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Probabilistic Maintenance Optimization for Fatigue-critical Components with Constraint in Repair Access and Logistics

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Abstract: There is a need to consider repair delay and incurred failure risk in maintenance optimization for some fatigue-critical structural details in marine and offshore structures. For example, in some cases immediate repair may not be feasible due to weather, geographical location and/or technical restrictions. Also, immediate repair may be much more expensive than well-organized delayed repair. Moreover, detected cracks may sometimes be left unattended until more cracks are found and repaired together. This paper investigates a probabilistic maintenance optimization method allowing for repair delay and the incurred failure risk. The maintenance strategy considering repair delay is optimized based on uncertainty modelling, reliability and life cycle cost analysis. Special features of the maintenance strategy and its impacts on fatigue reliability and life cycle costs are discussed on an illustrative example. A method to quantify the risk incurred by repair delay is proposed. It is found that repair delay can result in significant decrease in fatigue reliability if inspection is scheduled in the late stage of service life. The benefits of the maintenance strategy to fatigue reliability and life cycle costs are very sensitive to inspection method. The failure risk incurred by repair delay would be the predominate risk in the life cycle.

Keywords: Integrity Management, Probabilistic Methods, Risk-based Inspection, Maintenance Optimization, Decision Analysis.

1. INTRODUCTION

Fatigue cracking and crack propagation have been tough challenges to integrity management of structures and assets containing a large number of steel and metallic welded structural components. Welded joints are especially prone to fatigue cracking and are the weakest connections in structures due to welding notch, local stress concentration, welding residual stress and initial flaws introduced in the welding process[1]. Fatigue cracks are widespread in large engineering structures as well as in mechanical components, especially prone in the vicinity of welded joints. Crack initiation is usually abrupt and difficult to detect, let alone to predict and prevent. Crack propagation, although it seems to be well understood by Paris’ law, is subjected to high degree of uncertainties associated with material fracture property, initial crack size, stress range calculation, etc. In addition, the propagation stages of some cracks may be very short, and it can be too late to take remedial actions after detection before failure of the structure, given that the time for cracks to propagate from a detectable crack size to final failure is usually short[2]. If remedial actions are not taken in time, crack propagation could lead to a sudden failure of structure.

To address these challenges, inspection and maintenance programs are usually developed, in the very beginning of asset operation, to validate the structural integrity, identify and repair cracks and thus ensure operational safety and reliability. Inspections also make possible to identify potential human and gross errors in design and manufacture, which could not be prevented absolutely in the design stage, and thus to mitigate failure risk. Inspection results can be used to reduce uncertainties in both modelling and design, to update failure probability, and to validate smaller design safety margins and less operational inspections[3]. However, the costs of inspection and maintenance are usually very high and make up a significant part of life cycle costs. For large engineering structures, e.g. ship structures and offshore platforms, inspection and repair tasks are cumbersome and costly as a result of large areas involved and the huge economic losses in case of downtime.
Driven by high industrial relevance, inspection and maintenance optimization have been studied widely across many engineering disciplines and industries with the objective to reduce maintenance costs without compromising structural safety. Probabilistic methods have received much attention as fatigue loading, resistance, crack initiation and propagation process and inspection quality are subjected to high degree of uncertainty, which needs to be treated in a consistent way in safety assessment, for design as well as maintenance optimization. Reliability-based and risk-based inspection planning have been studied so that inspection efforts and resources can be allocated based on reliability and risk levels, and inspection activities can be prioritized[4-7]. In consideration of the costs, inspection and maintenance activities should not only place focus upon the benefits to structural safety and reliability, but also on the trade-off between structural safety and the costs, to achieve optimal design and maintenance decisions from the perspective of whole life cycle, which have been the objective of many studies[8-11].

Previous studies predominantly address the circumstances where repair actions are taken immediately following crack detection, ignoring the time delay between crack detection and repair. However, in practice, a repair delay may be inevitable for some structural details in marine and offshore structures due to difficulties of access and logistics and/or economic considerations. This paper investigates the costs and risks associated with a maintenance strategy where the time delay between detection and repair is taken into account. The strategy is optimized based on probabilistic uncertainty modelling, reliability analysis and life cycle cost analysis. Special characteristics and influences of this maintenance strategy on fatigue reliability and life cycle costs are illustrated. A method to quantify the risk caused by repair delay is proposed and recommendations on reliability, risk, cost-based decision-making are given for an optimal maintenance strategy.

2. FATIGUE DESIGN OF STRUCTURAL DETAILS

The selected structural component is a typical stiffened plate (Fig.1) in ship structures, which comprise of a large number of plates, stiffeners, pillars, and welding joints. One of the most common fatigue-prone details is the welded T joint, which can be found in most of steel engineering structures, e.g. ships, offshore platforms, bridges, and wind turbine foundations, etc. Although stability of the plate is greatly improved by stiffeners, fatigue performance of the welded T joints is likely to be a problem due to poor welding techniques and inclusion of initial flaws in welding materials. Fatigue of joints may lead to structural failure under cyclic stresses, much less than material tensile strength, with adverse financial and environmental consequences. Thus, welded joints represent weak areas in structural integrity, and the integrity and fatigue performance of welded joints should be checked in the design stage and managed in the operational stage[1].

Figure 1: The Structural Component (Stiffened Plate)
The required service life of the joints are 20 years. The ship is trading in the sea environment, in which the frequency of wave loading is about 0.16 Hz, which corresponds to approximately \( N_0 = 5 \times 10^6 \) cycles per year. The fatigue resistance of the joints is given by the following two-segments S-N curves:

\[
N_F \Delta \sigma^{m_1} = \overline{a_1} \quad N_F \leq 10^7
\]

\[
N_F \Delta \sigma^{m_2} = \overline{a_2} \quad N_F \geq 10^7
\]

(1)

where \( m_1 \) and \( m_2 \) are the fatigue strength exponents, and \( \overline{a_1} \) and \( \overline{a_2} \) are the fatigue strength coefficients. The parameters for S-N curve can be found in rules of ship classification societies.

In consideration of inspection accessibility and failure consequences, the joints have been designed for 20 years with a fatigue design factor equal to 3. The allowed maximal equivalent stress range is \( \sigma_e = 19.2 \) MPa, and the design plate thickness is \( T = 25 \) mm. The design parameters are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
<th>Value</th>
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<tr>
<td>( T_{SL} )</td>
<td>Year</td>
<td>20</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>Cycle</td>
<td>( 5 \times 10^6 )</td>
</tr>
<tr>
<td>( \log_{10} \overline{a_1} )</td>
<td>[N, mm]</td>
<td>11.855</td>
</tr>
<tr>
<td>( \log_{10} \overline{a_2} )</td>
<td>[N, mm]</td>
<td>15.091</td>
</tr>
<tr>
<td>( T )</td>
<td>mm</td>
<td>25</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>MPa</td>
<td>19.2</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>-</td>
<td>5</td>
</tr>
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3. MAINTENANCE STRATEGIES AND OPTIMIZATION

3.1. Operational Maintenance

Although the joints have been designed with a fatigue design factor larger than 1, fatigue failure cannot be avoided absolutely. There are some factors that are unforeseen or cannot be fully taken into account in the design stage, e.g., initial cracks or flaws in material, residual stresses in structures, accidental damages, human errors, etc., which necessitate operational maintenance. Operational maintenance help to increase fatigue reliability by carrying out inspections to validate structural integrity or to identify cracks and repair them. The data collected by inspections can be used to validate the design plan or to validate a smaller design safety margin, and thus can facilitate improvement of design codes and rules.

3.2. Maintenance Delay

Studies on maintenance strategy, inspection and maintenance planning and optimization usually assume that cracks detected by inspections are repaired immediately. However, this assumption may not be applicable to some marine and offshore structural details. For example, there are circumstances when immediate repair may be inaccessible after crack detection due to weather, geographical location and/or technical restrictions. Also, sometimes immediate repair may be much more expensive than delayed repair with substantial preparation time, and thus it may be more economical to carry out repair after a time delay when considering the logistics. Moreover, the number of cracks needs to be taken into account when carrying out repair work. Sometimes, detected cracks are not repaired immediately and are left unattended until more cracks are found, so that they can be repaired all together. Clearly, these circumstances of repair delayed in time are of interest for marine and offshore industries but they have not been adequately addressed in practice.

3.3. Maintenance Strategy
This paper pays attention to situations where repair cannot be implemented immediately following detection by inspection, with the objective of quantifying the additional failure risk caused by the repair delay as well as developing a probabilistic basis for optimizing maintenance decision in such scenarios. This kind of maintenance is depicted as MS4 in Table 2, compared with MS3, which is the commonly-used maintenance strategy where cracks detected by inspections are repaired immediately. The intention is to reveal the influences of repair delay on reliability and risk by comparative studies. Two other maintenance strategies without inspection are investigated: MS1 and MS2. MS1 is ‘do nothing’, which means to leave the structural detail as its as-built condition without maintenance intervention assigned. MS2 is ‘time-based repair’, which means to do repair work anyway without inspection at the maintenance intervention time and the intervention time is optimized. The latter is similar to the strategy of time-based replacement. In summary, inspection is assigned under MS3 and MS4, but not under MS1 and MS2. Repair is done immediately at the maintenance intervention time under MS2 and MS3. However, under MS4, at the maintenance intervention time, inspection will be implemented. If the inspection result is detection, repair work will be carried out after a constant repair delay time labelled as $T_d$. A time delay of 4 years is applied in the illustrative example.

### Table 2: Maintenance Strategies

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<tr>
<th>Maintenance Strategy</th>
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<td>MS1</td>
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<tr>
<td>MS2</td>
</tr>
<tr>
<td>MS3</td>
</tr>
<tr>
<td>MS4</td>
</tr>
</tbody>
</table>

#### 3.4. Maintenance Optimization

Herein only one maintenance intervention is planned during the service lives of the joints, to investigate explicitly the influences of repair delay on reliability and risk. In the context of multiple maintenance interventions, the influences of repair delay would be more obvious and significant. The adopted inspection methods are magnetic particle inspection (MPI) and visual inspection (VI). The timing for maintenance intervention is optimized under different maintenance strategies and different optimization objectives (reliability index, life cycle costs).

Prior information on the stochastic nature of crack propagation with time is necessary for probabilistic maintenance optimization. Based on the prior information, optimization objectives can be quantified under different maintenance strategies, and maximized (or minimized) to derive the optimal strategy.

#### 4. DETERIORATION MODEL FOR MAINTENANCE OPTIMIZATION

In this paper, the prior information on stochastic crack propagation is obtained by probabilistic crack propagation modelling, employing Fracture Mechanics (FM) and using Monte Carlo Simulation. According to FM, fatigue damage accumulates in terms of crack propagation under fatigue loading. FM considers the fatigue life of cracked structural details in consideration of initial cracks or flaws $a_0$ in materials. The relationship between the crack growth rate and the local stress range is given by Paris’s law

$$\frac{da}{dN} = C\Delta K^m, \quad \Delta K_{th} \leq \Delta K \leq K_{mat}$$

where $da/dN$ is crack propagation rate; $C$ and $m$ are material parameters; $\Delta K$ is stress intensity factor range; $K_{mat}$ is material fracture toughness; $\Delta K_{th}$ is threshold value for the stress intensity factor range. The stress intensity factor range $\Delta K$ is given by

$$\Delta K = \Delta \sigma Y(a)\sqrt{a}$$

where $Y(a)$ is geometry function and $\Delta \sigma$ is stress range.
Crack growth process is subjected to several sources of uncertainties, e.g., those associated with the initial crack size \(a_0\), calculation of stress range \(\Delta \sigma\), and the crack growth rate \(C\), etc. Meaningful statistical data on \(a_0\) for specific application is difficult to obtain due to measuring and sampling limitations. Here it is assumed that \(a_0\) follows an exponential distribution and the mean value \(E(a_0) = 0.043\). The uncertainties associated with calculation of \(\Delta \sigma\) originate from load description, the method used for calculation of structural response, stress concentration factor and the effect of welding notch, etc. In this paper, the uncertainties associated with calculation of \(\Delta \sigma\) are modelled as an additional variable \(B\), which follows a normal distribution. The statistical parameters for \(B\) are: the mean value \(E(B) = 1\) and standard deviation (SD) \(\mu(B) = 0.15\). Although affected by many factors, the crack growth rate \(C\) is often thought to be lognormally distributed and \(m\) in Equation (2) is adopted to be 3. The statistical characteristics for all variables are listed in Table 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Dimension</th>
<th>Mean</th>
<th>SD</th>
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<tr>
<td>(a_0)</td>
<td>Exponential</td>
<td>mm</td>
<td>0.043</td>
<td>0.043</td>
</tr>
<tr>
<td>(\log_{10} C)</td>
<td>Normal</td>
<td>[N, mm]</td>
<td>-12.74</td>
<td>0.11</td>
</tr>
<tr>
<td>(B)</td>
<td>Normal</td>
<td>-</td>
<td>1.00</td>
<td>0.15</td>
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**5 RELIABILITY-BASED OPTIMIZATION**

**5.1. Limit State Function**

Based on structural reliability theory, a limit state function can be formulated as structural capacity minus load effect or demand. The capacity of structural detail against fracture failure is defined based on serviceability analysis. It is thought that a structural detail is no longer serviceable if through-thickness crack occurs. So, the capacity against fracture failure (the critical crack size \(a_c\)) is often set to be equal to the plate thickness \(T\). The limit state function is given by

\[
L(t) = a_c - a(t)
\] (4)

where \(L(t) < 0\) signifies fracture failure. By definition, the failure probability \(p_f\) and reliability index \(\beta\) are given by

\[
p_f(t) = P(L(t) < 0)
\] (5)

\[
\beta(t) = -\Phi^{-1}(p_f(t))
\] (6)

where \(\Phi\) is cumulative distribution function of standard normal distribution. Event tree analysis and Monte Carlo simulation have been employed to calculate failure probabilities and reliability indexes under different maintenance strategies. It is checked that the samples are large enough so that the calculated failure probabilities and reliability indexes in the following sections are stable.

**5.2. Results and Discussions**

The objective of this paper is to investigate the influence of different maintenance strategies on fatigue reliability, so herein only one maintenance intervention is planned and optimized in the service life. It is noted that that if the reliability index with one maintenance intervention cannot meet the target reliability index, more maintenance interventions can be planned in the service life. The ‘reliability index’ in below figures refers to the fatigue reliability index at the end of service life \(\beta(t = 20)\), which is the smallest during the life cycle. The Figures 2 and 3 present the reliability against maintenance intervention time, adopting MPI and VI inspection methods respectively. It is found that:

1) In both figures, MS2, MS3 and MS4 can help to increase fatigue reliability, compared with ‘do nothing’ under MS1.

2) The optimal repair time under MS2 is \(t_r = 10\) years which is approximately at the middle of the required service life. This achieves a maximum reliability index of \(\beta = 2.40\).

3) The benefits of MS3 and MS4 to fatigue reliability are dependent on the adopted inspection method. The more accurate inspection method can bring a higher reliability index. Under MS3,
adoption MPI (Fig. 2), the maximum reliability index is $\beta = 2.62$ when inspection time $t_i = 9$ years; adopting VI (Fig. 3), the maximum reliability index is $\beta = 2.09$ when inspection time $t_i = 12.5$ years. Under MS4, adopting MPI (Fig. 2), the maximum reliability index is $\beta = 2.17$ when inspection time $t_i = 8$ years. Adopting VI (Fig. 3), the maximum reliability index is $\beta = 1.53$ when inspection time $t_i = 12$ years.

4) The optimal inspection times adopting VI is generally later than the optimal inspection times adopting MPI. Under both MS3 and MS4. This is because VI is more likely to detect the cracks which have propagated for a longer time.

5) Compared with MS3, the repair delay under MS4 in both figures leads to significant decrease in reliability index. Further it can be seen from both figures that there is a larger difference in fatigue reliability between MS3 and MS4 if the inspection is scheduled in the late stage of service life (i.e. after the 7th year). The difference is larger when the applied inspection method is VI. If the inspection is scheduled in the early stage of service life (within the first 7 years), adopting MS4 or MS3 would not result in much difference in fatigue reliability.

Figure 2: Reliability Index (Magnetic Particle Inspection (MPI))

![Figure 2: Reliability Index (Magnetic Particle Inspection (MPI))](image)

Figure 3: Reliability Index (Visual Inspection (VI))

![Figure 3: Reliability Index (Visual Inspection (VI))](image)

6. COST-BASED OPTIMIZATION

6.1. The Framework
As per previous section, planned maintenance activities help to improve fatigue reliability and thus help to reduce failure risk. Different maintenance strategies generally result in different reliability levels and involve different maintenance costs. To make economically-efficient maintenance decisions, the benefits as well as the costs associated with a maintenance strategy have to be quantified and assessed before maintenance work is implemented, and rational decision-making basis have to be developed so that different maintenance strategies can be assessed within one framework.

The decision-making framework is developed based on the concept of risk, which is defined as the product of failure probability and failure consequence. If the failure consequence is quantified in financial terms, then the failure risk can be formulated in financial terms and understood as potential loss associated with a structural (design and maintenance) plan. In such way, any design plan and maintenance activity can be linked with a certain level of failure risk and potential financial loss. The failure risk can be integrated into the life cycle total costs (LCC). Conversely, design plan and maintenance strategy can be assessed and optimized based on LCC.

This paper is focused on the operational maintenance and the design plan is not optimized. This is to say, design parameters are the same for all maintenance strategies, so the design costs are also the same and thus are not included in the LCC. LCC is comprised of inspection costs, repair costs, and failure risk, as follows:

\[
C_l = \sum_{k=1}^{N_f} p_i^k \cdot c_{i0}^k \cdot \frac{1}{(1+r)^{t_i^k}}
\]

\[
C_r = \sum_{k=1}^{N_R} p_r^k \cdot c_{r0}^k \cdot \frac{1}{(1+r)^{t_r^k}}
\]

\[
R = p_f \cdot c_f
\]

where \( R \) is failure risk, \( c_f \) is failure consequence; \( N_f \) and \( N_R \) are the number of inspections and repairs in the service life; \( c_{i0}^k \) and \( c_{r0}^k \) are costs for the \( k^{th} \) inspection and repair respectively; \( p_i^k \) and \( p_r^k \) are the probability of the \( k^{th} \) inspection and repair are actually performed; \( t_i^k \) and \( t_r^k \) are the timing of the \( k^{th} \) inspection and repair; \( r \) is average annual interest rate.

To reveal the influences of repair delay on fatigue reliability and LCC, one maintenance intervention is planned in the service life in this study, e.g. \( N_f = N_R = 1 \). In the context of multiple maintenance interventions, the influence of repair delay on total life cycle costs would be more significant. Other input parameters are set as: \( q_f = 100, c_{i0} = 10, c_{r0} = 0.1, r = 0 \). The interest rate is a socio-economic factor which can be taken into account when long duration of service life is considered. In this paper interest rate is not considered so that the LCC is influenced by maintenance strategy, inspection time and inspection method, rather than socio-economic factor.

6.2. Results and Discussions

Figures 4 and 5 give the LCC against maintenance intervention time while adopting MPI and VI inspection methods respectively. The unit of the LCC in the figures is dependent on the unit of an inspection cost. It is found that:

1) Generally, compared with MS1 ‘do nothing’, MS2, MS3 and MS4 can help to reduce LCC, if intervention time is scheduled properly within specific time periods, as indicated in the figures, where the LCCs under MS2, MS3 and MS4 are less than that under MS1. In addition, it can be seen that LCC is reduced by a larger margin under MS3 and MS4 than MS2, relative to MS1. Also, the LCC is reduced for a longer duration under MS3 and MS4 than MS2.

2) Based on LCC, the optimal inspection time while adopting MPI (Fig. 4), is \( t_i = 8 \) (LCC=2.99) under MS3 and \( t_i = 7.5 \) (LCC=3.56) under MS4. The optimal inspection time while adopting VI (Fig. 5), is \( t_i = 12.5 \) (LCC=3.24) under MS3 and \( t_i = 12 \) (LCC=7.11) under MS4.

3) To assess the significance of different optimization objectives of reliability and cost, the optimal maintenance intervention times in figures 2 are compared with those in figure 4, and figure 3 compared with figure 5. It can be seen from that optimal inspection times obtained by cost-based optimization while adopting MPI (Fig. 4) under both MS3 and MS4 are a little earlier than those...
obtained by reliability-based optimization (Fig. 2). However, the optimal inspection times obtained by cost-based optimization while adopting VI (Fig. 5) under both MS3 and MS4 are the same as those obtained by reliability-based optimization (Fig. 3). This is probably because that the probability of detection is very low while adopting VI, so the expected inspection and repair costs are very small compared with failure risk. In such circumstance, LLC minimization is almost in line with failure risk minimization and reliability maximization. However, as MPI is much more accurate than VI, the probability of detection while adopting MPI is much higher and the expected inspection and repair costs account for a significant part of LCC. It is fair to say that the cost based optimum inspection time is earlier than the reliability based optimum inspection time when the inspection method is accurate and inspection and repair costs are significant.

4) It is noticeable that the minimum LCC while adopting MPI under MS4 (LCC=3.56) is somewhat higher than that under MS3 (LCC=2.99), while the minimum LCC while adopting VI under MS4 (LCC=7.11) is a much higher than that under MS3 (LCC=3.24). This means that the cost savings brought by MS4, relative to MS1, are more sensitive to inspection method than MS3.

7. FAILURE RISK CAUSED BY REPAIR DELAY

7.1. The Influence of Repair Delay
Compared with MS3, the influence of repair delay under MS4 on failure risk and LCC lies in three aspects, all of which are relevant to the case of detection.

1) First of all, in case of detection, repair would not be done immediately under MS4, and there is a failure probability and risk during the repair delay time $T_d$, which would have been mitigated by repair under MS3.

2) Then, in case of detection, repair would be carried out but at different times under MS3 and MS4. Failure probabilities and risks of repaired structural detail under MS3 and MS4 would be different due to different time lengths of crack propagation.

3) In addition, the probabilities of repair and expected repair costs are different under MS3 and MS4, although the probabilities of detection are the same. In case of detection, the probability of repair under MS4 is lower than that under MS3 because under MS4 there is a failure probability during the repair delay time $T_d$, as mentioned in 1).

As discussed above, it should be noted that if cracks are detected and not repaired immediately, there would be a failure risk $R_{delay}$ during the repair delay time, compared with immediate repair. The risk incurred by repair delay may be influenced by the stochastic nature of crack propagation, inspection time, inspection method, failure consequence and the repair delay time. The following formulation is proposed to quantify the failure risk incurred by repair delay

$$R_{delay} = p^f_{delay}(t) \cdot c_f$$

where $p^f_{delay}(t)$ is the probability that at a planned inspection time $t$th year, the structure detail is survived and a crack is indicated by an inspection but not repaired, and by time $(t + T_d)$th year, the structure detail has failed.

The changes of the failure probability and risk of repaired structural detail under MS4, compared with those under MS3, are given by

$$\Delta p^r_{repaired} = p_f(T_{SL} - t - T_d) - p_f(T_{SL} - t)$$

$$\Delta R^r_{repaired} = \Delta p^r_{repaired}(t) \cdot c_f$$

Here it is assumed that the repaired structural detail returns to its initial damage state. In other words, the distribution of initial flaw size and stochastic of crack propagation of the repaired structural detail is the same as those of the original one.

The change of expected repair costs under MS4, compared with those under MS3, is given by

$$\Delta C_r = -\Delta p^f_{delay}(t) \cdot c_{r0}$$

### 7.2. Results and Discussions

Figures 6 and 7 give the failure risk during incurred by repair delay $\Delta R_{delay}$, the change of failure risk of repaired structural detail $\Delta R_{repaired}$ and the change of expected repair costs $\Delta C_r$ under MS4, compared with MS3, against maintenance intervention time while adopting MPI and VI respectively. The risks are understood as potential costs. It should be noted that a negative value in the figures indicates that compared with immediate repair (MS3), there is a lower expected repair cost under the maintenance strategy of delayed repair (MS4).

The results show that:

1) The values of $\Delta R_{repaired}$ and $\Delta C_r$ are very small and negligible compared with $R_{delay}$. Hence, attention should be paid to $R_{delay}$.

2) $R_{delay}$ increases with the maintenance intervention time (inspection time).

3) The value of $R_{delay}$ is not affected significantly by the inspection method.

In order to fully understand the failure risk incurred by repair delay $R_{delay}$ under MS4, it has been compared with the failure risk $R$ during the whole service life. The results are shown in Figures 8 and 9.
9 for MPI and VI respectively. It is interesting to note that $R_{delay}$ accounts for a major part of the minimum failure risk $R$. That is to say, if the maintenance intervention time is obtained by reliability-based optimization, the failure risk incurred by repair delay $R_{delay}$ would be the predominate risk in the life cycle. This tendency is especially pronounced while adopting MPI.

**Figure 6: Changes Caused by Repair Delay (MPI)**

**Figure 7: Changes Caused by Repair Delay (VI)**
8. CONCLUSIONS

The maintenance strategy considering repair delay has been investigated and optimized based on probabilistic uncertainty modelling, reliability analysis and life cycle cost analysis. Special features of the maintenance strategy and influences of repair delay on fatigue reliability and life cycle costs have been discussed on an illustrative example. This paper has proposed a probabilistic method to quantify the risk incurred by repair delay. Based on probabilistic investigations, the following conclusions have been made:

1) Repair delay can result in significant decrease in reliability index compared with immediate repair if inspection is scheduled in the late stage of service life. If the inspection is scheduled in the early stage of service life, there isn’t much difference in fatigue reliability whether repair is done immediately or not. This tendency is more pronounced when more accurate inspection method is adopted.

2) The optimal inspection time obtained based on life cycle cost minimization is earlier than that obtained by fatigue reliability minimization. This is true for all maintenance strategies and is pronounced by the higher accuracy of the inspection method.

3) The savings in life cycle total costs brought by the maintenance strategy of delayed repair are much more sensitive to inspection method than those brought by the maintenance strategy of immediate repair. The savings of life cycle total costs brought by the maintenance strategy of delayed repair drop very quickly with the reduced inspection quality.
4) Compared with immediate repair, delayed repair represents significant failure risk incurred by the repair delay; while the changes in the failure risk of repaired structure detail and in the expected repair costs are negligible.

5) The failure risk incurred by the repair delay increases with inspection time but is not affected significantly by the inspection method.

6) If the maintenance intervention time is obtained by reliability maximization, the failure risk incurred by the repair delay would be the predominate risk in the life cycle. This tendency is more pronounced while adopting more accurate inspection methods.

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