to the minimization of volume. Truss-type behaviour is assumed, i.e., joints/hinges are assumed at all nodes. While this is not realistic in an additive manufactured strut, the contribution of bending to the strength of the structure is considered to be small. This assumption will be tested in the near future as joint fixity may enhance the bucking strength of individual members.

Where the optimization process generates a member with small (or negative) area which results in stress greater than the material strength, the area is automatically increased to a value that gives stress just equal to the maximum. It is acknowledged that this violates non-redundancy principle of Gen and Cheng (2000), i.e., the desire for a 1-to-1 mapping between genotype and phenotype. In this case the genotype is the coding that defines the structure to the computer while the phenotype is the actual structure used to evaluate the fitness.

**Non-Linear Global Buckling**

A simplified non-linear analysis is carried out during the optimization process to evaluate the stability of each solution within the population. This was performed by analysing the solution three times. The displaced co-ordinates were saved after the first analysis and used as the node locations for the second analysis in which the same loading was applied. This was repeated for the third analysis. The desired result is a solution that has nodal displacements that stay constant or decrease relative to the previous analysis, i.e. a solution that is converging towards a stable equilibrium geometry and is not increasing in displacement with each analysis. It is assumed that three analyses are sufficient to ensure the stability of solutions.

To promote stable solutions in the population within the algorithm, a penalty was imposed on solutions for which displacements were not decreasing between analyses. The penalty applied is ‘soft’, i.e., small violations of the constraints are allowed as compensation for reduced volume. This penalty factor $\varphi$ was introduced to the fitness with a modification factor, $k_\varphi$, to scale the relative values of the two components. The scaling factor allows the penalty, $\varphi$, to be changed, as appropriate, to a meaningful magnitude in relation to the total volume of the solution.

The fitness, $\Omega$, for the $i^{th}$ solution in a population with consideration of volume and non-linear stability penalties for that solution is:
\[ \Omega = \sum (vol_i) + k_p \sum (\phi_j) \]  

(2)

where \( vol_i \) is the volume of member \( i \) and \( \phi_j \) is the penalty function for displacement at degree of freedom \( j \).

RESULTS

A case study of a 1000 mm high × 500 mm wide strut is considered to illustrate the process (Fig. 2). The strut is subject to a 1 kN vertical compressive force and a nominal horizontal perturbation loading equal to 5% of the vertical loading, i.e. 0.05 kN. This is imposed to encourage non-linear buckling action for the global structure being optimised.

Figure 2. The case study.

There are a total of 15 members joining 8 nodes. The algorithm was run for 180 generations, each consisting of 150 candidate solutions to the problem. Each solution consisted of the 15 genotype member areas. The fitness was calculated using Eqn. 1 with the phenotype member volumes and stability penalties. The progress of the solution fitness’s towards the optimum solution is illustrated in Figures 3(a). Where genotype area is too small to satisfy the corresponding strength constraint, the phenotype area is automatically set to the minimum required. As the volume is calculated from the phenotype, there is no saving in fitness. When the stability constraint is violated, i.e., displacement is seen to be growing strongly in successive analyses, it is penalized and the fitness increases. It can be seen that fitness of all members grows rapidly through the early generations and there is not much variation between the 150 solutions in each generation. Around the 100th generation, the standard deviation of the elite set has converged towards zero so it is reset and the process restarted. The restart generates a wider range of fitness values amongst the solutions, including some with higher fitness. The restart process is restarted again in the 159th generation. At this point, the improvement resulting from the restart is small so the process is terminated. The fitness of the most-fit solution in each generation is illustrated in Figure 3(b). As the most-fit solution is transferred from each generation to the next, this most-fit
fitness never reduces. The improvement in the early stages after each restart is apparent. It can also be seen that the magnitude of the gain in fitness reduces with successive restarts.

The load paths in the optimum solution, found after 180 generations, is illustrated graphically in Figure 4. As would be expected, the predominant load path is a straight line down from the point of application of the main load at node 7 through to the support at node 1. Some very small forces exist in a group of stabilizing members: 11, 10, 9, 6 and 2, although stability may be coming instead from the other diagonals: 14, 8, 4 and 3. The horizontal force is another possible reason why there are small forces in the latter and in other members associated with bending behaviour such as 7 and 15.

![Figure 3](image-url)

**Figure 3.** Convergence of solution fitness to the optimum. (a) Fitness of all 150 solutions in each of the 180 generations. (b) Fitness of the most-fit solution in each generation.

![Figure 4](image-url)

**Figure 4.** Distribution of internal forces in the optimized structure.

The low fitness of some solutions is due to the non-linear stability constraint. This is quadratic so it grows rapidly with the extent of the violation. The penalty was applied to ensure that increases in displacement diminish rapidly with successive analyses of a particular solution. Figure
5 illustrates a typical result for one node of one solution. In the 2nd and 3rd analyses, the geometry is revised to take account of the displacements at the nodes in the previous analysis. From the 1st to the 2nd analysis the displacement increases by $1.1025 \times 10^{-6}$ mm and from the 2nd to the 3rd, it increases by a further $0.206 \times 10^{-6}$ mm. This is considered to be a stable situation because, while displacement is clearly increasing, it is clearly converging with successive analyses. The penalty function was applied when the 2nd-to-3rd increase exceeded $x\%$ of the 1st-to-2nd increase. In this example, the value of $x$ was chosen, arbitrarily, to be $30\%$.

![Figure 5. Vertical displacement in 3 successive analyses at node No. 7 (The most fit solution from the last generation).](image)

**CONCLUSION**

This paper describes an optimization algorithm to find the optimum topology of a simple compression structure using a pre-defined truss-type ground structure. The Cross Entropy method of optimization is applied, an Evolutionary Algorithm that operates on a population of solutions through several generations. The method is shown to converge well to an optimum over 180 generations. A simplified non-linear analysis, involving three linear elastic analyses of each structure, is used to check the stability of each solution. A soft penalty function is applied to reduce the fitness of solutions with poor stability. The method works well and has considerable potential for the design of super-light structures. An experimental testing programme on additive manufactured samples is planned for the near future.

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**REFERENCES**

