A Probabilistic Approach for Joint Optimization of Fatigue Design, Inspection and Maintenance

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ABSTRACT

This paper addresses challenges in fatigue management of marine structural assets with a holistically approach, by jointly considering fatigue design, inspection and maintenance decisions, whilst taking into account sources of uncertainties affecting life cycle performance. A risk-informed and holistic approach is proposed for jointly optimizing fatigue design, inspection and maintenance based on the same fatigue deterioration model. The optimization parameters are fatigue design factor (FDF) and inspection intervals, while the objective is to minimize expected life cycle costs (LCC). The framework is to guide design process as well as to formulate optimal maintenance strategies. The proposed approach is exemplified for the marine industry through a fatigue-prone detail in a ship structure to obtain the life cycle optimal management solution that achieves a best compromise between structural safety and life cycle costs.

KEY WORDS: Integrity management; risk-based inspection; maintenance optimization; uncertainty management; probabilistic design optimization; decision analysis; life cycle engineering

INTRODUCTION

Marine structures are designed, constructed and managed to provide a variety of functions in support of transportation, production, leisure, etc. With the development of technology, functional requirements, budgeting control, safety and reliability are increasingly paramount. Local failures and structural collapse are normally avoided by design analysis and in-service inspections and maintenance, to achieve an acceptable failure probability. Deterioration factors, excessive deformations and vibrations are controlled so that structures are durable and serviceable within the required service lives. Other safety-related structural requirements concern redundancy, robustness and resilience (Faber, 2017). Besides, it is required that engineering design, inspection and maintenance activities are viable and sustainable, both economically and the environmentally. In order to ensure that structures meet the performance requirements, it is becoming normal practice to develop and identify safety check lists and formats to avert potential threats and failure modes in the design stage (HSE, 2001).

Fatigue crack growth is one of major threats for marine structures exposed to the sea environments, in which cyclic wave loading lead to deterioration in terms of crack initiation and growth in structures and if undetected lead to failure. Compared with other threats, fatigue cracks are safety-critical, as they can cause sudden rupture of structural cross-sections, leading to losses to human lives, commercial assets and the environment (Fricke, 2003). Crack initiation can be caused by several mechanisms, e.g. cyclic loading, local stress concentrations, corrosion, imperfections in materials, etc. Fatigue cracks are very common in marine assets operating at seas, and detecting and repairing fatigue cracks represent a substantial and expensive task. According to the characteristics of crack development, fatigue cracks are typically very small during a significant part of the service life, and therefore the time is usually very short for cracks to develop from a detectable size \( a_0 \) to the critical size \( a_c \) (Fig. 1). This poses a real challenge for detecting cracks reliably before they may cause catastrophic failures.

![Figure 1. Crack initiation and propagation](image-url)

Another challenge is that fatigue failures are difficult to predict accurately with existing analytical, numerical and experimental approaches. Fatigue resistance is affected by many factors that are only partially controllable, e.g. stress ranges, mean stresses, stress concentration factors, loading sequence, material properties, fabrication and welding techniques, environments, etc. (Fricke, 2003). Fatigue deterioration is associated with a high degree of uncertainty, e.g. those associated with fatigue loading, stress calculation, material properties, fatigue resistance data, fatigue accumulation model and crack growth.
model, etc. There are also uncertainties in the detection capabilities of non-destructive testing (NDT) methods and in the effects of repairing methods (Straub and Faber, 2006).

In consideration of these uncertainties, normally several steps are taken in the design, manufacture and operation stage respectively to safeguard marine assets against fatigue failure. These include setting appropriate safety factors, scantling, welding and quality control procedure, in-service inspections and repairs, etc. (Fig. 2). Trying to counteract negative effects of all sources of uncertainties by adopting relatively large safety factors, strict quality control measures, periodical inspections and conservative repair criterion for all structural details based on engineering experience, these measures are typically conservative and prescriptive, and result in significant increase in total costs of design, manufacture, inspection and maintenance. However, sometimes fatigue failures still cannot be fully avoided because local loading conditions and other specific factors are not taken into account and high-risk areas are not identified.

![Safety control measures during life cycles of marine assets](Image)

Figure 2. Safety control measures during life cycles of marine assets

Probabilistic techniques and reliability methods have been applied to uncertainty modelling and development of reliability-based design codes (Lee et al., 2016, Yeter et al., 2015, Akpan et al., 2015) and inspection planning frameworks (Lotsberg et al., 2016, Dong and Frangopol, 2016, Goyet et al., 2002, Zou et al., 2017a, Zou et al., 2017b, Faber, 2002). Reliability methods provide consistent uncertainty treatment and thus help to achieve consistent design criteria and safety level. In addition, inspection findings can be utilized to update design failure probability with Bayesian theorem. The distributions of model parameters can also be updated and thus modelling uncertainties are reduced and design, inspection and maintenance practice is improved.

Besides uncertainty identification, formal cost-benefit analysis needs to be done to achieve optimal budget allocation for the design, inspection and maintenance activities. This requires that failure probability and failure consequences are quantified explicitly. The concept of risk is introduced as the product of failure probability and failure consequences and widely used in probabilistic safety assessment. Risk based inspection planning (RBI) has been studied to achieve optimal investments in inspection and maintenance in offshore engineering (Dong and Frangopol, 2016, Faber et al., 2003, Akpan et al., 2015, Lassen and Recho, 2015). The benefits of a risk perspective are that failure consequences and costs of safety control measures are fully accounted for in the decision making process to ensure that the benefits brought by the safety control measures outweigh their costs and best compromise is achieved between structural safety and total costs.

The above studies have applied risk analysis to inspection planning, which are typically carried out on existing design plan or constructed structures. However, fatigue design has seldom been connected with inspection & maintenance planning. In their studies, design decision and inspection & maintenance decision are typically optimized separately and based on different theoretical models and inputs, and thus are far from optimal with respect to life cycle total costs. In this regard, a decision support tool is developed in this paper to support fatigue design, inspection and maintenance decision making process for marine assets based on integration of life cycle risk and cost analysis.

**ENGINEERING DECISIONS AGAINST FATIGUE**

**Design plan**

Structural design involves determination of materials, scantlings, joining methods and quality control procedure to ensure that the designed structure can survive identified loads with an accepted failure probability. Traditional deterministic design method adopts one safety factor to counteract the effects of all uncertainties. For fatigue design, the safety factor is labelled as fatigue design factor (FDF). Determination of safety factor relies largely on engineering experience and engineering judgement, not on theoretical basis due to degree of difficulty/complexity. In addition, operational inspection and maintenance is typically not considered in the design process.

In this paper, the decision on FDF is supported by a holistic management tool in which the FDF is optimized together with inspection and maintenance plan. The effects and benefits of planned inspection and maintenance on the structural performance and risk are assessed quantitatively before they are implemented. With explicit quantification of the benefits of operational inspection and maintenance, more rational decision making on the FDF can be achieved. The FDF determined by the holistic management tool is optimal with respect to life cycle total costs.

**Inspection plan**

Inspections are recognized as main means of identifying damages in structures. Apart from manufacture inspections, operational inspections help to validate structural design and to identify new damages developed in service. A complete operational inspection plan specifies:

- Where to inspect
- How many inspections in lifetime
- When to inspect
- How to inspect

Risk analysis provides a rational and consistent basis to quantify and compare the risks associated with failures in different areas of structural assets. Based on risk quantification, inspection tasks can be prioritized and inspection resources can be allocated and targeted at the areas with high failure risks.

The number of inspections and inspection times are main objectives addressed in an inspection plan. These are determined based on life cycle cost-benefit analysis. The risk reduction benefited from inspections and repairs are formulated explicitly. The costs of design, inspection and repair are also formulated. These formulations form the basis to calculate life cycle total costs. Based on minimization of the life cycle total costs, the optimal number of inspections and inspection times can be optimized.

**Repair plan**

As inspection results are unknown in the design stage, planned inspection cannot give any useful information that can alter structural
failure probability if no action is linked with inspection results. It is the repair activities that ultimately change the structural reliability. So following detection of cracks, decision makers have to decide on

- Whether to repair the detected cracks or not
- When to repair the detected cracks
- How to repair the detected cracks

Whether and when to repair, is dependent on the specified repair criterion. The criterion has to be explicit and easily judged such that it can be act on when the criterion is met. It is normally defined based on crack size. The repair criterion determines the probability of repair and has a significant effect on the accumulated failure probability and the life cycle total costs.

FRAMEWORK FOR JOINT OPTIMIZATION

The general idea of this paper is that assets are best managed with a holistic approach, jointly considering all the engineering decisions (design, inspection and maintenance) and the uncertainties affecting fatigue performance of structural details, based on the same fatigue deterioration model. It is generally acknowledged that it is not economical to design structures in such a way that operational inspection and maintenance are unnecessary, and operational inspection and maintenance are essential for structural integrity and reliability management. Developing inspection and maintenance programmes has been main tasks of structural integrity management (SIM), as illustrated in Fig. 3. Excessive damages can be found normally by inspection and repaired before they cause greater consequences. In case inspection has also detected human errors in design and manufacturing, remedy measures can be taken on time to rectify them. The costs are normally very high associated with inspection and unplanned maintenance due to downtime in service. Therefore there is merit in optimizing inspection and maintenance plans, taking into account uncertainties associated with fatigue loading, resistance and inspection performance. However, current structural integrity management normally only optimizes operational inspection and maintenance decisions and is disconnected from design process.

Figure 3. Typical structural integrity management

It is noted that the best opportunity for risk mitigation is in the conceptual design stage when it's possible to make major changes. So there is merit to consider operational inspections and maintenance in the design stage and optimize operational inspections and maintenance together with design plan, subjected to objective and constraint in safety and life cycle total costs. The effect of operational inspections and maintenance on failure probability can be taken into account early in the design stage with the joint optimization framework as illustrated by Fig. 4. The following points explain the idea of joint optimization in detail:

1. The design, inspection & maintenance decisions jointly determine the failure probability at the end of service life \( p_f \) and expected life cycle total costs (LCC), given by total sum of design costs, expected inspection costs, expected repair costs and failure risk in terms of potential financial loss.
2. Conversely, the decisions are optimized based on minimization of LCC with constraint on \( p_f \) or reliability index \( \beta \).
3. The failure probability \( p_f \) associated with a design plan is obtained based on a probabilistic fatigue deterioration model. The failure probability can be updated with planned inspections and repairs.
4. Inspection result is unknown in the design stage, but the distribution of possible results (detection, or no detection), i.e. the probability of detection or no detection, can be obtained based on prior (probabilistic) information on the detectability of inspection method.
5. A repair criterion is defined and the probability of repair can also be calculated. The repair criterion influences failure probability at the end of service life \( p_f \) and life cycle total costs LCC and are optimized.
6. Both the result of detection (and possible repair if the repair criterion is reached) and no detection contribute to lower failure probability.
7. In case of repair, the failure probability of repaired structure is obtained by a repair effect model, which specifies changes of structure by repair in terms of fatigue performance or damage extent.
8. The costs of design are related to the design plan; the expected inspection costs are related to the inspection times and methods adopted; the expected repair costs are related to the inspection times and methods, the repair methods and repair criteria; failure risk is related to the failure probability at the end of service life and failure consequence.

The difference between the proposed joint optimization approach (Fig. 4) and inspection planning in traditional SIM (Fig. 3) lies in that here
inspection & maintenance decisions are optimized together with design decision while in traditional SIM, the inspection & maintenance decisions are optimized on the basis of existing design plan or constructed structure. Existing design plan, even if optimal from the design perspective, may not be optimal from the whole life cycle perspective, considering operational inspections and maintenance.

FATIGUE DETERIORATION MODELS

S-N model

Fatigue resistance of structural details is given by S-N curves, which are obtained by specimen tests under controlled loadings and statistical analysis of specimen fatigue resistance data. A typical two-segment S-N is given by

\[
\begin{align*}
N_f \Delta \sigma^{m_1} &= \bar{a}_1 & N_f \leq 10^7 \\
N_f \Delta \sigma^{m_2} &= \bar{a}_2 & N_f \geq 10^7
\end{align*}
\]

where \(N_f\) is fatigue life, \(m_1\) and \(m_2\) are the fatigue strength exponents, and \(\bar{a}_1\) and \(\bar{a}_2\) are the fatigue strength coefficients.

Under cyclic wave loading, fatigue damage \(D\) accumulates in structures and failure occurs if the fatigue damage reaches a fatigue failure criterion \(\Delta\). A Limit state for fatigue failure can be formulated as

\[L_1 = \Delta - D\]  

(2)

The fatigue damage \(D\) can be calculated by Miner’s rule

\[D = \sum_{i=1}^{n} \frac{n_i}{N_{fi}}\]  

where \(n_i\) is number of load cycles at the \(i\) stress range level; \(N_{fi}\) is the fatigue life under the \(i\) stress range level; and \(n_b\) is the number of stress range levels.

Fracture mechanics (FM) model

FM approach represents a more rational approach for fatigue analysis, as it is based on the physics of crack propagation. From the perspective of fracture mechanics, fatigue deterioration begins with initiation of micro cracks, and then evolves into macro cracks. Propagation of macro cracks leads to final fracture. The relationship between the crack propagation rate and the local stress field is given by Paris’ law (Paris et al., 1961)

\[\frac{da}{dN} = C \Delta K^m, \quad \Delta K_{th} \leq \Delta K \leq K_{mat}\]  

where \(da/dN\) is crack propagation rate; \(C\) and \(m\) are material parameters; \(\Delta K\) is stress intensity factor range; \(K_{mat}\) is material fracture toughness; and \(\Delta K_{th}\) is threshold value for the stress intensity factor range. The stress intensity factor range \(\Delta K\) is given by

\[\Delta K = \Delta \sigma Y(a)/\sqrt{\pi a}\]  

where \(Y(a)\) is geometry function and \(\Delta \sigma\) is stress range.

A limit state function based on FM approach is formulated as

\[L_2(t) = a_c - a(t)\]  

where \(a_c\) is critical crack size and is usually set to be equal to plate thickness.

PROBABILISTIC INSPECTION MODELLING

No inspection is perfect with 100% accuracy, whether visual inspection or NDT methods. The true damage state and the damage state indicated by an inspection method are not always in agreement. Sometimes an existing crack cannot be identified, and an indicated crack does not actually exist. In addition, there is uncertainty associated with sizing of cracks.

In this paper, the uncertainties associated with an inspection method are identified and treated with probabilistic techniques so that the reliability of inspection is traceable. The reliability of detection capability is characterized with the concept of probability of detection (PoD). PoD refers to the probability that an existing crack can be detected by an inspection method. The PoD curve for an inspection method can be obtained by tests on inspectors under the same conditions repetitively. The PoD curve of an inspection method equals to the cumulative density function of detectable crack size \(a_d\), which is treated as a variable in probabilistic analysis. The detection event is formulated as

\[D(t) = a_d - a(t)\]  

where \(D(t) < 0\) signifies detection while \(D(t) > 0\) signifies no detection. It must be noted that both the capacity of inspection method \(a_d\) and the crack size \(a(t)\) are variables.

REPAIR EFFECT MODEL

Planned inspection without repair cannot improve structural reliability, because the inspection result is unknown in the planning stage and thus cannot provide any new information with respect to the structural damage state. Only when a criterion for repair decision is defined, which specifies when repair actions are taken following detection, structural reliability can be improved. The benefits of planned inspection and maintenance can be realised by adopting a ‘repair effect model’, by which the failure probability in case of crack detection and repair can be assessed. Depending on the specific repair method, it is often assumed that after repair, fatigue life is prolonged with an extended fatigue life or, that the distribution of initial crack size in the repaired structural detail is the same as the original initial crack size, but modelled as a new random variable in simulation. In this paper, the latter assumption is adopted, as the structural performance is indicated by the crack size (the damage state), not by fatigue life.

UNCERTAINTY MANAGEMENT

Uncertainties compromise the benefits of safety control measures, and should be properly managed in an explicit way, and ideally managed in a holistic approach, jointly considering all engineering decisions and uncertainties affecting structural fatigue performance (Faber, 2005). In traditional design codes, uncertainties are taken into account by introducing large safety factors. In this framework, uncertainties are modelled in probabilistic terms, so that optimal design, inspection and maintenance decisions can be made under different degree of uncertainties.

Some identified sources of uncertainties that affect fatigue design, inspection and maintenance decisions are those associated with:

- Fatigue resistance \(N_f\)
- Calculation of stress range \(\Delta \sigma\)
- Fatigue failure criterion \( \Delta \)
- Material property (crack growth rate \( C \))
- Initial flaw/crack size \( a_0 \)
- The smallest detectable crack size of an NDT method \( a_d \)
- Crack size measurement \( a_t \)

In face of uncertainties, engineering decisions on structural safety can be optimized based on probabilistic analysis results so that even safety levels and best compromise between structural safety and costs are achieved.

**RISK ANALYSIS**

In order to develop a rational decision support tool, it is important to take into account the potential loss and cost of failure. This is a prerequisite to quantify the extent of risk mitigation and the benefits of risk mitigation measures. To meet viability requirement, the benefits of risk mitigation measures should outweigh their costs.

**Calculation of probabilities**

The probabilities that need to be calculated are the failure probability at the end of service life, the probabilities of inspection at inspection times, and the probabilities of repairs after inspections. Inspection results are unknown in the design stage, and different results lead to different actions. If the repair criterion is reached, repair actions are implemented following the inspection; otherwise no action. The failure probability is determined by the design plan, inspection plan (inspection methods and times) and repair plan (repair criteria and effect), and the probabilistic characteristics of the crack development. The failure probability is calculated based on event tree analysis. Fig.5 shows an example of event tree analysis for one maintenance intervention, with the repair criterion that all detected cracks are repaired immediately.

![Event tree analysis for one maintenance intervention](image)

**Figure 5. Event tree analysis for one maintenance intervention**

**Failure consequence**

Failure consequence in terms of financial loss due to fatigue failure is quantified in the following way. Based on engineering practices, there is a known optimal fatigue design plan. The optimal design plan is based on fatigue analysis with S-N approach, without due considerations in crack presence, and operational inspection and maintenance. The known optimal design plan is used to derive failure consequence and serves as a reference design. The design plan is optimal from the perspective that it achieves best compromise between design costs and failure risk, i.e. the sum of design costs and failure risk in terms of potential financial loss is minimal. If the formulation for design costs is known, then the failure consequence can obtained based on derivation of Eq. 8.

\[
LCC = C_D + C_F = C_D + p_f \cdot c_f
\]  

(8)

\[
C_F = p_f \cdot c_f
\]  

(9)

where \( C_D \) is design costs, \( C_F \) is failure risk in terms of potential financial loss, \( c_f \) is failure consequence in terms of financial loss and \( p_f \) is failure probability.

With added consideration of the presence of in initial cracks and essential operational inspection and maintenance, the above design plan is no longer optimal with respect to life cycle total costs, and need to be re-optimized in order to achieve best compromise between the failure risk and the total costs of risk mitigation measures (sum of design costs, inspection costs and repair costs).

**LIFE CYCLE COST ANALYSIS**

As mentioned earlier, the optimal design, inspection and maintenance solution is supposed to provide the best compromise between the failure risk and the total costs of risk mitigation measures. The optimal solution is obtained based on derivation of Eq. 9.

\[
LCC = C_D + C_I + C_R + C_F
\]  

(10)

where \( C_I \) and \( C_R \) are costs of inspection and repair respectively.

A formulation for the design costs is proposed based on plate thickness and design failure probability (Eq. 11). The design failure probability is calculated with design fatigue analysis method, which can be S-N approach or FM approach.

\[
C_D = k \cdot \frac{t^2}{p_f^i}
\]  

(11)

The costs of inspection and repair are obtained by adding the cost of inspection and repair is each inspection together. The costs are discounted to the time point of design by an interest rate.

\[
C_I = \sum_{k=1}^{N_I} C_{insp}^k \cdot P_{insp}^k \cdot \frac{1}{(1+i)^t}
\]  

(12)

\[
C_R = \sum_{k=1}^{N_R} C_{rep}^k \cdot P_{rep}^k \cdot \frac{1}{(1+i)^t}
\]  

(13)

where \( N_I \) is the number of inspections in the life cycle; \( C_{insp}^k \) and \( C_{rep}^k \) are cost for the \( k \)th inspection and repair respectively; \( P_{insp}^k \) and \( P_{rep}^k \) are the probability of the \( k \)th inspection and repair are actually performed; \( t_k^i \) and \( t_k^r \) are the timing of the \( k \)th inspection and repair; \( r \) is average interest rate.

Failure risk is given by Eq. 9, and the failure risk associated with a design plan is given by Eq. 14.

\[
C_{F_d} = p_f^d \cdot c_f
\]  

(14)

where \( p_f^d \) is design failure probability, with no consideration of inspection and repair.

**AN ILLUSTRATIVE EXAMPLE**

An illustrative example is carried out regarding optimizing fatigue design, inspection and maintenance decisions for a typical fatigue-prone welded T joint in a ship structure. The annual wave loading is
approximately $N_0 = 5 \times 10^6$, and the required service life is $T_{SL} = 30$ years. A reference design is assumed as $T_o = 25\text{ mm}$, which corresponds to $FDF_o = 3$. The design plan is obtained by engineering experience and is regarded as optimal according to the design costs and failure risk associated with the design plan. However, taking into account operational inspections and maintenance, the design plan is no longer optimal with respect to life cycle total costs.

The decisions that need to be optimized are the fatigue design factor and the inspection interval ($FDF$ and $\Delta t_i$), to investigate their effects exclusively on the failure risk and life cycle costs. Decision on the inspection method is not optimized, and it is assumed that the same inspection method, magnetic particle inspection (MPI), is adopted in all inspections in the lifetime. The repairs criterion is that detected cracks are repaired immediately.

The following function is used for the PoD

$$PoD = F_{a_o}(a) = 1 - \exp(-\frac{a_d}{m_d})$$

(15)

where $m_d$ is mean value of $a_d$. The mean $m_d$ is about $0.89\text{ mm}$ for MPI (Dong and Frangopol, 2016).

Parameters and variables used in reliability analysis can be found in Table 1. The variables are material property $C$, initial flaw size $a_0$, uncertainty associated with calculation of stress range $B$, and the detectable crack size of MPI $a_d$.

### Table 1. Parameters and variables used in reliability analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Unit</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log_{10} C$</td>
<td>Normal</td>
<td>$N^{-1} \cdot \text{mm}^{5.5}$</td>
<td>-12.74</td>
<td>0.11</td>
</tr>
<tr>
<td>$m$</td>
<td>Deterministic</td>
<td>-</td>
<td>3.0</td>
<td>-</td>
</tr>
<tr>
<td>$B$</td>
<td>Normal</td>
<td>cycle</td>
<td>1.0</td>
<td>0.15</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Deterministic</td>
<td>cycle</td>
<td>$5 \times 10^6$</td>
<td>-</td>
</tr>
<tr>
<td>$\log_{10} a_1$</td>
<td>Deterministic</td>
<td>$N^4 \cdot \text{mm}^{-6}$</td>
<td>11.855</td>
<td>-</td>
</tr>
<tr>
<td>$\log_{10} a_2$</td>
<td>Deterministic</td>
<td>$N^4 \cdot \text{mm}^{-6}$</td>
<td>15.091</td>
<td>-</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Deterministic</td>
<td>-</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$m_2$</td>
<td>Deterministic</td>
<td>-</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>$T_{SL}$</td>
<td>Deterministic</td>
<td>year</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Deterministic</td>
<td>mm</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>$a_o$</td>
<td>Exponential</td>
<td>mm</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$a_d$</td>
<td>Exponential</td>
<td>mm</td>
<td>0.89</td>
<td>0.89</td>
</tr>
</tbody>
</table>

### Methods

Five cases are studied comparatively: S1, S2, S3, J1 and J2. These cases are different in terms of whether design and maintenance are jointly optimized (J1 and J2) or separately optimized (S1, S2 and S3), the approach used for design (SN approach for S1, FM approach for the other cases), and the approach for calculating the failure probability $p_f^D$ entering into the formulation for design costs (SN approach for S1, S2 and J1, FM approach for the other cases). The five cases are summarized in Table 2.

Here, joint optimization means that the design and maintenance solution are jointly optimized with the same approach (FM approach), while separate optimization means that the design solution is optimized with an approach (SN or FM approach) in the design stage, and maintenance solution is optimized with FM approach on the basis of existing optimal design plans.

Design costs $C_D$ are given by Eq. 11. Failure consequence $c_f$ is derived by the reference design plan. The unit repair cost is varied and the ratio $c_r/c_f$ is varied from 0.1 to 0.00001. It is assumed that the ratio of unit inspection cost to unit repair cost is a constant 0.1. I.e. Eq. (16) is used.

$$c_i = 0.1 \cdot c_r$$

(16)

### Table 2. The approach used for design, for operational inspection & maintenance and for calculation of design costs (S1, S2, S3, J1 and J2)

<table>
<thead>
<tr>
<th>Case</th>
<th>Approach used for design</th>
<th>Approach used for maintenance</th>
<th>Approach for calculation of design costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>SN</td>
<td>FM</td>
<td>SN</td>
</tr>
<tr>
<td>S2</td>
<td>FM</td>
<td>FM</td>
<td>SN</td>
</tr>
<tr>
<td>S3</td>
<td>FM</td>
<td>FM</td>
<td>FM</td>
</tr>
<tr>
<td>J1</td>
<td>FM</td>
<td>SN</td>
<td></td>
</tr>
<tr>
<td>J2</td>
<td>FM</td>
<td>FM</td>
<td></td>
</tr>
</tbody>
</table>

### Results and discussions

The obtained optimal design and maintenance solutions ($FDF$ and $\Delta t_i$) for the five cases are listed in Table 3-7, together with the achieved life cycle total costs and reliability indexes ($LCC$ and $\beta$) associated with the optimal solutions. Based on the tables, the following discussions are made.

#### Design approach without consideration of maintenance

1. The design failure probability in S2 is calculated with FM approach, while the formulation for design costs is based on the design failure probability calculated with SN approach. On the other hand, the design failure probability in S1 is calculated with SN approach, and the formulation for design costs is also based on the design failure probability calculated with SN approach. So the optimal design solution in S2 ($FDF=5$) is different from that in S1 ($FDF=3$).

2. Due to considering presence of cracks, the failure probability calculated with FM approach $p_f^D$ is larger than that with SN approach $p_f^SN$ for the same design solution. As the failure consequence $c_f$ is the same, so the failure risk with the FM approach is higher than that with SN approach if the same design solution is applied. In order to mitigate the higher failure risk in S2, a safer design solution than that in S1 ($FDF=3$) is optimal, from the perspective of minimizing the total costs of design and failure risk.

3. When the formulation for design costs is based on the design failure probability calculated with the approach used in design, the design approach in S3 (FM approach considering presence of cracks) is beneficial, compared to S1, for all cost structures. For all $c_r/c_f \leq 0.00001$, $0.1$, the expected life cycle costs in S3 is smaller than that in S1.

4. However, when the formulation for design costs is based on the design failure probability calculated with SN approach, it seems that the design approach in S2 (FM approach considering presence of cracks) is beneficial, compared to S1, only when the unit repair cost is high. Only when $c_r/c_f = 0.1$, $LCC=5.31e8$ in S2 is smaller than $LCC=6.05e8$ in S1.
Design & maintenance joint optimization

1. The design and maintenance solution in J1 are jointly optimized with probabilistic FM approach. The design solution in I2 is optimized with probabilistic FM approach, and then the maintenance solution is optimized with the same approach. Compared to J1, it can be seen that the design solutions in I2, although optimal with respect to design costs, are not optimal with respect to life cycle total costs. For example, for the case $c_r/c_f = 0.0001$, the optimal solution for I2 ($DFD=5, \Delta t_i = 6$) result in $LCC=1.19e8$ and $\beta = 4.239$; while for J1, the optimal solution ($DFD=2, \Delta t_i = 3$) result in $LCC=1.94e6$ and $\beta = 4.421$.

2. It is noted that with the decrease of $c_r/c_f$, cost savings of joint optimization in J1, compared with separate optimization in I2, is more obvious. This is because with the decrease of unit repair cost $c_r$, it is more economical to plan more inspections and repairs than to increase the FDF. However, the obtained optimal design solution in I2 is fixed $DFD=5$, which is uneconomical from the perspective of life cycle total costs if the unit repair cost $c_r$ is small.

Table 3. Optimal design and maintenance solution ($DFD$ and $\Delta t_i$) for case S1, and the $LCC$ and $\beta$ associated with the optimal solution

<table>
<thead>
<tr>
<th>$c_r/c_f$</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.0001</th>
<th>0.00001</th>
<th>0.000001</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(DFD, \Delta t_i)_{opt}$</td>
<td>(3,15)</td>
<td>(3,10)</td>
<td>(3,6)</td>
<td>(3,5)</td>
<td>(3,3)</td>
<td></td>
</tr>
<tr>
<td>$LCC$</td>
<td>6.05e8</td>
<td>9.27e7</td>
<td>1.35e7</td>
<td>3.45e6</td>
<td>2.19e6</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.009</td>
<td>2.782</td>
<td>3.647</td>
<td>3.968</td>
<td>&gt;3.968</td>
<td></td>
</tr>
<tr>
<td>$C_D$</td>
<td>2.036e3</td>
<td>2.036e3</td>
<td>2.036e3</td>
<td>2.036e3</td>
<td>2.036e3</td>
<td></td>
</tr>
<tr>
<td>$C_M$</td>
<td>4.56e8</td>
<td>7.28e7</td>
<td>1.06e7</td>
<td>1.18e6</td>
<td>1.57e6</td>
<td></td>
</tr>
<tr>
<td>$C_F$</td>
<td>1.47e8</td>
<td>1.79e7</td>
<td>8.79e5</td>
<td>2.40e5</td>
<td>&lt;2.40e5</td>
<td></td>
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</table>

Table 4. Optimal design and maintenance solution ($DFD$ and $\Delta t_i$) for case S2, and the $LCC$ and $\beta$ associated with the optimal solution

<table>
<thead>
<tr>
<th>$c_r/c_f$</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.0001</th>
<th>0.00001</th>
<th>0.000001</th>
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</thead>
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<tr>
<td>$(DFD, \Delta t_i)_{opt}$</td>
<td>(5,15)</td>
<td>(5,10)</td>
<td>(5,8)</td>
<td>(5,6)</td>
<td>(5,5)</td>
<td></td>
</tr>
<tr>
<td>$DFD$</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$C_D$</td>
<td>4.58e8</td>
<td>4.58e8</td>
<td>4.58e8</td>
<td>4.58e8</td>
<td>4.58e8</td>
<td></td>
</tr>
<tr>
<td>$LCC$</td>
<td>5.77e8</td>
<td>5.77e8</td>
<td>5.77e8</td>
<td>5.77e8</td>
<td>5.77e8</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.479</td>
<td>1.479</td>
<td>1.479</td>
<td>1.479</td>
<td>1.479</td>
<td></td>
</tr>
<tr>
<td>$\Delta t_i$</td>
<td>15</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$LCC$</td>
<td>5.31e8</td>
<td>1.82e8</td>
<td>1.27e8</td>
<td>1.19e8</td>
<td>1.18e8</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.622</td>
<td>3.353</td>
<td>3.724</td>
<td>4.239</td>
<td>&gt;4.239</td>
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<tr>
<td>$C_D$</td>
<td>1.18e8</td>
<td>1.18e8</td>
<td>1.18e8</td>
<td>1.18e8</td>
<td>1.18e8</td>
<td></td>
</tr>
<tr>
<td>$C_M$</td>
<td>3.84e8</td>
<td>6.10e7</td>
<td>8.05e6</td>
<td>8.93e5</td>
<td>1.00e5</td>
<td></td>
</tr>
<tr>
<td>$C_F$</td>
<td>2.89e7</td>
<td>2.64e6</td>
<td>6.49e5</td>
<td>7.44e4</td>
<td>&lt;7.44e4</td>
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Table 5. Optimal design and maintenance solution ($DFD$ and $\Delta t_i$) for case S3, and the $LCC$ and $\beta$ associated with the optimal solution

<table>
<thead>
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<th>$c_r/c_f$</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.0001</th>
<th>0.00001</th>
<th>0.000001</th>
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</thead>
<tbody>
<tr>
<td>$(DFD, \Delta t_i)_{opt}$</td>
<td>(15,15)</td>
<td>(15,15)</td>
<td>(15,15)</td>
<td>(15,10)</td>
<td>(15,8)</td>
<td></td>
</tr>
<tr>
<td>$DFD$</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>$C_D$</td>
<td>6.41e6</td>
<td>6.41e6</td>
<td>6.41e6</td>
<td>6.41e6</td>
<td>6.41e6</td>
<td></td>
</tr>
<tr>
<td>$LCC$</td>
<td>7.64e6</td>
<td>7.64e6</td>
<td>7.64e6</td>
<td>7.64e6</td>
<td>7.64e6</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>3.100</td>
<td>3.100</td>
<td>3.100</td>
<td>3.100</td>
<td>3.100</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Optimal design and maintenance solution ($DFD$ and $\Delta t_i$) for case J1, and the $LCC$ and $\beta$ associated with the optimal solution

<table>
<thead>
<tr>
<th>$c_r/c_f$</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.0001</th>
<th>0.00001</th>
<th>0.000001</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(DFD, \Delta t_i)_{opt}$</td>
<td>(4,15)</td>
<td>(4,10)</td>
<td>(3,6)</td>
<td>(2,3)</td>
<td>(1,2)</td>
<td></td>
</tr>
<tr>
<td>$LCC$</td>
<td>4.96e8</td>
<td>9.05e7</td>
<td>1.35e7</td>
<td>1.94e6</td>
<td>2.89e6</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.353</td>
<td>3.102</td>
<td>3.649</td>
<td>4.421</td>
<td>4.526</td>
<td></td>
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<tr>
<td>$C_D$</td>
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<td>1.81e7</td>
<td>2.03e6</td>
<td>1.46e5</td>
<td>5.11e3</td>
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<tr>
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<td>1.05e7</td>
<td>1.76e6</td>
<td>2.64e5</td>
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</tr>
<tr>
<td>$C_F$</td>
<td>6.17e7</td>
<td>6.36e6</td>
<td>8.72e5</td>
<td>3.24e4</td>
<td>1.99e4</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Optimal design and maintenance solution ($DFD$ and $\Delta t_i$) for case J2, and the $LCC$ and $\beta$ associated with the optimal solution

<table>
<thead>
<tr>
<th>$c_r/c_f$</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.0001</th>
<th>0.00001</th>
<th>0.000001</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(DFD, \Delta t_i)_{opt}$</td>
<td>(15,15)</td>
<td>(15,15)</td>
<td>(15,15)</td>
<td>(9,10)</td>
<td>(5,5)</td>
<td></td>
</tr>
<tr>
<td>$LCC$</td>
<td>2.46e8</td>
<td>2.61e7</td>
<td>3.73e6</td>
<td>7.36e5</td>
<td>1.11e5</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>3.843</td>
<td>3.843</td>
<td>3.756</td>
<td>4.079</td>
<td>&gt;4.079</td>
<td></td>
</tr>
<tr>
<td>$C_D$</td>
<td>1.29e6</td>
<td>1.25e6</td>
<td>5.67e5</td>
<td>9.72e4</td>
<td>1.10e4</td>
<td></td>
</tr>
<tr>
<td>$C_M$</td>
<td>2.45e8</td>
<td>2.45e7</td>
<td>2.59e6</td>
<td>4.89e5</td>
<td>1.00e5</td>
<td></td>
</tr>
<tr>
<td>$C_F$</td>
<td>4.02e5</td>
<td>4.02e5</td>
<td>5.72e5</td>
<td>1.50e5</td>
<td>&lt;1.50e5</td>
<td></td>
</tr>
</tbody>
</table>

CONCLUSIONS

A risk-informed and holistic approach for jointly optimizing fatigue design, inspection and maintenance decisions has been proposed based on integration of risk quantification and life cycle cost analysis. The holistic approach has been implemented on a fatigue-prone structural detail and compared with the separate optimization approach. The effects of design cost formulation and cost structure on the optimal design and maintenance solution and om life cycle costs have been investigated, as well as the deterioration model (approach) used for design. A formulation for design costs has been proposed based on plate thickness and design failure probability. A method has been used to obtain the failure consequence in terms of potential financial loss based on an existing known optimal design plan. It is shown that it is beneficial to adopt the holistic approach for jointly optimizing the design, inspection and maintenance decisions.

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REFERENCES


