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COST 345

Procedures Required for the Assessment of Highway Structures
Numerical Techniques for Safety and Serviceability Assessment
Report of Working Groups 4 and 5
EXECUTIVE SUMMARY

This document treats the following aspects of the assessment of existing highway structures:

- **Levels of assessment:** Five levels of assessment are recommended varying from simple but conservative to complex but accurate.

- **Uncertainty modelling:** An integrated approach to traffic loading, structure condition and structural response is described.

- **Load modelling:** There can be considerable unused capacity in highway structures that are not subjected to the full design levels of traffic loading. This can be calculated from traffic weight statistics obtained from a weigh-in-motion system.

- **Modelling materials for assessment:** The processes are reviewed by which material properties in existing structures can be estimated.

- **Structural response modelling:** The types of analysis appropriate to the five recommended levels of assessment are proposed.

- **Target reliability levels:** The levels of reliability considered appropriate for highway structure assessment are discussed.

- **Reliability analysis:** The available procedures for full reliability analysis of highway structures are reviewed.

All of these topics are covered in detail in the following chapters. It is not possible in a report of this nature to provide sufficient details for an engineer to use all of the methods by reading this report alone. The report aims to provide sufficient information for engineers and network managers and authorities to choose the appropriate methodology for assessing their structures. It also aims to inform Engineers charged with assessment about some of the procedures available. It is sincerely hoped that this report will contribute to the continued safety and serviceability of the land transport fixed assets in Europe and elsewhere.
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Chapter 1 INTRODUCTION

1.1 Procedures for assessing highway structures – project overview

A new European action, known as COST 345, started in 1999 to study procedures for the assessment of highway structures. It was supported by the European Commission and involved experts from 14 countries. The purpose was to identify the procedures and documentation required to inspect and assess the condition of structures such as bridges, earth retaining walls, tunnels and culverts. The action has also:

- defined the requirements for future research work,
- provided information on the stock of highway structures - this can be used as input to budgetary plans for maintenance works and operating cost models and also for establishing recommendations for construction options, and
- identified those structures not amenable to simple numerical analysis.

Bridges, earth retaining walls, tunnels, culverts and the like make up a substantial proportion of the fixed assets of the land based transportation infrastructure. With the expansion of the road and railway networks in the last two centuries, large stocks of such structures have built up in many countries around the world: some existing structures on the highway network predate the 20th century and a number of masonry arch bridges date back to the Roman times. Many of these structures are now suffering from deterioration and defects and their continued safety and serviceability need to be ensured through formal inspection and assessment regimes followed by repair and strengthening as necessary.

Whilst considerable effort has in recent decades been put into the development of new standards and codes for the design of new structures, comparatively little has been done on the development of guidance documents covering the assessment of existing structures. This European Commission action aims to redress that imbalance.

In the absence of adequate documentation for inspection and assessment the only available option for assessing structural adequacy will be to use design standards for new structures. However, such an approach may be inappropriate and over-conservative for a wide range of structures. In some cases it may lead to the unnecessary replacement or strengthening of existing structures with all the attendant costs of traffic delays. What is required, therefore, is a system of assessment within which whole life performance of the structures is qualitatively or quantitatively examined against the factors of safety specified in current design standards. The COST 345 action aims to identify what the features of such a procedure should be and what is required for it to be established.

In European terms, the development and application of reliable inspection, assessment and maintenance procedures for the highway network would ensure the continued high performance of the network and, potentially, could save billions of Euros in construction activities, maintenance and traffic delay costs.

The end-users of the results of this action will include international, national and local government highway organisations and agencies, construction companies and the technical and scientific world. At the international and national levels, the findings of this study could influence matters of policy regarding safety and the administration and operation of highway networks. It will also be of interest to different parts of the institutions for decision-making in the areas of transport policy, legislation, and research and development.
At a regional or local level, engineers charged with the upkeep of a section of a road network will benefit from the availability of information on methods of inspection, assessment and analysis, and from improved whole life cost models. Together these will improve the efficiency of operations, provide more reliable predictions of expenditure, and assist in the planning and execution of inspection and maintenance works. Such information will also be of benefit to road operators and contractors concerned with maintenance works.

This report deals with the use of numerical techniques for assessing the safety and serviceability of existing highway structures. It is an integral part of the COST 345 working group reports, following on from “Inspection” and “Condition Assessment” and providing the basis for decision making on “Remedial Measures”.

1.2 Purpose of assessment

When an existing highway structure is found to have deteriorated (corrosion of steel sections, concrete in poor condition etc.) or when a fault or damage is discovered, it is relatively easy to determine the type and extent of the repairs necessary. In the absence of a detailed assessment, the purpose of such repairs would be to bring the structure to its original state as far as practicable.

On the other hand, when an existing structure has no apparent problem, but for example, traffic weights and volumes have increased, or say an aspect of the original design rules is now found to be inadequate, it is very difficult to judge if the structure now needs to be strengthened or not. Any work done to a structure in perfect condition seems to be a waste of money and effort. Yet if nothing is done, the structure will be at some risk. Formal calculation based assessments are necessary to deal particularly with such cases.

The assessments are carried out to identify which structures are at unacceptably high level of risk so that the appropriate remedial actions, i.e. strengthening, weight or width restriction etc., can be taken. To illustrate the process, let us assume that, if nothing is done, one structure will one day collapse under load. To prevent this happening, say, 10 structures will be identified through assessments to be at unacceptable risk and are strengthened. In order to identify these ten structures, say, 200 structures which are considered to be potentially in the risk category are put in the assessment programme.

It may appear from the above hypothetical scenario that 9 structures are strengthened unnecessarily and 199 structures are assessed unnecessarily. This represents a considerable waste of money and effort, as each strengthening on average can cost about 100,000 € and each assessment may cost 3000 € on average. However, this is a very necessary process. The reasons are as follows:

(1) It is not possible in advance to know which particular structure will eventually collapse. It may not necessarily be the weakest structure as a slightly stronger structure instead may come under an extremely high load and collapse. Hence the assessments have to identify a number of structures which may collapse or be seriously damaged under the combined effects of extreme load and structural weakness.

(2) One option would be to do nothing and accept that an occasional structure will collapse from time to time. This is not advisable for the following reasons:

(a) It would be politically unacceptable and no public authority can knowingly subscribe to it with the potential risk of death and injury, as well as the loss of amenity, that it implies.

(b) There is no guarantee that only an odd structure will occasionally collapse if nothing is done. Evidence is that in infrastructure systems including highway structure stocks,
where the numbers of similar elements are large, problems come in groups. Certainly an increasing number of structures can be seriously damaged in a relatively short period of time if preventative actions are not taken, causing enormous problems for the maintaining authorities.

Hence, the realistic approach is to carry out assessments of any potentially at risk structure groups, but improve the assessment methods to make them as accurate as possible so that the wasted assessments and strengthening are kept to a minimum. All research and development work in this area through the years has been aimed at achieving this goal.

1.3 Significance of structural assessment

Structural failure is not acceptable to the public; hence the order of the probability of failure inherent in the assessment criteria is very small. When a structure is assessed to be sub-standard, it does not mean therefore that it will necessarily fail or collapse. However, if such structures were left in large numbers without remedial action, there may be an unacceptable risk that a collapse in service would occur. The assessments are based on probabilities and therefore it is impossible to know beforehand which structures would actually fail in practice.

The absence of any apparent signs of distress in a structure does not mean that it is structurally adequate. When the failure mode is likely to be brittle, there may be no early warning signs. Furthermore, end restraint or composite action which cannot be relied upon at all times in certain older structures, may temporarily prevent such a structure from showing distress.

Structure assessments are generally carried out using formal calculations based on standard specified rules. This has given the impression that the process is precise and the result must be followed without question. This approach can sometimes lead to inflexibility when implementing the results of assessments in difficult situations, for example when the closure of a particular structure is simply not possible. Many of the factors that bring about structural collapse, however, cannot be taken into account in calculation, e.g., undiscoverable condition, freak events. Furthermore, there are many approximations and uncertainties in the assessment process and these should be examined and rational methods developed to make the assessment process more comprehensive (i.e., inclusive of the component uncertainties) and flexible, and yet consistent when carried out by different engineers. Nevertheless, calculation-based assessments are the only practical means available at present for gaining assurance about the adequacy of the whole stock of highway structures. The methods necessary for carrying out such assessments are described in this report.

1.4 Numerical methods of assessment

Currently, the rules used in highway structure assessment are provided mainly by design standards with additional standards relating to testing methods, including load testing. In some countries, the design standards used can be either the current standards, or those that were current at the time of construction. In others, only current design loading specifications can be used, although these can be modified specifically for assessment and can include reduced load levels based on restricted traffic conditions. Additional requirements can be given regarding exceptional traffic loading.

It is important to note that the rules set down in a design code constitute a set of prescribed rules that are only valid within a certain context. For assessment, situations often exist which render design codes inapplicable either because of existing structural condition or because of the presence of non-conforming details. This is particularly true in the case of older bridges and current design codes have to be interpreted carefully before being used.
The design codes present safety margins that, in general, exceed those that are reasonable to accept for the assessment of existing structures. This is because the level of knowledge of existing structures and the actual traffic conditions can be determined with a greater degree of certainty, as they can be observed and/or measured. Thus partial safety factors can be reduced while maintaining the same level of structural safety. Knowledge of the structures can be increased by further investigations and this can justify further reductions in partial safety factors. The partial safety factors take into account variability in structural behaviour and loading. The required safety margin is reduced with age. Finally, the optimum safety level in a new design is greater than in an existing structure because of the large costs associated with rehabilitation relative to new construction.

The use of such principles leads to complexity in the assessment calculations. More importantly, however, is the selection of appropriate safety level that at present is very difficult because of the lack of rules on how such a choice is made. Selecting an appropriate level of safety may include consideration of socio-economic conditions as presented in Chapter 7.

It is clear that the establishment of principles and procedures to be used for the assessment of existing structures is needed because some aspects of assessment are substantially different from new design, and require knowledge and procedures beyond the scope of design codes. In addition, structure assessment should be carried out in stages of increasing sophistication, aiming at greater precision at each higher level. In order to save structures from unnecessary rehabilitation or replacement (and therefore to reduce owners’ expenditure), the engineer must use all the techniques, all the methods and all the information available in an efficient way. Simple analysis can be cost effective if it demonstrates that the structure is satisfactory, but if it does not, it can present major drawbacks regarding the structure under study and more advanced methods should be employed.
Chapter 2    LEVELS OF ASSESSMENT

2.1 General

The purpose of assessment is to check structures for their capacity to safely carry or resist specific loading levels and to identify those structures which have an unacceptable probability of failure, either in part or complete collapse, under extreme conditions of loading and material weakness.

If a structure is found to be inadequate in an assessment, it becomes necessary to replace or strengthen it to make it safe for the required loading. Otherwise, as a temporary measure, the loading needs to be restricted in some way in order to carry on using the structure. Repairs and strengthening and traffic disruption resulting from them can be costly to the owners and the users of the structures, and hence, the assessment of doubtful structures should be carried out as accurately as possible. At the same time, theoretically complex and rigorous assessments can themselves be very costly and time consuming. It is therefore advisable that, when a structure fails the initial assessment, the assessment should be taken to more rigorous levels, taking into account the cost and time implications of doing so and the likelihood of changing the result. In some cases, the end result will quickly become self-evident and the process can be terminated at an early stage.

Methods of assessment have been the subject of considerable research and development effort in recent years; as a result it is now possible to carry out assessments in five distinct levels. These levels of assessment are numbered 1 to 5, with Level 1 being the simplest and Level 5 the most sophisticated. Means for carrying out assessments at Levels 1, 2 and 3, are now generally available. Levels 4 and 5 involve structural reliability calculations and are currently only used by experts.

2.2 Level 1 assessment

This is the simplest level of assessment, giving a conservative estimate of load capacity. At this level, only simple analysis methods are necessary, and partial safety factors from the assessment standards are used.

2.3 Level 2 assessment

This next level of assessment involves the use of more refined analysis and better structural idealisation. The more refined analysis may include grillage analysis or possibly finite element analysis when it is considered that these may improve the result. Non-linear and plastic methods of analysis (e.g., yield line or orthotropic grillages) may also be used.

This level also includes the determination of characteristic strengths for materials based on existing available data. This may be in the form of existing mill test certificates or recent tests on another similar structure. No new tests would be carried out on the structure for a Level 2 assessment.

2.4 Level 3 assessment

Level 1 and Level 2 assessments make use of Assessment Live loadings given in the standards or estimated as applicable generally to the network. Level 3 assessment includes the option to determine and use structure-specific loading. For many bridges, particularly where they are located on a lightly trafficked road, the use of bridge-specific traffic loading can be quite beneficial.
Level 3 assessment may also make use of material testing to determine characteristic strength or yield stress. Furthermore, in Level 3, consideration may be given to the use of load testing in the form of diagnostic load tests. It should be noted however that, pending further research, proof load tests must be performed with the greatest possible care in order to avoid damage of the structures and are not recommended for inexperienced users.

2.5 Level 4 assessment

Levels 1 to 3 assessments are based on code implicit levels of safety, incorporated in the nominal values of loads and resistance parameters and the corresponding partial safety factors. The corresponding reliability is related by implication to past satisfactory performance of the structure stock or through calibrations where these have been carried out.

Any calibration involves an element of averaging which makes the results acceptable for the bulk of structures of the type concerned. Nevertheless, the resulting rules may be over conservative for a particular structure which may be significantly different in some way from the norm used in the calibration. Level 4 assessments can take account of any additional safety characteristic to that structure and amend the assessment criteria accordingly.

Any changes to the criteria used in this level may be determined through rigorous reliability analysis, or by judgemental changes to the partial safety factors.

In the deliberations involving Level 4 assessments, care should be taken not to double count structure-specific benefits which have already been taken into account. For instance, if system analysis based methods such as the yield line method have been used in Levels 2 or 3, system effects should not be utilised in Level 4.

Level 4 assessment may be particularly beneficial in the following circumstances:

1. The bridge assessment criteria have been primarily devised for longitudinal effects on main deck members. All other elements such as cantilever slabs, cross beams, pier heads etc. may be examined in Level 4 for determining element-specific target reliability.

2. The whole life reliability of a structure, in the absence of any significant deterioration, increases from the day it is constructed to the end of its functional life. This effect has not been taken into account in the present criteria.

3. The failure of a retaining wall adjacent to a minor road will obviously have much lesser consequences than the failure of a major bridge. Such considerations may be used in a Level 4 assessment.

2.6 Level 5 assessment

Level 5 assessment involves reliability analysis of particular structures or types of structure. Such analyses require statistical data for all the variables defined in the loading and resistance equations. The techniques for determining the probability of failure from such data are now available and can be undertaken relatively easily in modest time frames.

Level 5 assessment provides greater flexibility but it should be noted that the results are very sensitive to the statistical parameters and the methods of structural analysis used. At present therefore Level 5 assessment should not be used in conjunction with prescribed target reliability, as there is no guarantee of achieving consistency in different assessments. However Level 5 may be used if the target reliability is determined specifically by the same Assessing Organisation for a class of identical
structures or structural elements, e.g., pier cross-heads, taking the reliability of the structures as
designed in respect of the assessment load, as the target reliability.

Level 5 assessments require specialist knowledge and expertise.

2.7 Whole life assessment

2.7.1 General

It is to be noted that the assessment of the structural performance of highway components, as carried out according to currently used codes and standards, determines the adequacy of a structure at the time of the assessment and, more generally, at the design stage (new structures). However assessment of structures is an essential part of the management and operation of the road where conditions of safety and mobility must be guaranteed at all times: this implies the evaluation of future maintenance needs. To reach this goal it is necessary to predict the future performance of structural elements/components, in particular under different maintenance strategies, and to cost the various options using the principles of whole life costing (see Appendix A).

The whole life performance profile of a structure may be determined in terms of its available safety factor or load carrying capacity or reliability. Such a profile depends on the as-built capacity of the structure, material deterioration in future years, variations in loads and past maintenance activities.

A number of different sources of uncertainty are therefore inherent in this process, related to:

1. structural capacity and current loading,
2. time related performance and corresponding maintenance works,
3. amount of rehabilitation work,
4. unit cost of work.

Reliability analysis and probabilistic methods are useful tools for dealing with the uncertainties related to these values.

2.7.2 Principles of whole life costing of existing structures

Highway structures are an important part of a nation’s road system. Maintaining and repairing existing structures are a major economic concern for many governments and local authorities around the world. For the organisation and performance of structure management activities, different computerised management systems are used today across Europe by the road directorates of each country. Many structures were built with technical codes and requirements that are outdated today. The reason for this is that they were not designed for very long life spans and for the much higher loads we require today. It is generally expected that during their service life, highway structures can fulfil certain demands such as traffic safety, continuous traffic flow and a designed load carrying capacity. Regular and systematic inspection of the existing stock should be performed in order to verify that such demands are met at all times. These inspections, accurately documented, are essential for the road authority in the planning of the necessary maintenance and repair works, thus contributing to cost-effective structure management overall. Inspections are also used to establish the cause of structural deterioration and the collection of enough data to be incorporated in the assessment procedures. The time intervals for inspections, maintenance and repairs depend on the type of structure, the experience in the different countries, the financial resources available, the ADT value, and deterioration factors such as freeze-thaw cycles, chlorides, de-icing salts, marine environment, carbonation, and traffic load increase. Inspections, maintenance and repairs will of course constitute a part of the whole life costing for the owner of the bridge.
As an example, in Sweden all bridges are cleaned every year after the winter season and lightly surveyed. More detailed inspections are performed every three and six years. These kinds of measures will of course vary between different countries and different owners – see Appendix A, Table A.1.

Maintenance will of course always be needed. Typically railings, lampposts and other steel details need repainting regularly. Railings are often damaged by cars. The time intervals and the probability for these kinds of incidents are highly dependent on the structure type and the \textit{ADT}-value. Some suggested maintenance intervals, according to the Swedish National Road Administration, are listed in Appendix A, Table A.2.

Unsurprisingly, taxpayers want to get as much “value for money” as possible. The “value” is firstly a road system as effective as possible and with as few interruptions as possible for maintenance and repair. There are other values of importance concerning the environment, preserving energy and to use as little non-renewable material resources as possible. Very important concerns are also those of traffic security. Other “values” could be aesthetic or the preservation of old structures of historical interest.

The “money” in the “value for money” requirement could be investment cost, life cycle cost with or without user costs. There are many different views on how to calculate these kinds of cost. All factors including the effect on traffic, life of repair and the residual life of the structure must be taken into account in order to enable bridges to be maintained at minimum overall cost (see Appendix A where some important formulas for calculating life cycle cost are given).

For the cost assessment, the deterioration rate of the different parts of the structure must be known in order to be able to assume maintenance and repair time intervals. This is not an easy task; the best information for assuming the time intervals may be historical data from actual bridge inspections and repairs.

2.8 Inspection: Level 0 assessment

In general, the assessment of structures consists of quantitatively evaluating their structural performance. This can be done at different and increasing levels of detail and complexity, as described, and with the tools and methods reported in the rest of this document. However, it is worth mentioning at this point that the road owners and operators make extensive use of assessment based on visual inspections or based on measurements of physical/chemical parameters (results of testing) for the purposes of monitoring highway structures.

Even if such results are extremely conservative, they allow:

1. a rapid evaluation of the overall conditions of large populations (i.e., the entire bridge stock),
2. prediction of future trends based on past observations and experience,
3. easy collection of data for defining maintenance and repair strategies and their associated costs.

Visual observations (extent and severity of damage) and simple tests are used to assess the conditions of structures based on an arbitrary scale, generally ranging from “good” condition to “very poor” condition. Their main advantages are their simplicity and repeatability, the low cost and the easy link with maintenance strategies, as maintenance options may be directly associated with condition ratings and classes of visual deterioration.

One of the main disadvantages of visual inspections is the subjectivity of the assessment as it depends on the experience and judgement of the engineer. Moreover, visual observations cannot de-
tect latent defects or defects at early stages of deterioration (e.g., initiation of corrosion) and no direct information may be derived on the structural deterioration.

An approach based on the measurements of physical and/or chemical parameters has the advantage that knowledge of deterioration mechanisms is increased, as testing can be conducted regularly. As a consequence, a feedback for design is provided to improve durability and reduce deterioration rates.

Main disadvantages may be identified in the fact that often measurement techniques only account for a single mode of deterioration and each element of the structure may experience different deterioration processes at different stages in its life. Measurements may sometimes not be made at the exact points where deterioration is detected and taking samples and cores do disturb the structure.

Finally, due to the difficulty of accessing the structure, testing and in some cases inspection may have heavy consequences for the traffic flow.
Chapter 3  UNCERTAINTY MODELLING

3.1  General

If all information is known about a structure, including all of the material properties, all of the loads to which the structure is and will be subjected and how the structure does and will behave when subjected to these loads, an engineer can say whether or not a structure will survive for a certain period of time. Since it is not possible to know each of these exactly, engineers must make conservative approximations and estimations, which allow structures to be designed and assessed. Each approximation and estimation is associated with uncertainty. The sources of these uncertainties are often classified as either:

1. natural – uncertainties due to the unpredictability of loads, such as wind, earthquake, snow, etc…, and the differences in mechanical behaviour of the materials in a structure; or
2. human – uncertainties due to intended and unintended departures from the optimal design, such as approximations and calculation errors during the design phase or use of non-specified materials and changes without re-analysis during the construction phase.

In the assessment of existing structures, engineers do not have to work with the same uncertainties that existed during the design phase. As the structure exists, the loads to which it is subjected can be measured to give a more accurate portrayal of the extreme loads to which the structure is and will be subjected in the future. The material properties can be measured, which often has the effect of removing the conservative bias that the engineer had at the time of design. The overall structure can be tested to determine more accurately the structural behaviour and to verify the structural response models that were used.

The uncertainties in the evaluation of structures are due to inherent variability, imperfect modelling and estimation error. These uncertainties can be incorporated into the assessment processes using probabilistic methods.

3.2  Inherent variability

The assessment of highway structures involves the evaluation of many processes and phenomena that are inherently random, meaning that the values required for assessment (such as wind loads) due to these phenomena, are unpredictable. In such cases it can only be said that their values will be within a certain range of values and that some of these values are more likely to occur than others. There is significant variability in most engineering information.

Uncertainties due to inherent variability can be divided based on whether or not they can be easily affected by human intervention. Uncertainties that can be modified by human intervention include uncertainties associated with material properties, such as concrete or steel strength, and with element geometry, such as the dimensions of concrete deck slabs. These uncertainties are strongly affected by the use of production and quality control methods (Kerkson & Bradley 1991). Uncertainties that cannot easily be modified by human interventions include uncertainties associated with snow loads, wind speeds, and earthquake ground motion intensities.

3.2.1  Imperfect modelling and estimation error

Uncertainty associated with imperfect modelling and estimation error is introduced through the mathematical or simulation models used to represent real-life phenomena. The models used by en-
engineers are only imperfect representations (albeit in various degrees) of the real world. Consequently, the estimations made using these models contain uncertainty. Discussions of the models used to estimate loads, material properties, and structural response are given in chapters 4, 5 and 6, respectively.

Model uncertainty has both, a systematic component (bias) and a random component. The systematic component (e.g. a constant underestimation of material strength) is often built into design equations to ensure that engineers are conservative. The random component is due to the inability to define the model exactly. For example, the estimation of concrete strength in a bridge often requires using a mathematical model. As the selected parameters of such model are based on engineering judgement and on a number of representative in-situ tests, the mathematical model will always have some uncertainty due to the estimation error. Naturally, this uncertainty diminishes as number of tests increases.

3.3 Evaluating uncertainties

Probability is the conceptual and theoretical basis for modelling and analysing uncertainty. To describe the range of values that a variable may have and the likelihood that it may have each of the values within the range, likelihood of occurrence experiments are often conducted. This experimental data can then be shown graphically as a histogram or frequency diagram (Figure 3.1).

From this data probabilistic distributions can be determined to describe mathematically the likelihood of the variable having each of the values within a range of possible values. More information on these distributions can be found in almost any book on basic probability theory, such as that by Ang and Tang (1975) or Schneider (1997).

It should be noted that the availability of data and the quality of information will affect the degree of uncertainty when using probability. However, the lack of sufficient data does not lessen the usefulness of probability when assessing existing structures.

![Figure 3.1: Variation of steel yield strength, $f_y$, represented by (a) a probability distribution function and (b) a cumulative distribution function](image-url)
3.4 Reducing uncertainty

Uncertainty due to inherent variability often cannot be reduced. For example, the wind loads on a structure are inherently variable and cannot be modified by human intervention in a reasonable way. However, in some cases it is possible to reduce uncertainty due to inherent variability in the design phase by ensuring the quality control measures, e.g. of concrete strength. This is of little help when evaluating existing structures. Uncertainty associated with imperfect modelling and estimation error can be reduced by adopting a more accurate model or updating an existing model.

When adopting a new model, one must be aware that the most complex model is not necessarily the most accurate one. One way of determining if a model is more accurate is to test its validity using certain statistical tests, known as goodness of fit tests. These tests can be also used to distinguish between different models in the case that several distributions fit well with the test data. This is done by determining their relative degree of validity. Two commonly used goodness of the fit tests are the chi-square ($\chi^2$) and the Kolmogorov-Smirnov (K-S) tests. Detailed information on these methods is given by Ang (1974).

On the other hand, assessments of existing structures can benefit from using additional test data or information to update initial estimations or distributions. Initial estimations are often based on various sources, such as existing experimental results/measurements, physical reasoning and subjective reasoning. Although this estimation should be determined as accurately as possible it is unlikely that it will be error free. These errors of estimation can be reduced when new information, such as new data from tests of material properties or from load tests on a structure, becomes available.

The Bayesian approach may be used to systematically incorporate new information into an existing model. More information on the Bayesian approach can be found in Ang (1975). More information on the ability to increase the reliability of structures using existing data is given by a number of authors (Faber 1998, Stewart 1998, Adey 2002).

3.5 Common mistakes

The modelling of uncertainty must of course be done correctly. Some of the major sources of errors in the consideration of uncertainties using probabilistic methods are:

- lack of identification and separation of different statistical populations;
- inadequate test data;
- neglecting the systematic variations in observed variables (e.g., temperature fluctuations);
- excessive extrapolation of statistical information; and
- neglecting correlations between variables.

More information on these subjects can be found in various references on probabilistic analysis (e.g, Ang 1975 and 1984, Schneider 1997).

3.6 Conclusions

Engineers must deal with uncertainty due to inherent variability, imperfect modelling and estimation error. The greatest benefit of assessing an existing structure is that much of the uncertainty due to imperfect modelling and estimation error that existed during the design phase can be removed. The loads to which a structure is subjected as well as the structural response of the structure can be measured. The reduction of uncertainty when evaluating a highway structure can result in an improved reliability and thus may result in cancellation of a costly and unnecessary intervention.
Chapter 4  LOAD MODELLING

4.1 Introduction

The design and assessment of highway bridges and culverts has traditionally been based on conservative empirical methods. For bridge/culvert assessment, similar models can be used. However, in some cases of assessment, great savings can be made if it can be shown that, on a probabilistic basis, the bridge has sufficient capacity to carry the load to which it is subjected. In many cases, such an approach can be used to justify not strengthening the bridge or certainly a reduced rehabilitation requirement. An example is the 5 span Vilsund steel bridge in Denmark where the Danish Road Directorate saved 13.5 million € through rehabilitation based upon probabilistic methods (Enevoldsen at al 2000).

As with the design of a new bridge, the loads to which an existing structure is subjected include:

- dead and superimposed dead load,
- wind and temperature loading,
- differential settlement and earth pressure,
- traffic loading (both normal and abnormal as well as permit, e.g., UK HA & HB loading),
- earthquake, ship impact, ice, scour and flood etc.

In prescribing these loads bridge design codes specify the partial safety factors by which they should be magnified and combined in determination of load effects (i.e., bending moments, shear forces etc.) at the serviceability (SLS) and ultimate limit states (SLS) for a variety of loading combinations. The magnitudes of these partial safety factors reflect the uncertainty associated at the design stage with both material resistance and the combined load components. For example the British standard dealing with loading, BS5400 Part 4, specifies a dead load ULS partial safety factor, \( \gamma_f = 1.15 \) for concrete while the factor for steel is \( \gamma_f = 1.05 \), reflecting the relative uncertainties associated with these materials. In addition the ULS factor for superimposed dead load is \( \gamma_f = 1.75 \). Clearly, these factors attempt to represent the level of uncertainty facing the engineer at the design stage.

In the assessment of an existing structure a more accurate assessment of the loads to which the structure is subjected is possible. For example dead and superimposed dead loading can clearly be assessed to a higher degree of accuracy for an existing structure, e.g., through measurement of the actual thickness of the asphalt layer etc. The obvious consequence of more accurate load assessment is in the justified reduction of the associated load partial safety factors at the ultimate and serviceability limit states. In addition, for the existing structure, the effects of the construction process and subsequent life of the structure, during which it may have undergone alteration, deterioration and/or other changes to the as-designed state, must be taken into account. These factors are allowed for through the prescription of partial factors or other code provisions for actual variation in the basic variables describing actions, material properties, geometric data and model uncertainty (Holický 2001). Numerous national codes (BD44/95 1995, BD 21/97 1997, Danish Road Directorate 1996 etc.) and International Standards (ISO 2394 1999, ISO/CD 13822 1999, ISO 12491 1998) exist relating to the assessment of existing structures.

The main principles of these standards that should be considered when assessing existing structures are (Holický 2001):
• currently valid codes for verification of structural reliability should be applied; old codes valid in the period during which the structure was designed should only be used as guidance documents;

• actual characteristics of structural materials, actions, geometric data and structural behaviour should be considered; the original design documentation should be used as guidance only.

An additional consideration of the assessment process is in the combination of loads to determine overall effects. National and International codes of practice specify partial factors whose magnitude is dependent upon the loading combination in which they are considered. For example the British loading code BD 37/88 specifies in ULS Combination 1 a partial factor on traffic load of 1.5, with a factor of 0 on wind and temperature effects. In ULS Combination 3 the traffic load factor drops to 1.25 with wind 0.0 and temperature restraint effects factored by 1.3. One approach in assessment is to employ these combinations but to reduce the partial factors, as discussed, to reflect increased knowledge of loads. Alternatively, combination rules such as those of Turkstra (1980) or Borge (1971) may be employed. However, great care should be exercised in their use.

4.2 Load Types

Determination of loads for the assessment of an existing structure is a somewhat simpler task than for the design of a new structure. Accurate knowledge of the loads and of the condition of the structure permits an updating of load and resistance models, thereby resulting in more accurate modelling of the reliability/safety of the structure. The benefit of this is a justifiable reduction in the load partial safety factors for the various prescribed combinations (which are not envisaged to change from the design code) whilst at a minimum maintaining the required $\beta$-safety index for the structure.

It is important to stress however, that in the determination of an appropriate partial safety factor for assessment purposes, consideration should be given to whether the considered load is time invariant or time variant. For example in the case of dead load, the level of loading can be accurately assessed through measurement and may reasonably be expected to remain the same for the remaining serviceable structure life. On the other hand, Figure 4.1 illustrates the concept of time variant loading, which is seen to behave as a random variable during the lifetime of the structure. Examples of time variant loading are traffic, wind, temperature and earthquake loading which present time variant phenomena, which cannot be predicted to remain unchanged for the lifetime of the structure but will be represented by stochastic (i.e., random) variables. In attempting to provide values for these loads to be considered in design, codes of practice must provide for all variations and combinations. In the case of traffic loading, the codes must provide for the heaviest traffic on a high-density route. In assessing an existing structure, the same statistical principles as were used in code calibration may be employed. However, as site specific assessment is performed, more representative descriptions of traffic, wind, temperature and ground acceleration records may be available leading to more accurate/appropriate characteristic load effect prediction.

In terms of time variant and invariant loads it is also important to discuss the asymptotically time invariant loads such as differential settlement, earth pressures and creep and shrinkage effects. Clearly, these are all initially time variant phenomena, which behave asymptotically after some point in time, $t_s$ during the lifetime of the structure. Thus for example the creep induced variation in flexural stiffness ($EI$) in a reinforced or prestressed concrete member due to a load applied at age $t_c$ (e.g., dead load, pre- or post-tensioning) will approach zero with $t$ in the extreme (generally after 20 years, with creep coefficients calculated according to CEB –FIP Model code (1990). Figure 4.2 for example illustrates this behaviour for creep effects.
Clearly then, in assessing an existing structure, effort should be made to determine if these effects have indeed become asymptotic which will have implications for assessment of levels of prestress etc..

![Figure 4.1: Typical realisations of load effect S(t) with time](image)

**Figure 4.1:** Typical realisations of load effect $S(t)$ with time

![Figure 4.2: Creep strains for high-volume fly ash concrete (Sivasundaram 1989)](image)

**Figure 4.2:** Creep strains for high-volume fly ash concrete (Sivasundaram 1989)

### 4.2.1 Loading Data Required for Assessment

The data required for the assessment of an existing structure may be readily obtained through manual surveys etc. Any standard method may be used for collection of data relating to dead and superimposed dead loads, and once accurately determined these loads may be included in the assessment of the structure, without the need for significant further statistical analysis.

For most structures being assessed, a significant portion of the differential settlement will have already occurred and as such in the majority of cases its effects may effectively be ignored. This is
provided, of course, that there are to be no alterations to the structure which might induce additional settlements. Of increasing importance are the effects of passive and active earth pressures in integral structures and of the condition of fill behind integral abutments. Inspection of levels of compaction behind abutments etc. provides the engineer with a better indication of the level of elastic stiffness to assign to the soil in his/her model.

The concept of asymptotically time invariant loads has already been introduced as significant for bridge assessment. As an example, expected increases in the levels of creep strain may be assessed by employing models such as those proposed by the CEB MC-90. The level of future shrinkage strains may also be predicted with reference to the appropriate code. A significant implication of the accurate calculation/assessment of these parameters lies in the determination of current and future levels of prestress in an existing structure and consequently for structural resistance modelling.

It is recognised that the time variant live loads, such as traffic, wind, temperature and earthquake effects represent random phenomena and require statistical modelling to determine the magnitude of their characteristic effects. Extreme value distributions, such as the Gumbel family, amongst others, are fit to measured data, recorded over a period of time. Subsequent extrapolation of these distributions to a specified level of confidence or for a specified return period, yields a value of the given effect for a specified probability exceedance level.

The members of the Gumbel family of extreme value distributions are the Gumbel 1, Weibull and Frechet distributions given in equations 4.1, 4.2 and 4.3 respectively.

\[
G(x) = \exp \left[ - \exp \left( -\frac{x - \lambda}{\delta} \right) \right] \quad -\infty < x < \infty \quad \delta > 0
\] (4.1)

\[
G(x) = \exp \left[ - \left( \frac{\lambda - x}{\delta} \right)^\beta \right] \quad -\infty < x \leq \lambda
\] (4.2)

\[
G(x) = \exp \left[ - \left( \frac{\delta}{x - \lambda} \right)^\beta \right] \quad \lambda < x \leq \infty
\] (4.3)

The difference in these distributions lies in their tail behaviour and in modelling measured data for prediction of a characteristic load effect. Care should be taken to ensure that the most appropriate distribution has been selected. One way of doing this is by plotting the data on the probability paper relating to the chosen distribution. The degree of linearity and closeness of fit reflects the accuracy of the approximate distribution.

The duration of time over which data is collected to accurately model the extreme values depends on the effect being determined. For wind and temperature data, maximum and minimum values of the particular effect over a representative period of time (e.g. 50 years) and for a specific sampling frequency (e.g. monthly) should be collected. Typically such data is readily available from meteorological stations in the region of the structure under consideration. Earthquake data relating to ground acceleration etc. may be obtained from geological stations.

For traffic data, it is important to collect continuously recorded data in representative periods of time. The duration of recording is clearly dependent upon a number of factors, i.e., time, budget, location etc.. It is obviously desirable to have as much data as possible; however 1-2 week’s worth of continuously recorded data in conjunction with the results of manual surveys has been used for the
purposes of assessment (O’Connor 1998). Although this period of recording may seem inadequate when compared with the 50 year value indicated for wind and temperature data, it is important to point out that in performing traffic simulations, statistical techniques are employed to manipulate the recorded data and generate a large number of alternative flow patterns. This is further discussed in the next section.

4.2.2 Static Traffic Load Simulation for Assessment

Of the loads to be modelled, perhaps the most variable are those due to traffic, i.e., those induced by single vehicles and combinations of same, which traverse the structure. The characteristics of these vehicles vary widely with respect to their gross vehicle weight (GVW), axle spacing, distribution of load to axles, location in lane, velocity and in the likelihood of multiple presences of vehicles on the structure both longitudinally and transversely. Truck loading is a random phenomenon for which probabilistic models and statistical data are required. For assessment of existing structures, monitoring of traffic data using a weigh-in-motion (WIM) system can provide the necessary statistics to develop site specific loading models for ULS and SLS reliability assessment.

The value of using weigh-in-motion to collect traffic data and consequently update loading models for bridge assessment is clear when the traffic data from a number of alternative sites on the French motorway network, illustrated in figure 4.3, are considered. The variation in frequencies and intensities of traffic on the sites are evident as is the inherent conservatism of using the heaviest data to assess a structure on the route of least intense traffic.

There are three principal means by which trucks can be weighed while in motion:

1. **bar sensors** based on different piezo technologies or electrical capacitance are embedded in a groove in the road,
2. **bending plate or load-cell sensors**, made from steel plates (about 1.5 m × 0.5 m in size), or **capacitive mats** encased in a frame are embedded in a pit excavated in the pavement or,
3. **strain transducers** of a bridge weigh-in-motion system, which are attached to the soffit of an existing road bridge, measure the flexural strain in the bridge and are used to determine the weight of the truck on it.

In general, traffic records will only give information on normal traffic. The most critical situations for long spans appear when the traffic is congested while for short spans (i.e. <40 m) or local load effects, the heaviest individual axle (or group) or vehicle load is dominant. Therefore, it is necessary to combine realistic traffic scenarios (arrangements of vehicle, traffic types) such as free flowing and jammed traffic. It is important for subsequent extrapolation to ensure that the duration of each simulated scenario be retained for comparison with respect to its expected frequency during the lifetime of the bridge. A number of alternative traffic flow scenarios should be performed for both free flowing, jammed and mixed traffic, on the structure under consideration. It is often desirable to employ a technique such as Monte Carlo simulation or Poisson arrival processes to increase the number of simulated scenarios.

Simulations are performed and loads effects calculated, for the various traffic scenarios by passing the vehicles over influence lines/surfaces of the effects of interest and thereby compiling records of extreme load effects to be modelled by an appropriate Extreme Value distribution as previously discussed.

As traffic simulation is a computationally demanding process, three (perhaps more) levels of probabilistic based assessment methods can be envisaged:
L1. Simple load model: For short-span bridges and culverts, this would be an array of axles corresponding to a meeting event (or passing event) of two trucks. For longer bridges, it would correspond to a traffic jam.

L2. More complex model: This might involve a simulation of traffic loading on the structure using traffic with prescribed characteristics (mean and standard deviation of weight and axle spacing).

L3. Most complex model: This could involve a direct simulation of traffic loading on the structure using WIM data measured at or near the site.

Figure 4.3: Comparison of Gross Vehicle Weight distributions for four different sites

4.2.2.1 Dynamic Amplification of Static Load Effects

One main issue of contention in determining characteristic load effects is the application of dynamic amplification factors (DAF) to calculated effects determined from free and mixed traffic flow simulations. A number of issues may be raised concerning both the theoretical derivation and actual application of these amplification factors to extrapolated static load effects for bridge assessment:

1. The dynamic amplification factor is generally theoretically derived as the ratio between the dynamic and static values corresponding to the same fractile

\[ \phi_{cal} = \frac{E_{\text{dyn}(x-fractile)}}{E_{\text{stat}(x-fractile)}} \] (Bruls at al 1996). However, clearly the maximum dynamic effect will not necessarily correspond to the maximum static effect.

2. The factor is presented as a function of the influence surface, the span length and the number of lanes on the bridge. The factors take no account of the random variables describing either the vehicles themselves (i.e. their gross weight, speed, dynamic characteristics etc.) or of the relative dynamic interaction between the vehicles and the bridge. In addition research has demonstrated that the dynamic amplification is inversely proportional to the weight of the vehicle, i.e., as the gross vehicle weight increases, the dynamic amplification reduces (Nowak 1991, Nowak 1993, Nowak 1995).

The conservatism of the approach for the assessment of multiple lane bridges is best illustrated in a short example: for a two lane, bi-directional, short span structure (<40 m) the critical loading case
occurs when two vehicles meet on the bridge. The extreme static load over a period of one week may occur when two 35 tonne vehicles meet simultaneously on the span, resulting in a total load of 70 tonnes. When extrapolated to a larger time period, of say 1000 years, this may lead to an extreme value 10 - 20% in excess of the 1-week value. The Eurocode suggests that a dynamic factor of 1.2 be applied to this value. This presumes that the vehicles’ dynamics are in phase and that full frequency matching (i.e., resonance) with the bridge structure is achieved. This suggests an unrealistic scenario where a combination of a number of probabilities is achieved. A more realistic method would take account of the probabilities of arrival, vehicle frequency matching, vehicle-bridge interaction etc.

As in the prescription of static load effects for design code calibration, the prescription of dynamic amplification factors must attempt to provide a set of values that are applicable for a large range of structures. However, in the assessment of a particular structure, more accurate assessment of appropriate factors may be made through surveys of structural condition, road surface roughness, condition of joints at bridge extremes, natural frequency, etc., all of which contribute to the dynamic amplification factor. Such detailed work, which may also include detailed finite element modelling to take account of the probabilities of arrival, vehicle frequency matching, vehicle-bridge interaction etc. may only be applicable for significant capital projects. However, it is important to understand that the option is available. It is obvious that further research is required in this area.

4.3 Conclusion

In conclusion it is clear that load modelling for the assessment of an existing structure has the advantage of employing site-specific loads for the determination of load effects. Manual surveys may be performed to measure actual sizes for more realistic estimation of dead and super imposed dead loads, while data concerning wind, temperature and earthquake effects may be obtained from local meteorological and geological stations for required return periods. Traffic data may be collected at the site by employing weigh-in-motion technology in conjunction with vehicle classification data. The importance of having continuously recorded, representative traffic data has been stressed, and although the quantity of such data may be limited, statistical techniques may be employed to make the best use of what is available. The various concepts of static and dynamic traffic load simulation were also discussed, with varying degrees of complexity proposed. The advantages of such simulations are obvious when the site-specific parameters of traffic characteristics are considered. The concept of time invariant and variant loads was introduced and discussed with clear implications for structural assessment. In calculating extreme SLS and ULS load effects, load combination may be applied based upon existing codes of practice, with reduced partial safety factors, to reflect the reduced uncertainty associated with the applied loads. Alternatively, combination rules such as Turkstra’s or Borge’s rules may be employed. Extrapolation of load effects to determine extreme values may be performed using one of the Gumbel family of Extreme Value distributions, or an alternative distribution. Great care should be taken to ensure a good fit of the fitted distribution to measured or calculated effect values. Overall the clear advantages of site-specific load modelling for bridge assessment are clear.
Chapter 5  MODELLING MATERIALS FOR ASSESSMENT

5.1 General

In order to assess highway structures it is necessary to accurately model the resistance of their structural elements. This requires knowledge of the material properties in the structural elements, such as strength and stiffness, as well as the structural dimensions and how the various materials within the elements act together. It is also necessary to understand the influences on the material properties and structural dimensions, of time (i.e., the extent and strength changes due to deterioration mechanisms such as fatigue and corrosion), fabrication methods and quality control measures (such as construction and in-service inspections). The correlation effects between different properties and locations within the elements and structures must also be known.

This chapter addresses material properties in a general way that is applicable to the assessment of all materials that are used in highway structures, and gives some more specific details on the considerations required when modelling concrete and steel reinforcement. Section 5.2 explains variations in material properties and how they are modelled. Section 5.3 discusses the consideration of initial compliance controls. Sections 5.4 and 5.5 discuss, as examples, aspects to be considered when modelling the concrete and steel reinforcement that comprise concrete elements, respectively. The reader is referred to Appendix B for a more detailed look at the mathematical and probabilistic models proposed for material properties by various researchers.

5.2 Variations

Since there can be both, spatial and temporal variation in material properties and exact material properties at all locations and times within or between structures cannot be determined, there is uncertainty as to the material properties that are to be used to determine structural resistances. These uncertainties can be accounted for by modelling the material properties probabilistically.

Material properties within a structure vary both spatially and temporally. The material properties vary spatially because in each different location there is a different exact combination of components. For example, concrete at different places in a structure is made of different exact combinations of aggregate, cement and water and different exact configurations of these materials. The material properties vary temporally because of the loading of the structure and the physical processes at work in the materials that comprise the structural elements. For example, loading of steel reinforcement past the initial linear elastic portion of the steel into the strain hardening range changes the future yield stress of the steel, and the hydration process that occurs in concrete, results in increases in concrete strength.

In addition to these uncertainties the variation between material test specimens and the material in a structure must be considered. This variation has a systematic component due to bias in the predictions and a random component, which can be attributed to a lack of completeness in the models used for prediction, as well as differences in the materials used, qualities of workmanship and the effects of time.

Table 5.1 gives examples of systematic (bias) and random (coefficients of variations – COV) variations found in some common material properties. These values were taken from (CEB 1991) and (Ellingwood 1980). It must be noted that these are only examples and are not necessarily applicable in all cases.
### Table 5.1: Examples of systematic (bias) and random variations in material properties

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Bias</th>
<th>COV</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic limit of structural steel (welded)</td>
<td>$f_y$</td>
<td>1.25</td>
<td>0.08</td>
<td>(Ellingwood 1980)</td>
</tr>
<tr>
<td>Elastic limit of structural steel (rolled)</td>
<td>$f_y$</td>
<td>0.99</td>
<td>0.05</td>
<td>(Ellingwood 1980)</td>
</tr>
<tr>
<td>Compressive strength of concrete (20MPa-40MPa)</td>
<td>$f'_c$</td>
<td>1.31-1.19</td>
<td>0.14-0.09</td>
<td>(CEB, 1991)</td>
</tr>
<tr>
<td>Tensile strength of concrete (20 MPa – 40 MPa)</td>
<td>$f_t$</td>
<td>1.47-1.28</td>
<td>0.18-0.16</td>
<td>(CEB 1991)</td>
</tr>
<tr>
<td>Modulus of elasticity of concrete</td>
<td>$E_c$</td>
<td>1.18</td>
<td>0.10</td>
<td>(CEB 1991)</td>
</tr>
<tr>
<td>Tensile strength of reinforcing steel (400 MPa)</td>
<td>$f_y$</td>
<td>1.22</td>
<td>0.08</td>
<td>(CEB 1991)</td>
</tr>
<tr>
<td>Modulus of elasticity of reinforcing steel</td>
<td>$E_s$</td>
<td>1</td>
<td>0</td>
<td>(CEB 1991)</td>
</tr>
</tbody>
</table>

The modelling of material properties probabilistically involves the determination of representative probabilistic distributions. This requires a mathematical model and direct representation of the random variables in the mathematical model. The initial (or prior) distribution used in the model is based on existing historical data, test data, or expert opinion, or a combination of all three. It must be ensured that the information, on which the distribution is based, represents actual conditions, including environment, loading, fabrication, time effects, etc… For example, care must be taken to ensure that the deviations between the tested material property and the property that actually exists in the structure, due to the different conditions that exist for the test specimens and the material in-situ, are appropriately considered. The validity of the selected distribution should be verified and the prior distribution should be updated when new information becomes available. More information on testing the validity of distributions and on updating distributions when new information becomes available is given by Ang (1975).

### 5.3 Compliance

In the evaluation of existing structures the modelling of the material properties should take into consideration the compliance controls, if any, of the material at the time of construction. Compliance controls are often performed to ensure the material is of the desired quality. They affect the probability of having certain (low) material properties. For example if each structural member or group of specimens are tested and the ones that do not comply with the test are removed, the probability of having the undesired material properties accepted, and therefore, having them exist in the structure, are greatly reduced.

When taking into consideration the tests that are done for compliance control it must be considered that there are errors associated with these tests. The uncertainty incorporated into the compliance control tests depends on the exact tests and the procedure if a material does not pass a test. For example if a compliance control test is failed than the test specimen may be subjected to further testing or discarded immediately. The ability of compliance control tests to reduce the probability of having certain material properties depends on the ability of the compliance tests to determine whether or not a lot is inadequate (Kerkesen-Bradley 1991).
5.4 Considerations when modelling concrete

5.4.1 Sources of uncertainty

Sources of uncertainty in concrete properties are due to variations in the properties of the components of the concrete and proportion of concrete mix, variations in mixing, transporting, placing and curing methods, variations in testing procedures, and variations due to concrete being in a structure rather than in test specimens (Mirza 1979b). The concrete properties discussed herein are strength in compression and tension, modulus of elasticity in compression and tension, and creep and shrinkage.

5.4.2 Concrete strength in compression (in-situ)

In-situ concrete strength, $f_c$, is not the same as the concrete strength measured in test cylinders or cubes, $f'_c$. It is normally lower than $f'_c$ because of the different placing and curing procedures, the effects of vertical migration of water during the placement of concrete in deep members, the effects of difference in size and shape, the effects of different stress regimes, and the difference in directions of casting and loading of the structure and the specimens (Mirza 1979b). The mathematical models of in-situ concrete strength are predominantly constructed around transforming the concrete strength measured in test cylinders, considered as random variables, into the characteristic in-situ concrete strength. Different mathematical models consider spatial variation, temporal variation and the difference between test cylinders and in-situ strength in different ways. Some of the models that have been developed to determine the concrete strength in compression (in-situ) can be found in (Mirza 1979b; Bartlett 1996, ISO2394 1998, JCSS 2001). These references are by no means exhaustive.

5.4.2.1 Basic compressive strength $f'_c$

The concrete strength measured in test cylinders, the basic compressive strength of concrete, varies due to variations such as the exact composition and configuration of the constituents in each cylinder, the variations in the position of the cylinder in the test frame, and variations in the loading speeds. Normal and lognormal distributions are normally used to represent the basic compressive strength, although preference is sometimes given to lognormal distributions as they do not have negative values. Normal distributions give an increasingly conservative approach to the modelling of the low tail of $f'_c$ and lognormal distributions give unconservative estimates at the low tail (Balaguru 1995). The coefficient of variation is much smaller for the lognormal distribution (Balaguru 1995). The lognormal distribution gives a better fit than the normal distribution for concrete strength when the coefficient of variation is greater than 0.15-0.20 (Mirza 1979b).

In the JCSS probabilistic model code (JCSS 2001) it is suggested that the distribution of $x = \ln(f'_c)$ is normal if its parameters are obtained from an infinitely large sample, but because the sample is not infinite the parameters must be treated as random variables and $x$ has a student t-distribution.

5.4.2.2 Changes in concrete strength with time

Concrete strength changes with time due to the loads applied; increased loading causes micro cracks to grow and weakens the concrete, and the physical processes at work in the concrete, such as hydration (Neville 1997). In the JCSS probabilistic model code (JCSS 2001) it is recommended to take into consideration the concrete age at time of loading, $t$ (days) and the duration of the loading ($\tau$) by using a deterministic function. The average in-situ strength increases by about 25 –30 percent between 28 days and 1 year (Bartlett 1996, JCSS 2001).
5.4.2.3 Changes in concrete strength due to spatial variation

The concrete strength varies spatially in a structure due to variations in the properties of the components of the concrete at the different locations. To take into consideration spatial variation, it is recommended in (JCSS 2001) to use a standard normal variable. This variable is correlated within one structural element. It is uncorrelated for different elements. It is also recommended in (JCSS 2001) to use a log-normal variable to represent the additional variations in concrete strength due to the special placing, curing and hardening conditions of in-situ concrete. More information about the variation that may be expected between members and different batches of concrete may be found in (Bartlett 1996).

5.4.2.4 Degree of quality control

The degree of quality control affects the variation of the concrete material properties (Mirza 1979b, Stewart 1995). This can be used to determine improved probabilistic models of concrete properties by reducing the coefficient of variation of the distribution depending on the degree of quality control and observations from previously performed tests (Mirza 1979b) or it can be done directly in the mathematical model by considering the curing and compaction processes (Stewart 1995).

5.4.2.5 Effect of the speed of loading on concrete strength

The effect of loading rate on the in-situ concrete strength also affects the determination of in-situ strength (Mirza 1979b). The faster concrete is loaded the stronger the concrete is. The loading rate has little effect on the overall coefficient of variation of concrete (Mirza 1979b).

5.4.2.6 Concrete Strength in Tension

The relationship between tensile and compressive strengths of concrete depends on the size and type of aggregate, air entrainment, curing conditions, water/cement ratio, cement content, and age at the time of loading (Mirza 1979b). Models of tensile strength are proposed in (CEB-FIP 1991; JCSS 2001, Mirza 1979b). These are by no means exhaustive.

5.4.3 Modulus of elasticity

The modulus of elasticity of concrete, the relationship between stress and strain, depends on the modulus of elasticity of the aggregate and the volumetric proportion of aggregate in the concrete (Neville 1997). A model of modulus of elasticity is proposed in (JCSS 2001) that uses a deterministic creep coefficient, the ratio of the permanent load to total load and depends on the type of structure and a log-normal variable to represent the additional variations in the modulus of elasticity due to the special placing, curing and hardening conditions of in-situ concrete.

There is a high degree of correlation between initial tangent modulus and compressive strength (Mirza 1979b). The initial tangent modulus of elasticity of in-situ concrete can be described by a normal distribution (Mirza 1979b). There is little difference between modulus of concrete in compression and in tension (Mirza 1979b, Johnson 1928).

5.4.4 Concrete compression strain

A model of ultimate compression strain is suggested in (JCSS 2001). It is recommended to use a log-normal variable to represent the additional variations in the ultimate compression strain due to the special placing, curing and hardening conditions of in-situ concrete.
5.4.5 Drying shrinkage

Drying shrinkage of concrete is commonly defined as the time-dependent reduction of volume of hardened concrete, paste or mortar resulting from the loss of water. The rate of drying shrinkage depends on temperature and relative humidity in the concrete, the elastic properties of the paste and aggregate and their shrinkage as well as the restraint imposed by the aggregate and unhydrated cement, water-cement ratio, degree of hydration and admixture. Models are proposed by Madsen (1983).

5.4.6 Creep

Creep is the gradual increase in strain in concrete with time under load (Neville 1997). Creep can thus be defined as the increase in strain under a sustained stress and, because this increase can be several times as large as the strain under loading, creep is of considerable importance to structures. It has been found that the random variability of creep and shrinkage effects in concrete structures is often very large and should be accounted for in assessment (Madsen 1983). A model is proposed by Madsen (1983).

5.5 Considerations when modelling steel reinforcement

5.5.1 Uncertainty in steel reinforcement

The uncertainties in the determination of steel strength are due to the variation in the strength of the material, variation in cross section of the bar, effect of rate of loading, effect of bar diameter on properties of bar and effect of strain at which it is defined (Mirza 1979a). Effort must be made to ensure that distributions determined from test data are properly transformed to represent the in-situ conditions and the type of test performed. Different tests may sometimes be performed to measure the same property. For example, often there are two quoted steel strengths, the mill test strength and the static strength. The mill strength tests are done at a rapid rate of loading and use actual areas. The static strengths are determined based on nominal area and use a strain rate that is similar to what is expected in a structure.

5.5.2 Yield and ultimate strength

The yield strength of reinforcing steel is taken as the stress at a corresponding strain. This strain normally corresponds to the initial plastic deformation of the reinforcement. A model for the yield strength of reinforcing steel is proposed in (JCSS 2001), taking into consideration the variations in global mean of different mills, the variations in a mill from batch (melt) to bath and the variations within the melt. Normal or beta distributions can be used to represent yield strength (JCSS 2001, Mirza 1979a).

Strength fluctuations along bars are negligible (JCSS 2001, Woodward 1999). The yield force of a bundle of bars under static loading is the sum of the yield forces of each contributing bar. In general, it can be assumed that all reinforcing steel used at a job originates from a single mill. The correlation coefficient between yield forces of individual bars of the same diameter can be taken as 0.9 (Rackwitz 1996). The correlation coefficient between yield forces of bars of different diameter and between the yield forces in different cross sections in different beams in a structure can be taken as 0.4 (JCSS 2001).

The ultimate strength is often represented by normal or beta distributions (Mirza 1979a, JCSS 2001).
5.5.3 Variations in area of bar cross section
The actual areas of reinforcing bars tend to deviate from the nominal areas due to the rolling process. In general this value has been found to be less than 1 and that it can be represented by a normal distribution (Mirza 1979a, JCSS 2001, Allen 1972, Wiss 1973, American Society for testing materials 1972).

5.5.4 Modulus of elasticity
There is no difference in the modulus of elasticity of Grade 40 and 60 reinforcing steel (Mirza 1979a; CEB-FIP 1991).

5.5.5 Coefficients of correlation
Coefficients of correlation between reinforcement area, yield stress and ultimate strength are given in (JCSS 2001).

5.6 Conclusions
Material properties play an important role in the determination of the behaviour of highway structures. The uncertainties associated with material properties can be taken into consideration using probabilistic methods. When determining the values of material properties to be used in the assessment of an existing structure, the difference between test values and in-situ material properties must be considered, as well as the effects of compliance controls. A summary of proposed mathematical models and probabilistic models for modelling concrete and steel reinforcing material properties can be found in Appendix B.
Chapter 6  STRUCTURAL RESPONSE MODELLING

6.1 Introduction

The assessment of a highway structure requires the calculation of the response of a mathematical model of the structure to a complete range of loading conditions. This model should satisfy conditions of equilibrium and produce deformations compatible with the continuity of the structure and support conditions. It must be checked that reactions and internal forces/stresses at all sections of the structure are within reasonable safety levels. An assessment at level 1 (Section 2.2) is carried out with traditional methods of structural analysis (simple, convenient and ‘often’ conservative) while assessment at higher levels (Sections 2.3 to 2.6) will involve more refined methods of analysis.

Compared to the design stage, the assessment of a structure has the problem of determining what the physical structure really is. The designer has uncertainties concerning the relationship of actual peak loads in service to the ultimate loads assumed, the relationship of actual material properties to those assumed in design and the extent to which all the potential failure modes can be modelled by suitable mathematical relationships (Baker 1988). In the process of assessment, some of these uncertainties can be reduced through suitable field measurements. Therefore, partial factors used in design are inappropriate for assessment purposes. It must be acknowledged that the determination of partial factors for assessment will still be subjective to some extent, and regardless of the method of analysis chosen, there will be uncertainty in many of the parameters.

Separate or interdependent mathematical models of the structure and the soil can be established to determine the structural response. Hence, a particular model for a given structure will be influenced by the assumptions adopted for the foundation and the soil. If it is ensured that the ground can sustain the loading with acceptable displacements or provide appropriate stiffness, soil-structure interaction can be ignored in low-level studies (For bridges on stiff foundations, it is common practice to analyse the structure in isolation). The method of analysis to be used will depend on the following characteristics:

• behaviour of the structural material,
• structural geometry and boundary conditions and
• nature of the applied load.

Traditional methods of structural analysis are based on one- or two-dimensional models with elastic materials, geometric linearity and static loads. Other available techniques allow for three-dimensional modelling, a variety of non-linear response actions and dynamics. In higher levels of assessment, the method of analysis should ideally take account of all the significant aspects of the structural response to loads and imposed displacements. In the following pages, a number of currently available analysis techniques and the incorporation of field data into the structural models are reviewed. Finally, structural methods of analysis are classified into the five levels of assessment proposed in Chapter 2 for a number of highway structures (bridges, culverts, earth-retaining structures, reinforced soil and tunnels).

6.2 Methods of analysis

At first, structural assessment methods were purely based on experience. Then, findings in the 16th century allowed the use of criteria based on statics or elasticity. In the 19th century, the application of energy methods allowed for the use of methods based on allowable stresses. At present, struc-
tures are generally assessed with limit state methods (plastic methods, finite element methods and non-linear methods are employed in this calculation) and probabilistic approaches. The near future is orientated towards a reliability-based design/assessment approach.

A limit state is a condition beyond which a structure, or a part of a structure, would become unfit for its intended use. A limit state can be assessed on a deterministic or a probabilistic basis. A serviceability limit state denotes a loss of utility, e.g., due to cracking, exceeding displacements or vibrations. The ultimate limit state corresponds to the maximum load-carrying capacity of the structure or a section of the structure leading to collapse and it can be reached by:

- loss of equilibrium when a part or the whole of the structure is considered as a rigid body,
- excessive stresses in a section or the whole structure due to post-elastic or post-buckling behaviour and
- fatigue failure.

A first division of methods of analysis could be made into empirical, algebraic and numerical methods. Other divisions could be made according to the number of dimensions of the structural model (framed structures or walls and slabs), the behaviour of the structural material (elastic or plastic), the magnitude of the displacements with respect to the original geometry (linear or non-linear), the characteristics of the section (cracked or uncracked reinforced concrete section), the nature of the applied load (static or dynamic) or the definition of the structure (in deterministic or probabilistic terms).

6.2.1 Empirical, algebraic, and numerical methods

Empirical methods are simplified analytical tools, applicable to very specific cases. They have the advantage of providing a quick assessment of the structure, generally conservative. They only need a few geometric parameters. Their main disadvantage lies in the subjective appraisal of some parameters while ignoring many others. Algebraic methods are limited to cases where load distribution, section properties and boundary conditions can be described by simple mathematical expressions. Numerical methods provide a more practical means of analysis for complex structures. Unlike the subjective idealisations assumed in empirical methods, numerical methods can allow for:

- a definition of the real structural profile, preferably obtained from observation and measurements on site,
- a more accurate spatial localisation of the applied load and
- a structural model with strength properties equivalent to that of the material characteristics of the real structure, preferably taken from load tests.

The finite element method is the most popular numerical method. Other numerical methods are less general: e.g., the finite difference method, successfully applied to bridge decks that can be simulated with orthotropic plate theory, or the finite strip method, successfully applied to straight, skew and curved-plate and folded-plate structures.

When using the finite element method, the structures are subdivided into a finite number of simple elements, and the complex differential equations are then solved for the simple elements. In frames, trusses and grids, the elements are bars/beams connected at nodes. In walls, slabs, shells and mass structures, two- and three-dimensional continuous elements are used. Assemblage of the elements into a global matrix transforms the problem from a differential equations formulation over a continuum to a linear algebra problem. The solution is approximate but accuracy improves when a finer element size is used. Finite element analysis may be used for detailed stress analysis. Even though the use of finite element modelling still ignores a lot of uncertain characteristics of the structure, a
three-dimensional model incorporating the appropriate structural material and general geometric construction can be capable of predicting the structural response satisfactorily.

6.2.2 Frame and spatial analysis

Frame analysis is used for framed structures that are discretised as a set of one-dimensional members. Framed structures consist of members which are long in comparison to their cross section (e.g., beams, grids, plane and space frames or trusses). The displacement or stiffness method is well suited to analysing these structures by computer. When structures with two significant dimensions (e.g., a wide bridge deck) are studied with frame analysis, the effects of transverse load distribution or the transverse composition of the structural material cannot be taken into account. More advanced techniques of analysis appear to be necessary when the structure geometry has been grossly distorted (i.e., those structures most in need of accurate assessment).

In spatial analysis, the internal forces/moments generally have six components. Further assumptions are sometimes made in order to simplify the three-dimensional problem (e.g., in-plane behaviour).

6.2.3 Cracked or uncracked analysis

It is normal practice to analyse using gross section properties. More accurate analysis allows for cracking of sections. The rigidity of a section can be greatly reduced when allowing for cracking. Relative rigidity of cracked and uncracked sections might affect bending moment.

6.2.4 Elastic and plastic analysis

Elastic methods are commonly used to analyse the performance of a structure, especially concerning serviceability, while plastic methods are used to analyse the mechanism of collapse of a structure.

6.2.4.1 Elastic methods

Steel structures obey Hook’s law (linear elastic deformation). The same assumption is made for concrete subject to small displacements. When the stress-strain relationship is non-linear, it is necessary to develop an expression relating forces and deformations in terms of stress and strain, axial load and extension, or moment and curvature.

When deformations in a structure are proportional to the applied load, the principle of superposition applies and the internal forces can be determined by adding the effect of the forces applied separately. If the structure is statically indeterminate, the principle of superposition is valid only if Hook’s law is obeyed because the internal forces depend on the deformation of the members.

6.2.4.2 Plastic methods

The plastic approach is increasingly used in design, particularly for steel construction. The load is increased until yielding occurs at some locations, the structure undergoes elastic-plastic deformations, and on further increase a fully plastic condition is reached, at which a sufficient number of plastic hinges are formed to transform the structure into a mechanism. This method is limited by the effect of repeated loading and instability.

For slabs, the yield-line theory gives an upper bound of the ultimate load capacity of a reinforced concrete slab by studying assumed mechanisms of failure (Ghali 1989, Nielsen 1984). The strip method gives a lower bound solution to the collapse load. Neither the yield-line method nor the strip method of ultimate load guarantee safety against cracking or excessive deformation at service loads. Further, failure can occur prior to the occurrence of a mechanism if insufficient ductility ex-
6.2.5 Linear and non-linear geometry

In some cases, the geometry of the structure is substantially distorted by the applied loads, and equilibrium cannot be based on the original directions and relative position of loads and members. As a result, the structure behaves nonlinearly even if the stress-strain relationship of the material is linear. For instance, if axial forces are large, they can cause a change in bending stiffness (especially in slender members).

A non-linear analysis is also required in the cases of creep and shrinkage in concrete, accurate simulation of cyclic load effects, etc.

6.2.6 Static and dynamic analysis

From the point of view of the nature of the applied load, the methods of analysis can be static or dynamic. Static forces produce displacements that do not vary with time. Sub-sections 6.2.1 to 6.2.5 have introduced methods of analysis of structures subjected to static loads. Dynamic forces are time-dependent and cause vibration of the structure. These forces can be related to cyclic loading (analysed with methods of fatigue assessment), impact loading (analysed with empirical methods), seismic and wind loading (analysed using response spectra methods), or free/forced vibration due to traffic (analysed with finite element interaction models).

6.2.6.1 Dynamic analysis

Dynamic response analysis incorporates uncertainties regarding boundary conditions, imperfection effects, levels of damping and of excitation. At present, this area relies on a combination of empiricism and experience and there is a need to improve calculations of dynamic response and to incorporate them into design and assessment procedures. In order to solve the dynamic problem, the structure is generally discretised through lumped-mass, generalised displacements or finite element procedures (Clough & Penzien 1993). There are different types of dynamic analysis (MSC/NASTRAN 1997):

- **Real eigenvalue analysis**: This analysis is used to determine only the basic dynamic characteristics of a structure, this is, the frequencies and mode shapes at which the structure naturally tends to vibrate. There are a number of approaches to solve this problem: Givens Householder and modified Givens Householder methods (good methods for small, dense matrices), inverse power and Sturm modified inverse power (good for determining a few modes) and Lanczos (good for medium to large models).

- **Frequency response analysis**: This approach calculates the response of a structure to loads that vary as a function of frequency. It is an efficient method to find the response to an excitation explicitly defined in the frequency domain (frequency, amplitude and phase are specified). Two different methods can be used in frequency response analysis: the direct and the modal methods. The direct method solves the coupled equations of motion in terms of forcing frequency using complex algebra. The modal method utilises the mode shapes of the structure to reduce and uncouple the equations of motion (when modal or no damping is used); then, the solution is obtained by summation of the individual modal responses.

- **Transient response analysis**: This approach calculates the response of a structure to loads that vary as a function of time. The time-varying loading can include non-linear effects that are a function of displacement or velocity. As in frequency response analysis, direct and modal methods can be used depending upon the structure and the nature of the loading.
• **Others:** Response spectrum analysis, random response or non-linear transient response can be used in combination with one of the preceding methods of analysis.

The accurate analysis of earthquake and wind effects is highly complex (Gould & Abu-Sitta 1980). When the wind action is considered, the degree of sophistication of the analysis can be related to the probable maximum mean hourly wind speed appropriate to the return period, the fundamental natural frequencies and the wind-loaded lengths of critical members. In practice, only wind-sensitive and/or large bridges need to be investigated for interaction with wind (Liebenberg 1992).

Earthquake loading is a common application of enforced motion at a set of points in the structure for transient response. Rigorous dynamic analysis requires the use of characteristic earthquake accelerograms. The *large-mass method* can be used to model the action of an earthquake. Simpler deterministic methods based on *spectra response* can be used to estimate the maximum displacements of the structure. Structural models can be lumped or generalized coordinate, single or multi-degree of freedom, elastic or elasto-plastic systems, subjected to translation, rotation or multiple excitation. More complicated analysis, involving the random nature of the excitation and the non-linear nature of the response, may be desirable in some cases. If the soil is resting on rigid-base rock, the soil can be represented in the analytical model by combining a layer of soil with the structure model. Because stiffness and damping properties of the soil substructure are frequency dependent, the earthquake response analysis is more conveniently carried out in the frequency domain and then transformed back into the time domain (Clough & Penzien 1993).

The passing of a truck over a bridge is an enforced motion transient problem. The following techniques can be used to simulate bridge-vehicle dynamic interaction:

- **Lagrange multiplier techniques:** The Lagrange Multiplier formulation allows for the representation of the compatibility condition at the bridge/vehicle interface through a set of auxiliary functions. An entry into the assembled stiffness matrix of the vehicle-bridge system allows for the definition of the forces acting on the bridge due to the moving wheels. A compatibility condition between the vertical displacement of the wheel and the bridge at the contact point is also established (Cifuentes 1989).

- **Convolution methods:** The bridge and truck are modelled separately and combined in an iterative procedure. The method involves convolution of the vehicle loads either in the time domain or with modal responses of the bridge. The convolution integral is solved by transformation to the frequency domain using the fast Fourier transform. The method is then extended by an iterative procedure to include dynamic interaction between the bridge and a mathematical model of a vehicle (Green & Cebon 1994).

### 6.2.7 Fatigue assessment

The Palmgren-Miner rule is commonly used for fatigue damage calculation. Fatigue can be assessed by:

- simplified methods that are applicable to parts of bridges with classified details and which are subjected to standard loadings or
- methods using first principles that can be applied in all circumstances.

Palmgren-Miner rule can be used to compute the total lifetime of a new structure, but it does not allow the prediction of remaining lifetime of existing or partially damaged structures (Jacob 1998). A Fracture Mechanics approach, such as Paris-Erdogan’s law, can be used for this purpose, though they require knowledge of more parameters than Palmgren-Miner’s rule.
6.2.8 Impact assessment

Accidental collision impact loading is usually specified in the form of equivalent static loads to be applied at specified levels against balustrades and piers. A correct dynamic analysis is highly complex so that present designs are based on full-scale tests using a vehicle with appropriate impact characteristics. The difficulty of predicting response to impact loading makes it very difficult to give complete theoretical design criteria.

6.2.9 Deterministic and probabilistic analysis

Generally bridges are assessed using deterministic methods with elastic or plastic limit state analysis. Fully deterministic methods derive the loads from worst possible traffic conditions and nominal material strength values. However, probabilistic analysis can be considered in special cases, e.g., to check the need for bridge strengthening. In a probabilistic analysis, uncertain parameters concerning load (Chapter 4), resistance (Chapter 5) and the computer model are represented as stochastic variables with corresponding statistical distributions (Enevoldsen 2000). When probabilistically modelling the uncertain variables, the type of probability distribution, the distribution parameters and the calculation procedure for load effects, influence the results of the analysis significantly.

6.3 Bridge structures

When assessing a bridge structure, those failure modes against which the structure was originally designed must be checked (e.g., ultimate capacity of a structural member being exceeded as a result of overloading). Failure can occur due to yielding of the material at a sufficient number of locations to form a mechanism or due to buckling induced by axial compression or torsional-flexural buckling without stresses exceeding the elastic limit. Inspection strategies are used to assess the structure and prevent failure modes resulting from localised deterioration of critical components (e.g., corrosion of a prestressing cable) which are not considered herein (Woodward & Bevc 2003).

Elastic methods of analysis should be used to determine internal forces and deformations. Plastic methods of analysis (e.g., plastic hinge methods for beams, or yield line methods for slabs) may be used when they model the combined local and global effects adequately, though elastic methods generally lead to more conservative solutions. The structural response can be calculated by one overall analysis (e.g., using finite element analysis) or by separate analysis for global and local effects. All members must be assessed for the worst combinations of loading. The maximum load-carrying capacity of a structure is calculated for the ultimate limit state (instability, buckling, fatigue).

6.3.1 Deck section

The behaviour of the deck structure must be checked against different modes of failure. This procedure is generally assessed in successive steps as follows:

a) The response of the structure is checked first by linear elastic analysis. Moduli of elasticity and shear modulus values should be appropriate to the section material. In-plane shear flexibility should be allowed for in concrete flanges of box sections due to shear lag effects.

Primary stresses can be obtained from the combined effect of all the local load actions in producing bending, shearing or twisting of the structure. Conventional structures can be calculated using beam theory. However, more rigorous treatment allowing for second order effects (shear lag, warping, etc.) might be necessary for unconventional structures (e.g., thin-walled box-like structures). The ultimate capability of the structure can be calculated using plastic bending theory.
When assessing a structure, if the supports have moved compared to the design stage, they will induce internal forces in a statically indeterminate structure. A change in stress distribution within a section due to differences in temperature variation, shrinkage or creep can also be revealed during the assessment process.

b) Then, different parts of the structure can be analysed using elastic grillage theory, beam-and-slab models, finite element methods, etc. (Hambly 1991). Clearly, the assessment of three-dimensional effects can only be done accurately with three-dimensional models.

The grillage analogy involves idealising the structure as a number of longitudinal and transverse beam elements, rigidly connected at nodes. Transverse beams may be orthogonal or skewed with respect to the longitudinal beams. Each beam element represents either a composite section (e.g., main girder with associated slab) or a width of slab (e.g., a transverse beam may represent a width of slab equal to the spacing of the transverse beams). If a beam-and-slab model is used, plate-bending finite elements are added to the grillage. If the mesh is fine enough, this model allows for an analysis of local effects due to wheel loadings.

The bridge deck can be analysed with planar models. However, the use of effective flange widths is only approximate and it cannot address the issue of upstands. Hence, for accurate results, bridge decks with edge cantilevers, voided decks, cellular box or transverse diaphragms should be modelled in three-dimensions, but these models are considerably more complex. Upstand grillage and finite element methods adapt planar models to allow for effects such as shear lag, but they must be used with caution (O’Brien & Keogh 1999). Brick type elements can be used to describe the geometry of highly complex bridge decks very accurately.

c) Finally, discontinuities and details can be analysed by elastic analysis to determine the detailed stress distribution using finite element methods, etc. Load actions near discontinuities will be taken into consideration. Further stresses as a result of this stress concentration can result in fracture and a fatigue analysis is required.

6.4 Culverts

The culvert will respond differently if it is made of corrugated steel (flexible) or reinforced concrete (rigid). While steel structures deflect longitudinally to conform with the surrounding foundation, reinforced-concrete structures tend to behave as a beam due to the stiff nature of its box-type structure.

Flexible culverts are thin-walled structures which integrity mainly depends on the confining capability of the surrounding soil. Techniques to incorporate the effect of soil-structure interaction will be introduced in the following section. They can be represented with a two-dimensional model. However, the response of a three-dimensional finite element culvert model involving soil-structure interaction might differ significantly from a two-dimensional approach. The interaction between embankment, culvert and foundation soil can only be made in detail with a three-dimensional finite-element model for the foundation. In soil-structure systems incorporating rigid culverts, the stiffness of the culvert will be well in excess of the stiffness of the surrounding soil mass and interaction effects are much less important.

6.5 Earth-retaining structures

There are two main types of retaining walls:

- Non-embedded walls: Stiff structures for which the soil-structure interaction is relatively simple (e.g., gravity, counterfort or cantilever walls).
• Embedded walls: Flexible structures for which the soil-structure interaction has a strong influence on its behaviour (e.g., embedded cantilever wall, propped or anchored cantilever wall).

Retaining walls and soil are mutually interdependent. The soil does not only generate loading but also adjusts and distributes earth pressures to accommodate small movements.

6.5.1 Simple models

In a first analysis of an earth retaining structure, soil-structure interaction can be ignored and bending moments in the wall can be calculated from the assumed earth and water pressure diagrams. Although the behaviour of the wall is not truly represented, this method provides an adequate factor of safety in terms of stability, and it is necessary before moving to other methods to take into account the relative stiffness between soil and structure. Limit equilibrium (Coulomb wedge analysis), stress field (Rankine) and limit analysis (upper and lower theorems of plasticity) are simple methods of analysing retaining walls (Potts 1992). All of these methods assume the soil to be everywhere at failure. Empirical factors have to be used to allow for wall flexibility and surcharges have to be made in an approximate manner. A grillage analysis of edge corner effects can be used for studying three-dimensional structures (e.g., in an abutment).

6.5.2 Sophisticated models

More elaborate models of earth retaining structures allow for soil-structure interaction, but they require information on the stiffness characteristics of the wall, the soil and the props or ground anchors, the shear strength parameters and water conditions, in addition to the initial in situ soil stresses. The difficulty of assessing these parameters reliably limits the accuracy of the predictions obtained with these methods.

6.5.2.1 Beam on elastic foundation (Winkler springs)

This model requires appropriate values for the spring constants to represent the ground behaviour. Complex retaining structures are reduced to a single isolated wall and much of the soil-structure interaction is not considered. The wall is represented using either finite differences or finite elements. Winkler models are suitable for determining internal forces, but, if displacements around the excavation are to be predicted, a continuum model is required. A beam on springs requires less computer resources than finite element methods, but computer capacity is generally not a limiting factor today.

6.5.2.2 Continuum models

As in the beam on springs approach, general ground movements are not allowed in continuum model calculations. The advantage of a continuum model is the small computational effort required when compared to more sophisticated finite element models. Continuum models use interaction coefficients derived from finite element analyses or boundary integral equations. They are commonly applied in the case of embedded walls.

6.5.2.3 Finite element method

The finite element method takes account of the interaction between all the components within the retaining wall. Geometry, soil parameters and boundary conditions are defined. In the case of overconsolidated clay, linear elastic finite-element methods might achieve good predictions of overall ground and wall movements. More sophisticated models will be necessary to predict the magnitude of the movements behind the wall. In the case of soft clays and sands, yield in shear should be included in the finite elements modelling the soil. However, a major source of uncertainty arises from a lack of knowledge of the pressure due to compaction of the retained soil.
6.6 Reinforced-soil structures

Reinforced soil can be used as an alternative to earth-retaining structures. A simplistic approach assesses the internal and external stability of reinforced soil through the use of ultimate properties. Additionally, internal stability can be assessed with a permissible-stress approach.

When using the finite-element method, the structure is commonly modelled in two dimensions. Strip reinforcements can be treated as sheets with equivalent tensile and frictional characteristics or alternatively, as a single material with properties representative of both soil and reinforcement. Due to the significant spacing between reinforcement and the difficulties of analysing collapse in a discretised system, the finite element method is not capable of providing reliable detailed behaviour. However, it can be useful in those situations where conventional methods are not feasible (i.e., analysis of reinforced soil in combination with a structure).

6.7 Tunnels

Methods of analysis of tunnels range from simple beam-and-spring models to finite element models incorporating bedding, fracture planes and other elaborate features. Beam-and-spring models represent the tunnel lining as a string of interconnected pin-ended structural beams, and the ground as a series of radial springs. The cohesion or internal friction of the ground is not represented in these models. A finite element mesh can be used to represent the ground with internal friction and cohesion properties and linear elastic axial and shearing stiffness. Some models allow for elastoplastic behaviour and ground properties are varied in different layers (Bickel 1996). However, the use of analytical methods is less reliable than for other types of structure due to the complexity of the system and the variability of the ground. Thus, the use of three-dimensional finite element modelling and plasticity is limited to research, and empirical methods have been developed to cover a wide range of circumstances.

6.8 Integration of field data and structural models

In order to represent the structural response correctly, accurate field measurements must be taken. The quality of the output depends on the quality of the input. Accordingly, complex analytical tools can only be justified if a realistic assessment of the material properties and overall condition of the existing structure can be made. Then, structural models can be improved by measuring dynamic effects (impact factor, damping, etc.) or by measuring other results of load testing, including observed cracks or the true distribution of loads, etc.

6.8.1 Visual inspection

It is necessary to carry out a visual inspection of the structure being assessed (Woodward and Bevc 2003). This inspection might reveal:

- scouring of piers and/or abutment supports,
- cracks in a section of the structure,
- quality and condition of the structural material,
- deformations of the profile,
- condition of the joints,
- damping devices.

Calculations can vary as result of observation. Additionally, a number of reduction factors relating to the condition of the bridge can be adopted based on observation. There is a need for a rational
basis for these reduction factors. The structure dimensions should be measured with appropriate surveying equipment on site and in the case of observed deformations, the new profile should be considered in the analysis.

6.8.2 Material and live load testing

The assessment of a structure might require more data than purely the observation of the visible portion of the structure. Concrete tests include cover depth, rebound hammer, ultrasonics, impact echo, permeability, carbonation, thermography, radar, slot cutting, instrumented coring and others. Testing of reinforcement corrosion includes half-cell potentials, resistivity and rate of corrosion, chloride concentration and monitoring. Post-tensioning tendons can be tested with exploratory hole drilling, radiography, ultrasonics or through monitoring. Other tests are related to the determination of in-situ stress (Mallett 1994).

Load testing must be carried out with caution and must protect the structure from further deterioration. Garas (1987) verify by testing some of the methods of analysis at realistic scales, which cannot be achieved in the laboratory. The passage of heavily loaded trucks can be used to determine the actual live-load behaviour of the structure and to predict maximum live-load stresses. Forced vibration (controlled excitation with a shaker, a hammer, rockets or the quick release of forced displacements) or ambient vibration methods (due to natural causes such as wind, micro tremors and traffic) are typical dynamic tests to determine the frequencies and mode shapes of vibration of a bridge (Deger 1996). As tests at full scale are expensive and limited, scaled physical models using measurements from testing on the real structure, could also be used for assessment purposes.

The original structural design might have been altered not only due to aging and the application of loads, but grouting, saddling, guniting or post-tensioning in previous maintenance programmes. Housner (1997) discuss control systems, sensors for structural control, health monitoring and damage detection of Civil Engineering structures. Strains or displacements of the structure are generally measured under the application of a load of known characteristics (static or dynamic). These measurements can give more realistic values for:

- support stiffness, joint condition, restraints,
- behaviour of the cross section,
- elastic properties of the structural material,
- behaviour of the foundation,
- fill and structural material density,
- road profile (i.e., a bump, rutting, a pot-hole, etc.) and its effect on the traffic load and on the structure,
- natural frequencies and damping,
- stiffness matrix.

Then, these characteristics can be incorporated into the structural model.

6.8.3 Calibration of the structural model

The structural model is only as accurate as the assumptions made for its response to the application of a load. A combination of experimental data and a structural model can provide an insight into why a structure is behaving as observed. Optimisation techniques are commonly used for adjusting parameters of the structural models to field measurements. Parameter values are determined by comparing the measured and predicted response (Znidarič 1998, Quilligan 2002). A unique solution is not always ensured and it is beneficial to have the best possible initial model (i.e., clearly defining the geometry). Data might be taken from design drawings but should be verified by in situ measurements, especially for critical members, before starting the optimisation procedure. Then, the up-
dated models can be used to more accurately predict and assess the behaviour of the structure under different static or dynamic loading conditions. In a structural reliability model, the uncertainties in the design parameters are modelled probabilistically.

The process of identifying the behaviour of a given structure is summarised in the following steps (Doebling & Farrar 1999):

- Definition of the model chosen to predict the structural behaviour and the parameters of the model to be identified. Sophisticated finite element models require parameters such as strain-displacement relationships, material constitutive properties, structural connectivity, geometric distribution of mass and structural damping. Assumptions must be made, i.e., linearity, time-invariance of model parameters or, for more complicated models: non-linearity, properties defined in terms of a probability distribution, etc.
- Definition and acquisition of the experimental data. There are two types: response measurements (static or dynamic) and excitation measurements.
- Definition of the objective function and the constraints.
- Implementation of the optimisation technique to determine the identified parameters (Friswell & Mottershead 1995). The most common technique is least-squares minimisation. This approach calculates the structural properties such as stiffness, elastic modulus, density and thickness, which minimise the sum of squares of differences between the model and the measurements.

### 6.9 Levels of assessment

Methods of analysis are established for each structure and for five different levels of assessment. The levels reflect level of sophistication of the analysis or time available to the assessor (Sections 2.1 to 2.6). Level 1 of assessment corresponds to more simple/conservative methods, while higher levels will be used for more rigorous modelling. The number of parameters required increases with the level of assessment. Therefore, parameters for lower levels of assessment can be based on visual observation, but parameters for higher levels of assessment should be estimated from load testing. The same methods of structural analysis are used for level 2 and higher levels, but specific material properties and loading can be included in higher levels (Chapters 4 and 5). Hence, full partial factors from assessment standards (smaller than at the design stage) can be used for level 1, but characteristic strengths of materials must be based on existing data (from the same or a similar structure) for level 2 and on load tests on the structure being assessed for level 3 or higher. Level 4 uses modified partial safety factors to account for any additional safety characteristics specific to the structure being assessed and level 5 uses structural reliability analysis instead of partial safety factors (Chapter 8). Theoretically, the output of higher levels of assessment could be used as a diagnostic tool to prevent weaknesses at localised points and/or information on safety values.

All categories are summarised in Table 6.1. A stability analysis is also to be considered in level 1. An assessment associated with complex mathematical modelling should be used with considerable caution. The analysis of a special load (i.e., the dynamic response of a bridge to the crossing of a truck) might require some numerical manipulation (i.e., convolution or Lagrange technique) of these structural models.
<table>
<thead>
<tr>
<th>Structure Type</th>
<th>Level of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Not skew Beam</td>
<td>1-D or 2-D linear elastic (beam theory or plane frame analysis)</td>
</tr>
<tr>
<td>Not skew Slab</td>
<td>1-D or 2-D linear elastic (beam theory or plane frame analysis)</td>
</tr>
<tr>
<td>Not skew Beam &amp; Slab</td>
<td>2-D linear elastic with modified modulus of elasticity for the cables</td>
</tr>
<tr>
<td>Not skew Cellular</td>
<td>Simple equilibrium method of analysis</td>
</tr>
<tr>
<td>Skew, tapered and curved</td>
<td>2-D linear elastic allowing for soil-structure interaction (beam &amp; spring)</td>
</tr>
<tr>
<td>Bridges</td>
<td>Empirical or 2-D linear elastic arch frame</td>
</tr>
<tr>
<td>Rigid</td>
<td>Frame linear elastic</td>
</tr>
<tr>
<td>Earth-retaining walls</td>
<td>Frame linear elastic with modified modulus of elasticity for the cables</td>
</tr>
<tr>
<td>Reinforced soil</td>
<td>Simple equilibrium method of analysis</td>
</tr>
<tr>
<td>Tunnels</td>
<td>2- or 3-D FEM; linear or non-linear; elastic-plastic</td>
</tr>
<tr>
<td>Culverts</td>
<td>2- or 3-D FEM linear or non-linear; elastic or plastic; allowing for soil-structure interaction, cracking</td>
</tr>
<tr>
<td>Flexible</td>
<td>Simple equilibrium method of analysis</td>
</tr>
</tbody>
</table>
Chapter 7  TARGET RELIABILITY LEVELS

7.1  Introduction

The target reliability level is the level of reliability required to ensure acceptable safety and serviceability of a structure. The selection of the target reliability depends on different parameters such as the type and the importance of the structure, possible failure consequences, socio-economic factors etc. Thus, the requirements for safety and serviceability for the assessment of existing structures are in principle the same as for the design of new structures. The main differences are:

- **economic considerations**: the incremental cost between acceptance and upgrading an existing structure can be very large whereas the cost increment of increasing the safety of a new structure is generally very small; consequently conservative criteria are used in the design standards for new structures

- **social considerations**: these include disruption (or displacement) of occupants and activities as well as heritage values, considerations that do not affect the structural design of new structures

- **sustainability considerations**: considerations relating to reduction of waste and recycling, are more prevalent in the rehabilitation of existing structures

As a consequence the goal of minimum structural intervention which makes as much use of the existing materials in the structure as possible, applies for most existing structures of normal occupancy and use.

7.1.1  Formats for specifying target reliability levels

In order to be able to evaluate the results of an assessment and to judge whether a structure is deemed to be safe or not, target reliability levels must be specified by the authorities or bridge owner. These target reliability levels can be explicitly or implicitly specified in a code in different ways:

- **Level A**: Global safety factor formats and allowable stress formats. With the Level A format, only one safety factor is applied resulting in a lack of flexibility to adjust the safety margin according to differences in load dispersion, load combinations, consequences of failure and uncertainties in material modelling, load modelling and response modelling. Therefore, Level A formats cannot be recommended – a rational set up for working with these formats and reducing uncertainties cannot be established. Furthermore, Level A formats must be very conservative in order to cover all practical cases.

- **Level B**: Semi-probabilistic load and resistance factor formats using partial safety factors; limit state design. The verification of the required safety is handled by applying limit states in which the relevant load, strength and geometrical parameters are specified as characteristic values, each associated with a safety factor, i.e., partial safety factors. These partial safety factors should be specified so that the actual knowledge of the uncertain parameters in the assessment is reflected in the prescribed value of the partial safety factors. Level B formats are the core in any modern design code and are highly recommended as the format for establishing a general code for the assessment of existing structures.

- **Level C**: Probability-based formats, reliability index formats, probability of failure formats. These formats are also based on limit states. However, the uncertainties in the loads, strength and geometry as well as model uncertainties are reflected directly in the modelling of the stochastic variables. The result of the analysis is then the formal probability of failure with a specified reference period. The target levels must be specified as requirements for the probabil-
ity of failure. In order to have a cohesive format, specifications on the modelling of uncertainties including modelling uncertainties must also be provided in the format. Level C formats are the basis for a more refined safety analysis which is recommended if it is believed that a Level B based assessment is too conservative.

- Level D: Formats taking economical considerations into account. These are basically formats in which the partial safety factors in Level B or the target probabilities of failure in Level C are modified taking economic considerations into account. The format is then based on, for example, decision-theory or life cycle cost.

7.2 Determination of the target reliability levels

Before some requirements for determining the target reliability levels for assessment of existing structures are described, it is important to stress that all the formats are examples of a formal set of verification rules which cannot be reflected in occurrence probabilities. It is therefore very important to be aware of the fact that only documented safety and approaches to improve the degree or level of documentation are covered by the approaches presented. Topics such as the “real” safety or the “real” load carrying capacity cannot be included formally. One of the reasons for this is that all the methods presented do not take into account the effects of possible gross human errors. These need to be addressed by appropriate counteracting strategies developed in the field of Quality Assurance. Quality Assurance strategies are outside the scope of this study.

7.2.1 Level A formats for assessment of existing structures

The target reliability level used can be taken as the level of reliability implied by acceptance criteria defined in proven and accepted design codes. The calibration based on existing codes assumes that existing practice is optimal and that a correct application of the valid codes and standards results in a safe structure. Traditional deterministic codes employing allowable stress or general safety factor formats are still in use in some countries. However, applying these codes and their load combination rules to the assessment of structures can lead to major inconsistencies in dealing with safety checking. Structural design codes usually deal with only one type of material or form of construction, such as steel, reinforced concrete, prestressed concrete or timber. Each of the traditional deterministic codes has different sets of load combination requirements, which are seldom consistent from one code to the next. The inconsistencies encountered when designing new structures with traditional deterministic codes are also valid if these codes are used as a basis for assessment. Rational consistent safety checks for existing structures based on traditional non-limit state codes are almost impossible.

7.2.2 Level B formats for assessment of existing structures

The partial safety factors in Level B formats as known from many design codes, can basically be obtained by applying two approaches: 1) judgment or guesstimation and 2) code calibration. In (1) which is the most commonly applied (as for example in some Eurocodes) the partial safety factors are based on experience and knowledge from previous formats or safety levels which have been proven to work in practice. The code calibration is more rationally based on probabilistic analysis, i.e., the Level B format is established based on a Level C format.

In contrast to codes for new structures, formats for assessment should make allowance for matters such as the quality of inspection, the extent and quality of on-site measurements, potential failure modes and possible consequences of failure. Thus, for the assessment of existing structures, the required number or sets of partial safety factors are considerably larger than for the design of new structures. The partial safety factors should reflect the uncertainties or knowledge at the specific level.
of assessment (see Chapter 1). It is clear that the applied partial safety factors in general should be
greater in a crude Level 1 assessment than in a refined Level 4 assessment. The partial safety factors
provided, reflecting the degree of uncertainty in the knowledge, should allow for better knowledge
and reward the effort of obtaining higher quality and less uncertain knowledge by introducing lower
partial safety factors. The partial safety factors are decomposed according to the basic sources of uncertainty. Basically the knowledge on material strength (e.g., default, as-built or testing), the load modelling, the geometry, structural modelling and failure consequences should be reflected in the specified sets of partial safety factors. It is recommended that these partial safety factors be obtained by applying code calibration and Level C formats, i.e., probabilistic analysis.

7.2.3 Level C formats for assessment of existing structures

Level B formats must by nature be a generalisation in order to work for many types of bridge and
for many types of material. Taking this uncertainty due to generalisation into account, the Level B
format is in many cases conservative. It can therefore be worthwhile for the individual bridge to ap-
ply a Level C format using probability based assessment. Such a Level C code format does basically
include requirements for a) target reliability levels (e.g., maximum formal probability of failure), b)
typical statistical distributions and finally c) model uncertainties.

The uncertainties are physical uncertainties (identification of materials, traffic load model), statistical un-
certainties and uncertainties due to simplifications in the structural evaluation model. The uncertainties
are modelled as random variables which are the input parameters for the limit state. The inclusion of the
so-called model uncertainty, which accounts for simplifications in the load and resistance models, into
the limit state formulation, is also very important. The evaluation of the limit state can be used directly
to determine the formal annual probability of failure or the directly related reliability index, $\beta$, applying
standard techniques such as the First Order Reliability Method (FORM). Other reliability assessment
techniques as described in Section 8 are also possible. The Level C approach allows a determination of
the actual reliability of a certain structure using realistic input parameters. A structure, which can be
proven to have a reliability index higher than the respective minimum reliability index, can be considered
safe enough. In many of the modern codes, the overall safety requirements are specified in terms of re-
liability indices or probabilities of failure (see section 7.4). The values stated in the codes can be consid-
ered as minimum reliability levels.

It is often discussed whether this Level C format could be directly applied. However, it is clear that if
the Level B format is calibrated based on a Level C format, the Level B format (which in general is
accepted) by definition reflects or generalises the requirements in Level C; the argument for applying
the Level C format is clear. An assessment based on a Level C format does not fulfil all the specific
requirements of the level B format but neither does it compromise the requirement for the structural
safety given in the underlying codes because these are a direct part of the Level C format.

It is highly recommended to base the Level B formats for the Level 1 to 4 analysis on code calibra-
tion based on a clear and documented Level C format which both makes a basis for rational re-
quirements in assessment Levels 1 to 4 and can be applied directly for Level 5 assessments.

7.2.4 Level D formats for assessment of existing structures

All the methods to determine target reliability levels presented above do not take into account eco-
nomic aspects of maintenance and failure of a structure and thus, the very important parameter of
costs. However, the target probability of failure could be obtained from an optimisation of overall
costs including the costs of failure in such a way that the overall cost accumulated throughout the
life of a structure is minimal. These overall costs include the cost of planning and of execution as
well as operation and maintenance of a structure. Furthermore, costs of demolition and restoration
of the original state and the costs of failure of a structure during its service life, e.g., described by
7.2.4.1 Decision-theory based criteria

In making a decision about an existing structure there are three main courses of action:

- leave the structure unchanged,
- strengthen the structure or change its use,
- demolish the structure and replace it with a new structure.

Two criteria can then be derived which include the sum of all costs of failure $c_{\text{fail}}$ and the estimated cost for creating a new structure $c_{\text{new}}$:

\[
\text{do nothing: } p_{fA} - \frac{1}{1 + c_{\text{fail}} / c_{\text{new}}} < p_{\beta}
\]

\[
\text{demolish: } p_{fA} - \frac{1}{1 + c_{\text{fail}} / c_{\text{new}}} \geq p_{\beta}
\]

$p_{fA}$ is the probability after assessing the structure, $p_{\beta}$ is the target probability of failure (e.g., code specified value).

7.2.4.2 Life-cycle decision approach - concept of the minimum total expected cost

The approach to making decisions about the acceptability of existing structures presented above leads directly to the concept of optimal inspection and repair policies so as to minimise total expected costs including repair and expected costs as a consequence of failures (see Appendix A). Re-assessments become more likely to be necessary as a structure gets older. When the estimated reliability falls below an acceptable level, immediate action is required such as closing the road section or reducing the load.

7.3 Acceptable risk criteria

In general, acceptance criteria have been formulated as risk acceptance criteria or sometimes, risk tolerance criteria. In defining acceptable risk criteria, it is possible to take into account acceptable or tolerable risk levels for other risks in society.

7.3.1 Risks in society

To determine the target reliability level it is possible to compare the calculated probability of failure with other risks in society (Table 7.1) and from these to infer acceptable risks for structures. There is a great difference between voluntary and involuntary risks. Also, the risk depends on the degree of exposure to a hazard as well as on the potential consequences. Engineering structures are used by people in the expectation that they will not fail; thus, the probability of structural failure may be related to involuntary risk.

The number of fatalities and the associated frequencies are therefore critical results of a risk analysis. The possible consequences as well as the accumulated frequencies can be shown graphically on a double-logarithmic diagram; the so-called $FN$-curve. This curve can be helpful to the decision maker. If two systems have the same expected risk, the system with the steeper curve should be preferred as this implies relatively fewer accidents with great consequences.
Table 7.1: Selected risks in society

<table>
<thead>
<tr>
<th>Activity</th>
<th>Approximate death rate ($10^{-9}$ deaths/hour exposure)</th>
<th>Typical exposure (h/year)</th>
<th>Typical risk of death ($10^{-6}$/year rounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpine climbing</td>
<td>30000-40000</td>
<td>50</td>
<td>1500-2000</td>
</tr>
<tr>
<td>Boating</td>
<td>1500</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>Swimming</td>
<td>3500</td>
<td>50</td>
<td>170</td>
</tr>
<tr>
<td>Cigarette smoking</td>
<td>2500</td>
<td>400</td>
<td>1000</td>
</tr>
<tr>
<td>Air travel</td>
<td>1200</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Car travel</td>
<td>700</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>Construction work</td>
<td>70-200</td>
<td>2200</td>
<td>150-440</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>20</td>
<td>2000</td>
<td>40</td>
</tr>
<tr>
<td>Building fires</td>
<td>1.3</td>
<td>8000</td>
<td>8.24</td>
</tr>
<tr>
<td>Structural failures</td>
<td>0.02</td>
<td>6000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

7.3.2 Acceptable or tolerable risk levels

From the risks that are encountered in society, various bodies such as regulators of hazardous industries (nuclear or chemical facilities), have developed acceptable or tolerable risk levels related to the consequences of a failure. One approach is the concept of ALARP (as low as reasonably practical) defining an upper limit to the risk, where greater risk can not be tolerated and a lower limit below which is of no practical interest. Between these two limits the risk must be reduced (e.g., through spending money) to a level which is as low as reasonably practical.

7.4 Comparison of target reliability levels

In the following section the target reliability indices of various codes and standards currently in use are compared. The distribution types which were used for the derivation of the reliability levels are described where available. The designer dealing with the assessment of an existing structure may decide among the following tables which of the values are most suited and best applied to the solution of the problem at hand. When comparing the values in the tables presented in the following chapters and deciding on a reliability level, one must always consider the different reference periods used in the various documents (e.g., one year, life-time of the structure, etc.).

7.4.1 ISO/CD 13822:1999

In the ISO/CD 13822:1999 “Bases for Design of Structures – Assessment of Existing Structures” code, the target reliability mainly depends on the type of limit state examined as well as on the consequences of failure. As Table 7.2 shows, the target reliability index ranges from 2.3 for very low consequences of a structural failure to 4.3 for structures whose failure would have very high consequences. Thus, for the assessment of highway structures in the ultimate limit state, a value of 4.3 would be suitable for most cases.

7.4.2 ISO 2394:1998

In ISO 2394:1998 “General Principles on Reliability for Structures” the target reliability index to be chosen for assessment of existing structures depends on the consequences of a structural failure as well as the costs of a safety measure (Table 7.3). The following distribution types were used for the derivation of the reliability level:
Table 7.2: ISO/CD 13822:1999 - Target reliabilities

<table>
<thead>
<tr>
<th>Limit states</th>
<th>Target reliability index $\beta$</th>
<th>Reference period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serviceability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>reversible</td>
<td>0.0</td>
<td>intended remaining working life</td>
</tr>
<tr>
<td>irreversible</td>
<td>1.5</td>
<td>intended remaining working life</td>
</tr>
<tr>
<td>Fatigue</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inspectable</td>
<td>2.3</td>
<td>intended remaining working life</td>
</tr>
<tr>
<td>not inspectable</td>
<td>3.1</td>
<td>intended remaining working life</td>
</tr>
<tr>
<td>Ultimate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>very low consequences of failure</td>
<td>2.3</td>
<td>$L_s \text{ years}^*$</td>
</tr>
<tr>
<td>low consequences of failure</td>
<td>3.1</td>
<td>$L_s \text{ years}^*$</td>
</tr>
<tr>
<td>medium consequences of failure</td>
<td>3.8</td>
<td>$L_s \text{ years}^*$</td>
</tr>
<tr>
<td>high consequences of failure</td>
<td>4.3</td>
<td>$L_s \text{ years}^*$</td>
</tr>
</tbody>
</table>

$L_s$ is a minimum standard period of safety (e.g. 50 years)

- Resistance: Lognormal or Weibull distributions
- Permanent loads: Gaussian distributions
- Time-varying loads: Gumbel Extreme Value distributions

7.4.3 Eurocode 1:1993

The target reliability indices presented in draft Eurocode 1 “Basis of Design and Actions on Structures” only depend on the type of limit state examined (Table 7.4). Neither the consequences of failure, nor economic considerations as far as the costs of certain safety measures are concerned, are taken into account.

7.4.4 NKB Report No. 36: 1978

The NKB Report No. 36 “Guidelines for Loading and Safety regulations for Structural Design” gives reliability indices depending on the failure type and consequence. The values recommended for the ultimate limit state for a reference period of one year are given in Table 7.5. For the serviceability limit state NKB recommends values of $\beta = 1$ to 2. The values presented in Table 7.5 are also the basis of the PIARC report “Reliability Based Assessment of Highway Bridges” (PIARC 2000).

7.4.5 JCSS 2000

The publication of the Joint Committee of Structural Safety “Probabilistic Evaluation of Existing Structures” (JCSS 2000) is devoted directly to existing structures and probabilistic evaluation. The target reliability indices given for the ultimate limit state and a reference period of one year depend on the failure consequence and the costs of safety measures similar to ISO 2394:1998 (Table 7.6). For the serviceability limit state, values of $\beta = 1$ to 2 are recommended.

From these target reliability indices the standard code calibration process can be applied to obtain modified partial safety factors.

7.4.6 CSA 1981

The Canadian Standards Association (CSA 1981) uses a different and slightly more complicated approach than the documents presented above. To determine the target reliability factors such as the
element or system behaviour, the inspectability or the traffic category are considered to determine the appropriate reliability index (Table 7.7). It should be noted that the reliability indices given in the table are valid for a reference period equal to the life-time of the structure.

Table 7.3: ISO/CD 13822:1999 - Target reliabilities

<table>
<thead>
<tr>
<th>Relative costs of safety measures</th>
<th>Consequences of failure</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small</td>
<td>some</td>
<td>moderate</td>
<td>great</td>
</tr>
<tr>
<td>High</td>
<td>0</td>
<td>1.5 (A)*</td>
<td>2.3</td>
<td>3.1 (B)*</td>
</tr>
<tr>
<td>Moderate</td>
<td>1.3</td>
<td>2.3</td>
<td>3.1</td>
<td>3.8 (C)*</td>
</tr>
<tr>
<td>Low</td>
<td>2.3</td>
<td>3.1</td>
<td>3.8</td>
<td>4.3</td>
</tr>
</tbody>
</table>

*Notes:  
(A): for SLS, use $\beta = 0$ for reversible and $\beta = 1.5$ for irreversible limit states  
(B): for Fatigue Limit State, use $\beta = 2.3$ to $\beta = 3.1$ depending on the possibility of inspection  
(C): for ULS, use $\beta = 3.1, 3.8$ and 4.3

Table 7.4: Eurocode 1:1993 - Target reliabilities

<table>
<thead>
<tr>
<th>Limit states</th>
<th>Target reliability index $\beta$ (design working life: bridges 100 years)</th>
<th>Target reliability index $\beta$ (1 year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serviceability</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Fatigue</td>
<td>1.5 ÷ 3.8</td>
<td>-</td>
</tr>
<tr>
<td>Ultimate</td>
<td>3.8</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Table 7.5: NKB Report No. 36:1978 - Target reliabilities, ultimate limit state

<table>
<thead>
<tr>
<th>Failure Consequences</th>
<th>Failure Type</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ductile with extra carrying capacity</td>
<td>ductile without extra carrying capacity</td>
<td>brittle</td>
<td></td>
</tr>
<tr>
<td>Less Serious</td>
<td>3.1</td>
<td>3.7</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Serious</td>
<td>3.7</td>
<td>4.2</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>Very Serious</td>
<td>4.2</td>
<td>4.7</td>
<td>5.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.6: JCSS - Target reliabilities, ultimate limit state

<table>
<thead>
<tr>
<th>Relative cost of safety measure</th>
<th>Minor consequences of failure</th>
<th>Moderate consequences of failure</th>
<th>Large consequences of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>3.1</td>
<td>3.3</td>
<td>3.7</td>
</tr>
<tr>
<td>Normal</td>
<td>3.7</td>
<td>4.2</td>
<td>4.4</td>
</tr>
<tr>
<td>Small</td>
<td>4.2</td>
<td>4.4</td>
<td>4.7</td>
</tr>
</tbody>
</table>
Table 7.7: CSA - Target reliabilities, ultimate limit state

\[ \beta = 3.5 - (\Delta_E + \Delta_S + \Delta_I + \Delta_{PC}) \geq 2.0 \]

<table>
<thead>
<tr>
<th>Adjustment for element behaviour</th>
<th>( \Delta_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sudden loss of capacity with little or no warning</td>
<td>0.0</td>
</tr>
<tr>
<td>sudden failure with little or no warning but retention of post-failure capacity</td>
<td>0.25</td>
</tr>
<tr>
<td>Gradual failure with probable warning</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adjustment of system behaviour</th>
<th>( \Delta_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>element failure leads to total collapse</td>
<td>0.0</td>
</tr>
<tr>
<td>element failure probably does not lead to total collapse</td>
<td>0.25</td>
</tr>
<tr>
<td>element failure leads to local failure only</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adjustment for inspection level</th>
<th>( \Delta_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>component not inspectable</td>
<td>-0.25</td>
</tr>
<tr>
<td>component regularly inspectable</td>
<td>0.0</td>
</tr>
<tr>
<td>critical component inspected by evaluator</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adjustment for traffic category</th>
<th>( \Delta_{PC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>all traffic categories except Permit Controlled</td>
<td>0.0</td>
</tr>
<tr>
<td>Traffic category PC</td>
<td>0.6</td>
</tr>
</tbody>
</table>

7.5 CONCLUSIONS

When a reliability assessment of an existing structure is performed, it has to be decided if the probability of failure is acceptable. As shown in this chapter there is no easy answer to that question. The Engineer carrying out the assessment of the structure has to decide which of the values are most suited and best applied to the problem at hand as the estimated probability of failure associated with a project is very much a function of the understanding of the issues, the modelling of the data, etc. Furthermore, it depends on costs as well as consequences of failure. Still, the target reliability indices presented in the sections above can be helpful when a decision on the acceptable probability of failure has to be made.
Chapter 8 RELIABILITY ANALYSIS

8.1 Reliability analysis methods

In this chapter, the different formats presented in Chapter 7 are described in more detail, focusing on the reliability analysis method applied within each format. Only a short overview of the principal methods is given here as the concepts of reliability analysis are thoroughly presented in the ISO code 2394 (1998) as well as the ISO/CD 13822 (1999). Detailed background information can also be found in many textbooks. For an easy to understand introduction to reliability analysis and basic methods, the books by Schneider (1997) and Thoft-Christensen & Baker (1982) are recommended. For more advanced problems the books by Ditlevsen (1981), Ditlevsen and Madsen (1996), Melchers (1999) as well as Ang and Tang (1984) might be helpful.

8.1.1 Global safety factor format

The traditional method to define structural safety is through a general factor of safety, which may be selected on the basis of experiments, practical experience, economic as well as political considerations. Global safety factor formats were the basis for most of the former codes and standards used throughout Europe. The general safety factor format is often associated with elastic stress analysis and requires:

\[
S \leq R_a = \frac{R_f}{\gamma_g}
\]

(8.1)

where \(S\) is the applied stress and \(R_a\) is the allowable stress, which is derived by dividing of the so-called failure stress \(R_f\) of the material by a global safety factor \(\gamma_g\), set conventionally. Thus, the safety principle consists of verifying that the maximum stresses calculated in any section of any part of the structure under worst case loading remain lower than the allowable stress. Failure of a structure occurs when a part of it reaches the local allowable stress. The values for the allowable stresses are set more or less arbitrarily based on the mechanical properties of the material used. Whether failure actually occurs depends entirely on how well represented is the actual stress in the structure at the critical cross-section and how the actual material failure is represented. It is well known that observed stresses do not always correspond well to the stresses calculated by linear elastic analysis. Stress redistribution, stress concentration as well as changes due to boundary effects or the physical size effect of members all contribute to discrepancies which are not reflected by this method.

8.1.2 Partial safety factor format

The partial safety factor format is the basis of many codes and standards, such as the Eurocodes, currently in use. The partial safety factor format is claimed to be semi-probabilistic, considering the application of statistics and probability in the evaluation of input data, the formulation of assessment criteria and the determination of load and resistance factors. However, from the user’s point of view, the application of the partial safety factor format is still deterministic. Thus, the partial safety factor format does not provide information that would allow the user to assess the actual risk or reserve carrying capacity of structures.

The semi-probabilistic partial safety factor format replaces actual probability calculations as described in Section 7.2 by the verification of a criterion involving characteristic values of the resis-
Reliability Analysis

tance R and the stress S, denoted as $R_k$ and $S_k$, as well as partial safety factors $\gamma_R$ and $\gamma_S$ and can be described by the following formal limit state:

$$S_k \cdot \gamma_S \leq \frac{R_k}{\gamma_R} \quad (8.2)$$

The reliability of a given structure is ensured by certain requirements for the limit state, the characteristic values and the partial safety factors. These requirements are for example stated in the codes using this approach. Partial safety factors are designed to cover a large number of uncertainties and may therefore not be very representative for evaluating the reliability of a particular structure. Partial safety factors should be calibrated using probabilistic methods and idealised reliability formats, but in most of the countries where semi-probabilistic codes are used, the actual values for the partial safety factors are still influenced by experience as well as economic and political considerations.

### 8.1.3 Reliability formats

Using reliability formats the stress S applied and the resistance R describing the strength of the structural element are described by stochastic variables because their values are not perfectly known. If the verification of the criterion related to the limit-state results in the inequality:

$$S \leq R \quad (8.3)$$

the structure is considered safe. The difference, $R - S$, is called safety margin $M$. Figure 8.1 shows the problem with the variables $R$, $S$ and $M$. As the sum of two variables, the safety margin $M$ is also a variable and is Normally distributed if the variables $R$ and $S$ are Normally distributed. $\beta$ is the so-called reliability index and is determined as $
abla = \mu_M / \sigma_M$.

The safety margin $M$ distinguishes three states:

- the safe state or safety domain with $M > 0$,
- the limit state with $M = 0$ and
- the unsafe state or failure domain with $M < 0$.

The probability of failure $p_f$ of $S \leq R$ characterises the reliability level of a structure with regard to the limit state considered:

$$p_f = P(R - S \leq 0) = P(M \leq 0) \quad (8.4)$$

In Figure 8.2 R and S are plotted as marginal probability density functions on the r and s axes. The limit state equation $M = R - S$ separates the safe from the unsafe region by dividing the "hump" into two parts. The volume of the part cut away and defined by $s > r$ corresponds to the probability of failure $p_f$. The design point $(r^*, s^*)$ lies on the line defined by $R - S = 0$ where the joint probability density is greatest. If failure occurs it is likely to be near there.

If more than two variables are considered and if the safety margin is expressed by a non-linear function of the different variables, the probability of failure is:

$$p_f = \int_{M \leq 0} f_x(x_1, ..., x_n)dx_1...dx_n \quad (8.5)$$

with $M$ being the safety margin composed of $n$ variables represented as components of the vector $x$ and $M \leq 0$ representing the failure domain.
Reliability methods taking into account uncertainties of variables are the main criteria for a realistic safety assessment. Thus, reliability formats using probabilistic methods are an important alternative to semi-probabilistic approaches. Reliability formats are based on:

- the definition of a limit-state criterion,
- the identification of all variables influencing the limit-state criterion,
- the statistical description of these variables and the consideration of stochastic (in)dependency,
- the derivation of the probability density and its moments for each basic variable,
- the calculation of the probability that the limit-state criterion is not satisfied and
- the comparison of the calculated probability to a target probability.

If the assumptions on the variables are not based on adequate data, estimates of reliability can be misleading. Therefore, it is essential to ensure the quality of data and validity of assumptions when using probabilistic methods to make a decision on the reliability of a specific structure. This can be assured by standardising the approach and by setting requirements on how to use data with it. When
modelling the variables it is also very important to take into account what design codes, design methods and assumptions the Engineer has used during the original design of the structure. Furthermore, old codes and standards are often a valuable source of information when parameters of the distributions have to be determined.

The evaluation of equation 8.5 is a very difficult task, except for linear limit states and Gaussian variables. A direct analytical solution or numerical integration are often not possible. Thus, two methods, i.e., the reliability index methods and simulation methods, are introduced which allow the calculation of the probability of failure, even for complicated functions.

8.1.3.1 Reliability index method

Reliability index methods such as FORM (First Order Reliability Method) or SORM (Second Order Reliability Method) approximate the calculation of the probability of failure. The first step consists of transforming the problem into a space of standard Normal Distributions (Figure 8.3). In the standardised space the nearest point from the origin to the transformed limit state is called the design point and its distance from the origin is the reliability index $\beta$.

In FORM the failure surface is approximated by a tangent hyperplane at the design point and the probability of failure can be approximated by:

$$p_f = \Phi(-\beta)$$  \hspace{1cm} (8.6)

where $\Phi$ is the probability function of the standard Normal variable.

8.1.3.2 Simulation methods

The most important sampling methods are Monte-Carlo sampling (Melchers 1999) where the probability density function and the associated statistical parameters of the safety margin are estimated approximately employing random sampling. This method is very time-consuming for the solution of real Engineering problems.

Figure 8.3: Transformation to the standardised space
Advanced simulation methods such as importance or directional sampling try to reduce computational time by reducing the sample size required for the estimation of the probability of failure. These methods can be used instead of or together with reliability index methods especially in cases where it becomes important to check the accuracy of reliability index methods, such as multi-mode or multi-component failure.

8.1.4 Socio-economic formats

Socio-economic formats are a combination of reliability formats with socio-economical considerations. Failure costs are introduced to determine the required probabilities of failure or reliability indices.

8.2 Calculation of failure probabilities for time-invariant problems

Only for very simple cases can the probability of failure be determined by exact analytical methods or numerical integration. Direct numerical integration is only possible in some very special cases. For limit-state functions of more general form than linear functions and random variables that are non-Gaussian distributed, integration methods are not used in reliability computations due to the rapidly increasing computational demands as the number of dimensions increases (curse of dimensionality).

For most of the problems with quite a large number of random variables and different types of distribution, approximate methods, such as FORM or SORM, simulation methods or a combination of both, have to be used. The most common techniques are described in the following sections. More detailed information is given by, for example, Ditlevsen & Madsen (1996), Melchers (1999), Madsen (1987) and Ang & Tang (1984).

8.2.1 Simulation techniques

There are two different types of simulation method. The first type consists of zero-one indicator based methods which are non-analytical and semi-analytical conditional expectation methods. Direct Monte Carlo simulation with a sampling density equal to the original probability density, importance sampling where the Monte Carlo method is used with a fictitious density function close to the design point as well as adaptive sampling in which importance sampling is applied and the density function is updated successively, are zero-one indicator methods.

Direct Monte Carlo simulation is not likely to be used for Structural Engineering problems. For such practical problems many samples are required to estimate the probability of failure with an appropriate degree of confidence with the required number of samples increasing as the probability of failure decreases. The probability of failure of a structure is usually a very small quantity. For \( p_f = 10^{-6} \) the necessary number of samples \( N \) could be estimated as \( N > \frac{1000}{p_f} \).

The second type of simulation, importance sampling, is a more advanced sampling technique. Its objective is to reduce the size of the sample required. Importance sampling is a very robust and efficient approach for single limit state problems.

Conditional expectation methods comprise directional sampling as well as axis orthogonal simulation. Directional sampling is applied for unions of events, whereas axis orthogonal simulation is suitable for intersections of events. The objective of both methods is again to reduce the size of the sample required.
8.2.2 Second-moment and transformation techniques

8.2.2.1 First Order Reliability Method (FORM)

Using FORM the limit-state surface is linearised at the design point. The procedure to determine the probability of failure is straightforward even for non-linear limit-state functions. FORM includes also non-Gaussian random variables. It is quite a robust method and difficulties might only arise in very extreme cases where the linearisation of the transformed limit state equation leads to inaccurate results.

FORM uses the derivatives of the limit-state function. For simple examples the derivatives can be expressed explicitly. However, when the limit-state function is complex and dependent on structural behaviour or analysis, other numerical procedures are necessary which increases the computational effort as the number of basic variables increases.

The main steps in the FORM method with \( \mathbf{X} \) being the vector of the basic variables are:

- transformation of the variables \( \mathbf{X} \) into a space of standard Normal variables, \( \mathbf{U} \), and a corresponding transformation of the failure surface \( g(\mathbf{X}) = 0 \) into \( g^c(\mathbf{U}) = 0 \),
- approximation of the failure function \( g(\mathbf{U}) \) by a tangent hyperplane at the design point, which is the point on \( g(\mathbf{U}) \) closest to the origin,
- calculation of the probability of failure \( p_f = \Phi(-\beta) \), where \( \beta \) is the distance from the origin to the design point.

8.2.2.2 Second Order Reliability Method (SORM)

The use of a linear approximation for the limit state surface becomes less accurate as the limit state function becomes more curved. Methods dealing with the non-linearity of the limit-state have been termed second order methods. In SORM the limit-state surface is approximated by a parabolic, quadratic or higher surface at the design point.

8.3 Time-variant problems

Time-variant problems are characterised by the variability of actions and/or strength over time (degradation). These time-variant quantities need to be represented by stochastic processes. Two classes of time dependent problems may be relevant when structures are assessed, i.e., overload (first passage) failure and fatigue or other cumulative failure.

For solving time-dependent problems three possible ways have been proposed so far which are associated with simulation and with FORM/SORM respectively:

- importance and conditional sampling,
- directional simulation in the load process space,
- FORM for unconditional failure probability.

The simulation-based approaches are a natural extension of time-independent analysis once the outcrossing rate can be estimated efficiently. FORM applied to time-dependent problems tend to be far more difficult than for time independent problems and often numerical techniques have to be used to solve the resulting formulations. In this context importance sampling has been found to be particularly useful.

In comparison to time independent reliability techniques, there has been rather little comparison of the various approaches for time dependent solution techniques. One cause might be the excessive
computational times required for fundamental comparison (e.g., crude Monte Carlo) when stochastic processes are involved.

For practical Structural Engineering problems, a fully time-dependent approach would only be required when the resistance variables are time dependent or when more than one loading case has to be considered. Due to the complexity of the application of time-dependent approaches, these problems are often simplified.

8.4 Reliability analysis software

In practice reliability analysis and assessment of existing structures is in most cases not possible without appropriate software tools. In this section the most common software products are listed along with their main features and implementation algorithms. The main features are summarised in Table 8.1. Other software certainly exists or is currently under development. Thus the reader should keep the situation under review and use this list as a source for further information.

8.4.1 CALREL
Source: NISEE/Computer Applications, 404A Davis Hall, University of California, Berkeley, California, 94720, USA, [http://www.nisee.berkeley.edu](http://www.nisee.berkeley.edu).

- Program for structural reliability analysis using FORM/SORM for components and systems
- Possibility of sensitivity analysis, Monte Carlo simulation and directional simulation
- Large library of probability distributions for independent and dependent variables
- Can be operated as a shell program in conjunction with other programs

8.4.2 ISPUD

- Structural Reliability Package based on the use of search- or adaptive importance sampling
- limit-state function either explicit or implicit as a response surface
- Extreme value approach for time-dependent problems

8.4.3 PROBAN
Source: DNV Software, Veritasveien 1, PO Box 300, N-1332 Hovik, Norway, [http://www.dnv.com/](http://www.dnv.com/).

- FORM/SORM analysis
- Library of distribution functions
- Importance sampling and directional simulation methods
- First passage problems
- Can handle many types of stochastic material and system models
- Allows sensitivity and parametric sensitivity analysis

8.4.4 STRUREL

- Programs for component and system reliability, numerical analysis and structural analysis
• Package for statistical analysis
• Search routines within FORM
• Facilities for time variant analysis (outcrossing rate determination)

8.4.5 VAP

Source: IBK, ETH Zurich, CH-8093 Zurich, Switzerland, http://www.ibk.baum.ethz.ch/proserv/vap.html/.

• Windows-based
• All input introduced by means of clearly arranged windows
• All input values can easily be changed at any time
• Includes FORM, crude Monte Carlo method and a numerical integration method

8.4.6 Stochastic finite element software

There are also some programs available which combined reliability and finite element analysis. When using stochastic finite elements special attention should be given to the definition of the size of the random field mesh in comparison to the finite element mesh.

8.4.6.1 COSSAN


• Finite element code (only a few element types) and response surface method
• Advanced Monte Carlo simulation techniques, e.g., importance sampling and adaptive methods
• Everything integrated through a language interpretation package

8.4.6.2 NESSUS


• Uses probabilistic finite element and boundary element codes
• Advanced mean value first order algorithm
• Fast probability integration (importance sampling)

8.5 Conclusions

In this chapter different methods for determining the reliability of an existing structure were summarised. A broad range of methods is available. Which method to use greatly depends on the structure and complexity of the problem at hand.

For practical structural engineering problems the First or Second Order Reliability Methods (FORM or SORM) in most cases give good results. These standard methods have the advantages of being quite robust and not very time-consuming.
Table 8.1: Software tools for reliability analysis

<table>
<thead>
<tr>
<th>Name of Program</th>
<th>CALREL</th>
<th>ISPUD</th>
<th>NESSUS</th>
<th>PROBAN</th>
<th>STRUREL</th>
<th>VAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphic User Interface</td>
<td>-</td>
<td>-</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Platform</td>
<td>W/S/PC</td>
<td>PC</td>
<td>W/S/PC</td>
<td>W/S/PC</td>
<td>PC</td>
<td>PC</td>
</tr>
<tr>
<td>Symbolic coding</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>FORM</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>SORM</td>
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<td>yes</td>
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<td>yes</td>
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<td>yes</td>
<td>-</td>
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<td>Crude Monte Carlo</td>
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<td>yes</td>
<td>yes</td>
<td>-</td>
<td>yes</td>
<td>yes</td>
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<td>Adaptive sampling</td>
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<td>Latin hypercube sampling</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>Number of Distributions</td>
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<td>10</td>
<td>10</td>
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<td>45</td>
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<td>yes</td>
<td>-</td>
<td>yes</td>
<td>yes</td>
<td>-</td>
</tr>
<tr>
<td>Integration with FEA code</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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</tr>
</tbody>
</table>
APPENDICES
Appendix A  Whole life costing formulas

A1  Life cycle costing formulas

A1.1  Owner costs

The standard method for calculating life cycle costs is by discounting the different future costs to present values. The “present” time might differ, but usually the time used is the time of inauguration of the project. The life cycle cost (LCC) is then the sum:

\[
LCC_{\text{owner}} = \sum_{t=0}^{T} \frac{C_t}{(1 + p)^t}
\]

where:
- \(C_t\) = the sum of all costs incurred at time \(t\),
- \(p\) = the real interest rate or a rate taking into account changes in the benefit of the structure,
- \(T\) = the time period studied; typically for a structure, the expected life span.

The most important factor in equation A.1 is, except of course the costs, the interest rate \(p\). The real interest rate is usually calculated as the difference between the actual discount rate for long loans \((p_L)\) and the inflation \((p_i)\) as:

\[
p = \frac{p_L - p_i}{1 + p_i}
\]

The effect of the factor in the denominator is, taking the uncertainties into consideration, negligible.

If there is a change in the benefit of the structure, i.e., an increase in the traffic using the bridge, this could approximately be taken into consideration by using the formula:

\[
p = \frac{p_L - p_i - p_c}{1 + p_i}
\]

where \(p_c\) is the increase in traffic volume using the structure. If there is a risk for the opposite, a decrease in the usefulness of the structure, this factor should be given a negative sign. This could i.e. be accomplished by building the structure at the wrong place or on a road with decreasing traffic.

Taking all factors into account the \(p\)-value should be called “calculation interest rate” or likewise.

Typical values for \(p\) are in the order from 3% to 8%.

Equation A.1 is usually used to calculate the owners cost for investment, operation, inspection, maintenance, repair and disposal. The costs \(C_t\) at the time of inauguration are usually not too complicated to assume for the necessary above-mentioned steps in the management of a structure. There
is a great uncertainty in choosing the \( p \)-value, but still more uncertain is the calculation of the time intervals between the different maintenance works and repairs.

A1.2 Costs for the society

Typical costs, not clearly visible for the owner are costs occurring due to damage to the environment, the usage of non-renewable materials and society costs for health-care and deaths due to traffic accidents.

Most construction materials consume energy for production and transportation. One way to take this into account is by multiplying all costs for materials for construction and repair with some factor due to energy consumption for manufacturing and transportation.

The use of non-renewable materials might be taken into consideration by involving costs for reproducing or reusing materials when the structure is decommissioned.

Costs for health-care due to accidents and deaths is probably only actual when two different types of structures are compared and when the risks for accidents differs between the two concepts, or costs for accidents due to roadwork. The accident costs for roadwork can be calculated using:


\[
LCC_{\text{society, accident}} = \sum_{t=0}^{T} (A_n - A_r) ADT_t \cdot N_t \cdot C_{\text{acc}} \frac{1}{(1 + p)^t}
\]  

where:

\( A_n \) = the normal accident rate per vehicle-kilometres,

\( A_r \) = the accident rate during roadwork,

\( C_{\text{acc}} \) = the cost for each accident for the society,

\( ADT_t \) = the average daily traffic, measured in numbers of cars per day at time \( t \),

\( N_t \) = the number of days of roadwork at time \( t \).

The costs should be calculated to present value and added up for all foreseen maintenance and repair works for the studied time interval \( T \).

As an example the Swedish Road Administration uses a cost of about 1.6 million € for deaths and a third of that sum for serious accidents.

A1.3 User costs

User costs are typically costs for drivers, the cars and transported goods on or under the bridge due to delays due to roadwork. Driver delay cost is the cost to the drivers who are delayed by the roadwork, vehicle operating cost is capital cost for the vehicle which are delayed by roadwork and cost for goods are all kinds of costs for delaying the time for delivering the goods in time. Other user costs might be cost of damage to the vehicles and humans due to roadwork not included in the cost for the society. Travel (or driver) delay costs can be computed as:


\[
LCC_{\text{user, delay}} = \sum_{t=0}^{T} \left( \frac{L}{v_r} - \frac{L}{v_n} \right) ADT_t \cdot N_t \left( r_L w_L + (1 - r_L)w_D \right) \frac{1}{(1 + p)^t}
\]  

where:

\( v_r \) = the average speed of the traffic during the repair work measured in km/h,

\( v_n \) = the normal average speed measured in km/h,
Whole life costing formulas

\[ L = \text{the length of affected roadway or which cars drive,} \]
\[ v_r = \text{the traffic speed during bridge work activity,} \]
\[ v_n = \text{the normal traffic speed,} \]
\[ ADT_t = \text{the average daily traffic, measured in numbers of cars per day at time } t, \]
\[ N_t = \text{the number of days of road work at time } t, \]
\[ r_L = \text{the amount of commercial traffic,} \]
\[ w_L = \text{the hourly time value for commercial traffic,} \]
\[ w_D = \text{the hourly time value for drivers.} \]

The costs should be calculated to present value and added up for all foreseen maintenance and repair work for the studied time interval \( T. \)

Vehicle operating costs and costs for transported goods can be calculated as:

\[
LCC_{\text{user,operating}} = \sum_{t=0}^{T} \left( \frac{L}{v_r} - \frac{L}{v_n} \right) ADT_t \cdot N_t \left( r_L (o_L + o_G) + (1 - r_L) o_D \right) \frac{1}{(1 + p)^t} \tag{A.6}
\]

In equation A.6 the same parameters are used as in equation A.5 except for \( o_L \) which are operating cost for the commercial traffic vehicles, \( o_G \) operating cost for transported goods and \( o_D \) operating cost for cars.

There is usually an accident cost for roadwork for the user not included in the cost for the society. Equation A.4 could be used also for this by just adjusting the cost parameter for this case.

A1.4 Failure costs

There is a small risk for the total failure of a structure. To get the cost for failure one has to calculate all costs for the failure, accidents, rebuilding, user delay costs and so on and then multiply these costs with the probability for failure and with the appropriate present value factor.

A2 Time intervals to be used in whole life costing analysis

To be able to calculate costs incurring at different times and then be able to discounting these costs to present values, one has to assume the time intervals for different measures that has to be taken during the life span\(^1\) of a structure. Many parameters in a life cycle costing analysis are prescribed by the authorities, typically “calculation interest rates”, societal costs for accidents, costs for damage to the environment etc. The most important factor left to the structural engineer analysing the life cycle cost is the rate of deterioration and thus the necessary time intervals for maintenance and repair. Practical figures for the deterioration rates are very difficult to find in the literature and theoretical formulas based on the chemical or physical processes seem not very useful when calculating the life cycle cost. This is because many of these processes interact and influence with each other and with physical attacks i.e. car smashes, scribbling, sabotage etc. One could describe this as a “domino”-effect. The best way to determine maintenance and reparation time intervals is to study

---

\(^1\) For assessing an existing structure, the remaining life span is used instead.
the field data collected from inspections, maintenance and repairs as such data give good indications on the deterioration rate of the different parts of the structure.

Tables A.1 and A.2 below give inspection time intervals adopted in some countries and some important standard maintenance intervals.

**Table A.1: Inspection intervals in some countries**

<table>
<thead>
<tr>
<th>Country</th>
<th>General inspection</th>
<th>Major inspection</th>
<th>Special inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>1 year</td>
<td>3 years</td>
<td>Depends on results from major inspection</td>
</tr>
<tr>
<td>Denmark</td>
<td>1-6 years</td>
<td>Depends on general inspection results</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>1 year</td>
<td>5 years</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>3 months</td>
<td>1 year</td>
<td></td>
</tr>
<tr>
<td>Canada (Ontario)</td>
<td></td>
<td></td>
<td>Defines by the owner (2 years is recommended)</td>
</tr>
<tr>
<td>Slovenia</td>
<td>2 years</td>
<td>6 years</td>
<td>Depends on results from major inspection</td>
</tr>
<tr>
<td>Switzerland</td>
<td>15 months</td>
<td>5 years</td>
<td>when needed</td>
</tr>
<tr>
<td>Sweden</td>
<td>1 year</td>
<td>3 years</td>
<td>6 years</td>
</tr>
<tr>
<td>Germany</td>
<td>3 months</td>
<td>3 years</td>
<td>6 years</td>
</tr>
<tr>
<td>USA (national bridges)</td>
<td></td>
<td>2 years</td>
<td></td>
</tr>
</tbody>
</table>

**Table A.2: Some maintenance intervals according to the Swedish Road Administration**

<table>
<thead>
<tr>
<th>Preventive maintenance</th>
<th>Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impregnation of edge beams</td>
<td>10 years</td>
</tr>
<tr>
<td>Painting maintenance</td>
<td>20-30 years</td>
</tr>
<tr>
<td>Exchange of rubber lists in expansion joints</td>
<td>15 years</td>
</tr>
<tr>
<td>Water cleaning for removal of de-icing salts etc.</td>
<td>1 year</td>
</tr>
<tr>
<td>Rehabilitation of erosion protection</td>
<td>6 years</td>
</tr>
<tr>
<td>Cleaning of the drainage system</td>
<td>1 year</td>
</tr>
<tr>
<td>Cleaning from vegetation</td>
<td>5 years</td>
</tr>
</tbody>
</table>
Appendix B  A collection of proposed mathematical and probabilistic models of concrete and steel reinforcement

B1  Concrete

B1.1  General
The concrete properties discussed herein are strength in compression and tension, modulus of elasticity in compression and tension, and creep and shrinkage. Sources of uncertainty in the determination of these concrete properties are due to variations in the properties of the components of the concrete and proportion of concrete mix, variations in mixing, transporting, placing and curing methods, variations in testing procedures, and variations due to concrete being in a structure rather than in control specimens (Mirza 1979b).

B1.2  Concrete strength in compression (in-situ)
Mathematical models used to determine concrete strength in-situ are predominantly constructed around transforming the concrete strength measured in test cylinders into the in-situ concrete strength. In the number of methods investigated different ways to consider changes in time, spatial variation and loading speed of test cylinders are seen. In-situ concrete strength, $f_c$, is not the same as the concrete strength measured in test cylinders, $f'_c$. It is normally lower than the $f'_c$ because of the different placing and curing procedures, the effects of vertical migration of water during the placement of concrete in deep members, the effects of difference in size and shape, the effects of different stress regimes, and the difference in directions of casting and loading of the structure and the specimens (Mirza 1979b).

B1.2.1  Proposed models

Mirza, Hatzinikolas and MacGregor (Mirza 1979b)
It was reported that average ratios of in-situ concrete strength (measured using cores) to standard cylinder strength from (Bloem 1968, Campbell 1967 and Petersons 1968) varied from 0.74 to 0.9. It was proposed to determine the mean and coefficients of variation (herein, COV) of the $f_c$ from those of the $f'_c$ (at a loading rate of 0.24 MPa/sec for an average quality control) by equations B.1 and B.2.

$$f_c = 0.675 \cdot f'_c + 7.6 \leq 1.15 f'_c \quad \text{MPa}$$  \hspace{1cm} (B.1)

$$V_c^2 = V_{cyl}^2 + 0.0084$$ \hspace{1cm} (B.2)

where $cyl$ is compressive cylinders.
Changes in time were not considered. The speed of loading of the compression cylinders was to be considered directly as shown in equation 13.

**Bartlett and MacGregor (Bartlett 1996)**

In 1996 it was proposed that the mean and COV of \( f_c \) be determined as a probabilistic function of the ratio of average strength of concrete \( f'_c \) to the specified strength \( f_{spec} \), \( F_1 \), and the ratio of average \( f'_c \) to average \( f'_{c} \), \( F_2 \) (equations B.3 and B.4, respectively).

\[
f_c = F_1 \cdot F_2 \cdot f_{spec} \quad \text{(B.3)}
\]

\[
V^2_c = V^2_{cF1} + V^2_{cF2} \quad \text{(B.4)}
\]

The probabilistic models of \( F_1 \) and \( F_2 \) determined in (Bartlett 1996) are given in Table B.1 and Table B.2. It was found that ready mix concrete and precast concrete have different ratios of average strength to specified strength. It was found that the values of \( F_2 \) were consistently larger for columns and walls than those for beams and slabs, thus the distinction at 450 mm. It was suggested that the relative strength of the cores for elements thinner than 450 mm may have been because the cores contained relatively weak concrete from near the top of the element, or because of the greater sensitivity of the in-place slabs to poor curing practices.

**Table B.1: Probabilistic models of \( F_1 \) (Bartlett 1996)**

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Distribution</th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>Distribution</th>
<th>( \mu )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ready mix</td>
<td>Normal</td>
<td>1.25</td>
<td>0.13</td>
<td>Lognormal</td>
<td>0.234</td>
<td>0.180</td>
</tr>
<tr>
<td>Pre-cast</td>
<td>Normal</td>
<td>1.19</td>
<td>0.06</td>
<td>Lognormal</td>
<td>0.122</td>
<td>0.053</td>
</tr>
</tbody>
</table>

*The specified strengths were determined according to (CSA 1990)*.

**Table B.2: Probabilistic models of \( F_2 \) (Bartlett 1996)**

<table>
<thead>
<tr>
<th>Concrete elements</th>
<th>Distribution</th>
<th>( \mu )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt; 450 \text{ mm high})</td>
<td>Normal</td>
<td>0.95</td>
<td>0.14</td>
</tr>
<tr>
<td>(\geq 450 \text{ mm high})</td>
<td>Normal</td>
<td>1.03</td>
<td>0.14</td>
</tr>
</tbody>
</table>

*Changes in time were not explicitly taken into consideration.*

**ISO2394 (ISO 1986)**

In ISO2394 the mathematical model is simple (equation B.5), there is only one factor that converts the strength of the control cylinder to the in-situ strength, \( \eta \). The mean and coefficients of variations (COV) of \( \eta \) are given in Table B.3 for C20 and C40 concrete.

\[
f_c = \eta \cdot f'_{c} \quad \text{(B.5)}
\]
Table B.3: Parameters for $\eta$ in equation B.5 (ISO 1986)

<table>
<thead>
<tr>
<th>Concrete Grade</th>
<th>$\mu_\eta$</th>
<th>$\sigma_\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20</td>
<td>0.90</td>
<td>0.274</td>
</tr>
<tr>
<td>C40</td>
<td>0.85</td>
<td>0.316</td>
</tr>
</tbody>
</table>

where $\mu$ is the mean and $\sigma$ is the standard deviation.

JCCS (JCSS 2001)

The mathematical model proposed in (JCSS 2001) is the most elaborate of those reviewed. For in-situ concrete strength in compression at a particular point in the structure the model proposed is:

$$f'_c = \alpha(t, \tau) \cdot (f'_c)^i \cdot Y_i$$ (B.6)

where:

- $\alpha(t, \tau)$ = a deterministic function which takes into account the concrete age at the loading time $t$ (days) and the duration of loading $\tau$.
- $f'_c$ = a log-normal variable representing basic compressive strength, independent of $Y_i$.
- $\lambda$ = a log-normal variable to take into account the system variation of in-situ compressive strength and strength of standard tests. It is suggested to have a mean of 0.96 and COV of 0.005. It is also noted that this may be taken deterministically.
- $Y_i$ = a variable representing additional variations due to the special placing, curing and hardening conditions of in-situ concrete. It can also be taken as spatially varying random field whose mean value function takes account of systematic influences in space.

B1.2.2 Basic compressive strength $f'_c$

The probabilistic model of basic compressive strength, found using test cylinders, is crucial in all of the mathematical models used to determine the in-situ concrete strength. (Mirza 1979b) that “the majority of researchers have represented the distribution of concrete compressive strengths with a normal distribution (Julian 1955, Rüsch 1969 and Shalon 1955)”, but suggested that the lognormal distribution gives a better fit for concrete strength in which the quality control is poorer than average (COV > 0.15-0.20). It was found in a literature review done by (Mirza 1979b) that the average COV of concrete strengths in compression from various previous tests (Allan 1970, Komlos 1970, Malhorta 1969, Orr 1970, Ramesh 1960 and Wright, 1955) were 0.057. In 1991 it was found that in Sweden the COV of concrete obtained from compression test cubes decreases with increasing grade of concrete (CEB-FIP 1991).

Normal distributions were found in Balaguru (1995) to give an increasingly conservative approach to the modelling of the low tail of the $f'_c$ for the 2.5, 1.0 and 0.5 percent low chi-square tests. Lognormal distributions were found to give unconservative estimates at the 1.0 and 0.5 percent low tests. It was found that overall prediction accuracy for low tests 25 – 0.5 percent low tests is better if lognormal distributions are used. The COV is also much smaller for the lognormal distribution.
It is suggested in (JCSS 2001) that the distribution of $x_i = \ln(f')$ is normal if its parameters $\mu$ and $\sigma$ are obtained from an infinitely large sample, but because the sample is not infinite $\mu$ and $\sigma$ must be treated as random variables and $x_i$ has a student t-distribution according to:

$$F_s(x) = F_{tf}[\frac{\ln(x/\mu)}{\sigma} \left(1 + \frac{1}{n}\right)^{-0.5}]$$  \hspace{1cm} (B.7)

where:

$F_{tf} = $ the student t-distribution with $f$ degrees of freedom.

$n = $ sample size.

The $f'$ can therefore be represented as:

$$f' = \exp\left(\mu + t_f \cdot \sigma \cdot \left(1 + \frac{1}{n}\right)^{0.5}\right) \text{ Mpa}$$  \hspace{1cm} (B.8)

where $t_f$ the student t-density function for $f$ degrees of freedom.

Table B.4 gives the values suggested by (JCSS 2001) to determine $f'$ if there is no information available. The prior parameters may depend on the geographical area and the technology with which the concrete is produced. The distributions for the values given Table B.4, and the approximations given by equation B.8 are shown in Figures B.1 and B.2 for ready-mix concrete and pre-cast elements.

It is also suggested in (JCSS 2001) that $f_c$ can be approximated by the log-normal distribution with mean $\mu$ and standard deviation $\sigma \cdot \sqrt{\frac{n}{n-1} \cdot \frac{f}{f-2}}$ if $nf > 10$.

It is also found that similar values are taken in other references (Ellingwood 1980, Nowak 1994).

**Table B.4: Values of $\mu$, $n$, $\sigma$, and $f$ if no information is available (JCSS 2001)**

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Concrete grade</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$n$</td>
</tr>
<tr>
<td>Ready mix</td>
<td>C15</td>
<td>3.40</td>
</tr>
<tr>
<td></td>
<td>C25</td>
<td>3.65</td>
</tr>
<tr>
<td></td>
<td>C35</td>
<td>3.85</td>
</tr>
<tr>
<td></td>
<td>C45</td>
<td>3.98</td>
</tr>
<tr>
<td></td>
<td>C55</td>
<td>-</td>
</tr>
<tr>
<td>Pre-cast elements</td>
<td>C15</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>C25</td>
<td>3.80</td>
</tr>
<tr>
<td></td>
<td>C35</td>
<td>3.95</td>
</tr>
<tr>
<td></td>
<td>C45</td>
<td>4.08</td>
</tr>
<tr>
<td></td>
<td>C55</td>
<td>4.15</td>
</tr>
</tbody>
</table>
A collection of proposed mathematical and probabilistic models of concrete and steel reinforcement

Figure B.1. Density functions to represent $f'_{c}$ for ready mix concrete

Figure B.2. Density functions to represent $f'_{c}$ for precast concrete

Table B.5: Parameters used for $f'_{c}$ by various researchers

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{spc}$ MPa</th>
<th>$\mu$</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ellingwood 1980, Nowak 1994)</td>
<td>20.7</td>
<td>19.0</td>
<td>0.18</td>
</tr>
<tr>
<td>(Ellingwood 1980)</td>
<td>27.5</td>
<td>23.4</td>
<td>0.18</td>
</tr>
<tr>
<td>(Ellingwood 1980, Nowak 1994)</td>
<td>34.5</td>
<td>27.8</td>
<td>0.15</td>
</tr>
</tbody>
</table>

where the nominal concrete strength taken to be $0.75 f'_{c}$.

B1.2.3 Changes in concrete strength with time $\alpha(t, \tau)$

Concrete strength changes with time due to the loads applied and the physical processes at work in the concrete. It is recommended in (JCSS 2001) to take into consideration the concrete age at time of loading, $t$ (days) and the duration of the loading ($\tau$) by using a deterministic function, $\alpha(t, \tau)$, (equation B.9).

$$\alpha(t, \tau) = \alpha_1(\tau) \cdot \alpha_2(t)$$ (B.9)

where:

$$\alpha_1(\tau) = \alpha_3(\infty) + [1 - \alpha_3(\infty)] \exp[-a_1\tau] \text{ with } \alpha_3(\infty) \approx 0.8 \text{ and } a_1\tau = 0.4$$ (B.10)

$$\alpha_2(t) = a + b \cdot \ln(t)$$ (B.11)

It was found in (Bartlett 1996) that the average in-situ strength increases by about 25 percent between 28 days and 1 year. This is in approximate agreement with what is proposed by (JCSS 2001), 30 percent between 28 days and 1 year.
B1.2.4 Changes in concrete strength due to spatial variation, $U_{ij}$, $Y_{ij}$

To take into consideration the variation of concrete strength spatially within one structure, it is recommended in (JCSS 2001) to use a standard normal variable, $U_{ij}$. It is proposed that the variables $U_{ij}$ and $U_{kj}$ are correlated within one member by:

$$\rho(U_{ij}, U_{kj}) = \rho + (1 - \rho) \cdot \exp \left( \frac{r_{ij} - r_{kj}}{d_c^2} \right)$$  \hspace{1cm} (B.12)

where $d_c = 5$ m and $\rho = 0.5$. For different jobs $U_{ij}$ and $U_{kj}$ are uncorrelated.

It is recommended in (JCSS 2001) to use a log-normal variable to represent the additional variations in concrete strength due to the special placing, curing and hardening conditions of in-situ concrete at job $j$, $Y_{ij}$, with a mean of 1 and a COV of 0.06.

In (Bartlett 1996) it was found that for Canadian practice that for one member cast from a single batch of concrete the COV was about 0.07 and for many members cast from a number of batches of concrete about 0.13. It was also estimated that the COV for in-situ concrete strength for yet to be placed concrete (e.g. the design estimate) was about 0.23 (Bartlett 1996).

B1.2.5 Degree of quality control

It is possible to take into consideration the degree of quality control in the determination of the $f' c$. (Mirza 1979b) found that the COV of $f' c$ changes for different strength levels and qualities of control. They also reported that an unpublished analysis of concrete cylinders across Canada showed that the COV of cylinder strength were approximately 0.12, 0.15 and 0.18 respectively. The coves suggested are illustrated in Figure B.5 in the form of COV and $\sigma$. The ACI214 recommended that the level of control within the batch tests could be divided into three classes with corresponding COV as 0.04-0.05 for good control, 0.056 for average control and 0.06 for poor control Table B.6, Figure B.6) (Mirza 1979b).

In (Stewart 1995) it is proposed that the strength of the control cylinders given by $f' c$ be estimated, noting that it is a function of curing, $k_{cr}$, and compaction, $k_{cp}$, of concrete:

$$f_{spec} = k_{cp} \cdot k_{cs} \left( f' c + 1.65 \cdot \sigma_c \right)$$  \hspace{1cm} (B.13)
where the term $\left( f'_c + 1.65 \cdot \sigma_c \right)$ represents the mean compressive strength of perfect control cylinders, $\sigma_c$ is the standard deviation of the between batch concrete strengths (control). The parameters for the probabilistic models of $k_{cr}$ and $k_{cp}$ are functions of workmanship and quality control (performance) and are given in Table B.7.

Table B.6: Variation of the parameter of $f'_c$ for different quality control

<table>
<thead>
<tr>
<th>Source</th>
<th>(Mirza 1979b)</th>
<th>(ACI214 reported in Mirza 1979b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>COV ($f'_c &lt; 28$ MPa)</td>
<td>$\sigma (28 &lt; f'_c &lt; 50$ MPa)</td>
</tr>
<tr>
<td>Excellent</td>
<td>0.10</td>
<td>2.80</td>
</tr>
<tr>
<td>Average</td>
<td>0.15</td>
<td>4.20</td>
</tr>
<tr>
<td>Poor</td>
<td>0.20</td>
<td>5.60</td>
</tr>
</tbody>
</table>

Figure B.5. Density functions to represent $f'_c$ suggested by (Mirza 1979b)

Figure B.6. Density functions to represent $f'_c$ suggested by ACI214
Table B.7: Statistical parameters for \(k_{cp}\) and \(k_{cr}\) (Stewart 1995).

<table>
<thead>
<tr>
<th>Performance</th>
<th>(k_{cp}) (\mu)</th>
<th>(k_{cp}) COV</th>
<th>(k_{cr}) (\mu)</th>
<th>(k_{cr}) COV</th>
<th>(k_{cr}) (\mu)</th>
<th>(k_{cr}) COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor</td>
<td>0.80</td>
<td>0.06</td>
<td>0.66</td>
<td>0.05</td>
<td>0.66</td>
<td>0.05</td>
</tr>
<tr>
<td>Fair</td>
<td>0.87</td>
<td>0.06</td>
<td>0.84</td>
<td>0.05</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Good</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

B1.2.6 Effect of the speed of loading on concrete strength

To take into account the effect of loading rate on the in-situ concrete strength equation B.14 is proposed by (Mirza 1979b). It was found that speed has little effect on the overall COV of concrete. Equation B.14 is plotted in Figure B.7 for \(f_c = 20\) and 40 MPa.

\[
f_{cr} = f_c \cdot \left[0.89 \cdot (1 + 0.08 \cdot \log(145 \cdot \text{R}))\right] \text{ MPa} \quad (B.14)
\]

where R – rate of loading in MPa/sec

B1.3 Concrete Strength in Tension

The relationship between tensile and compressive strengths of concrete depend on the size and type of aggregate, air entrainment, curing conditions, w/c ratio, cement content, age at the time of loading (Mirza 1979b).

The mathematical model of tensile strength, \(f_{ct}\) proposed in (CEB-FIP 1991) is:

\[
f_{ct} = C \cdot f_c^{0.7} \quad (B.15)
\]

where C has the mean value 0.17 and the COV 0.15. The \(\mu\) and COV in tension of C20 and C40 grade concrete is shown in Table B.8.

![Figure B.7. Variation in \(f_c\) as a function of loading rate](image-url)
A collection of proposed mathematical and probabilistic models of concrete and steel reinforcement

Table B.8: Parameters of C20 and C40 grade concrete as determined in (CEB-FIP 1991)

<table>
<thead>
<tr>
<th>Concrete grade</th>
<th>μf_{ct}</th>
<th>cov_{f_{ct}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20</td>
<td>2.2</td>
<td>0.18</td>
</tr>
<tr>
<td>C40</td>
<td>3.2</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The mathematical model of tensile strength, $f_{ct,ij}$ proposed in (JCSS 2001) is:

$$f_{ct,ij} = 0.3 \cdot f_{c,ij}^{2/3} \cdot Y_{2,j}$$  \hfill (B.16)

It is recommended to use a log-normal variable to represent the additional variations in the tensile strength due to the special placing, curing and hardening conditions of in-situ concrete at job j, $Y_{2,s}$, with a mean of 1 and a COV of 0.3.

The probabilistic model of the splitting tensile strength of concrete in a structure, proposed by (Mirza 1979b) was a normal distribution with mean value given by equations B.17 and the COV by B.18.

$$f_{spsR} = 77.1 \cdot f_{c}^{1/2} \cdot \left[0.96 \cdot (1 + 0.11 \cdot \log(145 \cdot R))\right] \text{ MPa}$$  \hfill (B.17)

$$V_{spsR}^{2} = \frac{V_{cyl}^{2}}{4} + 0.0190 \geq V_{cstR}^{2}$$  \hfill (B.18)

It was found in a literature review done by (Mirza 1979b) that the average COV of concrete strengths in different tension tests from various previous tests (Allan 1972, Komlos 1970, Malhorta 1969, Orr 1970, Ramesh 1960 and Wright, 1955) is 0.067.

The values found by (Ellingwood 1980) are shown in Table B.9.

Table B.9: Values found in (Ellingwood 1980)

<table>
<thead>
<tr>
<th>f_{spec} (comp) MPa</th>
<th>μ (tension)</th>
<th>cov_{f_{t}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.7</td>
<td>2.11</td>
<td>0.18</td>
</tr>
<tr>
<td>27.5</td>
<td>2.34</td>
<td>0.18</td>
</tr>
<tr>
<td>34.5</td>
<td>2.52</td>
<td>0.18</td>
</tr>
</tbody>
</table>

where nominal concrete strength taken to be $0.75 f'_{c}^{B1.4}$

B.4 Modulus of Elasticity in Compression

The mathematical model of modulus of elasticity proposed in (JCSS 2001) is:

$$E_{c,ij} = 10.5 \cdot f_{c,ij}^{1/3} \cdot Y_{3,j} \left(1 + \beta_{a} \varphi(t, \tau)\right)$$  \hfill (B.19)

where:

$$\varphi(t, \tau) = \text{deterministic creep coefficient}$$
\[ \beta_d = \text{the ratio of the permanent load to total load and depends on the type of structure; generally between 0.6-0.8.} \]

\[ Y_{s,j} = \text{is a log-normal variable to represent the additional variations in the modulus of elasticity due to the special placing, curing and hardening conditions of in-situ concrete at job j, with a mean of 1 and a COV of 0.15} \]

It was found in (Mirza 1979b) that there is a high degree of correlation between initial tangent modulus and compressive strength. It is proposed that the initial tangent modulus of elasticity of in-situ concrete be described by a normal distribution with a \( \mu \) and COV are given by equations B.20 and B.21.

\[ E_{cR} = 5015 \cdot f_{c}^{1/2} \cdot (1.16 - 0.08\log t) \text{ MPa} \quad (B.20) \]

where \( t = \text{loading duration in seconds} \)

\[ V_{cR}^2 = \frac{V_{cR}^2}{4} + 0.0085 \quad (B.21) \]

### B1.5 Modulus of Elasticity in Tension

It is hypothesized in (Mirza 1979b) that the mean values of the modulus of concrete in compression and in tension are the same. (Johnson 1928) found that there was little difference between the two modulus.

### B1.6 Concrete compression strain

The mathematical model of ultimate compression strain suggested in (JCSS 2001) is:

\[ \varepsilon_{u,j} = 10^{-3} \cdot f_{c}^{-1/6} \cdot Y_{s,j} \cdot (1 + \beta_d \phi(t, \tau)) \quad (B.22) \]

It is recommended to use a log-normal variable to represent the additional variations in the ultimate compression strain due to the special placing, curing and hardening conditions of in-situ concrete at job j, \( Y_{s,p} \), with a mean of 1 and a COV of 0.15.

### B1.7 Creep and Shrinkage

Probabilistic descriptions of creep and shrinkage properties can be found in (Madsen 1983).

### B2 Steel Reinforcement

#### B2.1 General

The uncertainties in the determination of steel strength are due to the variation in the strength of the material, variation in cross section of the bar, effect of rate of loading, effect of bar diameter on properties of bar and effect of strain at which, is defined (Mirza 1979a).
B2.2 Yield strength

B2.2.1 Probabilistic models of mill test strengths

According to (JCSS 2001) the yield strength, denoted by $X_1$, can be taken as the sum of three independent Gaussian variables:

$$X_1(d) = X_{11} + X_{12} + X_{13} \text{ MPa}$$  \hspace{1cm} (B.23)

where:

- $X_{11} = N(u_{11}(d), \sigma_{11})$ represents the variations in global mean of different mills,
- $X_{12} = N(0, \sigma_{12})$ represents the variations in a mill from batch (melt) to bath and
- $X_{13} = N(0, \sigma_{13})$ represents the variations within the melt

$d$ = the nominal bar diameter (mm).

In (JCSS 2001) it is suggested that 19 MPa, 22 MPa, and 8 MPa be used for $\sigma_{11}$, $\sigma_{12}$, $\sigma_{13}$, respectively for high standard steel production. It is also noted in (JCSS 2001 and Woodward 1999) that strength fluctuations along bars are negligible. The value $\mu_1(d)$ is defined as the overall mean from the entire production given a particular bar diameter

$$\mu_1(d) = \mu_1(0.87 + 0.13\exp[-0.08 \cdot d])^{-1} \text{ MPa}$$  \hspace{1cm} (B.24)

It is suggested by (JCSS 2001) that the yield strength be represented by a normal distribution with a mean equal to the nominal strength plus two standard deviations. The standard deviation should equal 30 MPa.

In 1980 a summary of selected studies (Allan 1972, Bannister 1968, Wiss 1973, Helgason 1975, Julian 1957 and American Society for Testing Materials 1972) on Grade 40 and 60 steel bars ($f_y = 275$ MPa, $f_y = 414$ MPa) showed adequate agreement with a normal distribution in the range from about the 5th to the 95th percentile (Mirza 1979a). The COVs found in (Mirza 1979a) are given in Table B.10. The coefficient of correlation between yield strengths of individual bars is around 0.9 (Rackwitz 1996).

It was found in (Mirza 1979a) that beta distributions, such as shown in equation B.25 using the values in Table B.11, correlate well with the data for mill test strength of Grade 40 and Grade 60 reinforcing bars. The distributions are plotted in Figure B.8.

$$f_{F_y}(x) = A \cdot \left(\frac{F_y - a}{c}\right)^b \cdot \left(\frac{b - F_y}{c}\right)^c$$  \hspace{1cm} (B.25)

<table>
<thead>
<tr>
<th>Table B.10: The COVs found in (Mirza 1979a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual bar sizes</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>COVs one source</td>
</tr>
<tr>
<td>COVs multiple source</td>
</tr>
</tbody>
</table>
Table B.11: Coefficients for equation 24 mill yield strength

<table>
<thead>
<tr>
<th>Grade</th>
<th>Mean</th>
<th>COV</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>337</td>
<td>0.107</td>
<td>3.721</td>
<td>2.21</td>
<td>3.82</td>
<td>248</td>
<td>469</td>
<td>221</td>
<td>MPa</td>
</tr>
<tr>
<td>60</td>
<td>490</td>
<td>0.093</td>
<td>7.141</td>
<td>2.02</td>
<td>6.95</td>
<td>393</td>
<td>745</td>
<td>352</td>
<td>MPa</td>
</tr>
</tbody>
</table>

B2.2.2 Probabilistic models of static yield strength

The probabilistic models of static yield strength proposed in (Mirza 1979a) based on nominal area of cross section of reinforcing bars can be represented by the beta distributions given in equation B.25 with the values given in Table B.12.

It was suggested in (Mirza 1979a) that the difference in the static yield strength and mill test strengths could be represented with N(0.24, 0.032) MPa for both Grade 40 and 60. In (CEB-FIP 1991) it was reported that the COV for the strength of all bar sizes was approximately 0.08.

B2.3 Yield strength of bundles of bars

The yield force of bundles of bars under static loading is the sum of the yield forces of each contributing bar. In general, it can be assumed that all reinforcing steel used at a job originates from a single mill. The correlation coefficient between yield forces of individual bars of the same diameter can be taken as 0.9. The correlation coefficient between yield forces of bars of different diameter and between the yield forces in different cross sections in different beams in a structure can be taken as 0.4. Along structural members the correlation is unity within the distances of roughly 10m and vanishes outside (JCSS 2001).

Figure B.8. The density functions for mill test yield strength proposed by (Mirza 1979a).

Table B.12: Coefficients for equation 24, static yield strength

<table>
<thead>
<tr>
<th>Grade</th>
<th>Mean</th>
<th>COV</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>308</td>
<td>0.106</td>
<td>4.106</td>
<td>2.21</td>
<td>3.82</td>
<td>228</td>
<td>428</td>
<td>200</td>
<td>MPa</td>
</tr>
<tr>
<td>60</td>
<td>463</td>
<td>0.092</td>
<td>7.587</td>
<td>2.02</td>
<td>6.95</td>
<td>372</td>
<td>703</td>
<td>331</td>
<td>MPa</td>
</tr>
</tbody>
</table>
B2.4 Ultimate strength

B2.4.1 Probabilistic models of mill test strengths

It is proposed in (Mirza 1979a) that the mill test ultimate strength of Grade 40 and Grade 60 reinforcing steel be represented by the beta distributions given in equation B.25 and the values given in Table B.13. The density functions are shown in Figure B.10.

Table B.13: Coefficients for equation 24 ultimate mill test strength

<table>
<thead>
<tr>
<th>Grade</th>
<th>Mean</th>
<th>COV</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>517</td>
<td>0.109</td>
<td>2.381</td>
<td>2.21</td>
<td>3.82</td>
<td>379</td>
<td>724</td>
<td>344</td>
<td>MPa</td>
</tr>
<tr>
<td>60</td>
<td>738</td>
<td>0.085</td>
<td>4.61</td>
<td>2.02</td>
<td>6.95</td>
<td>607</td>
<td>1151</td>
<td>545</td>
<td>MPa</td>
</tr>
</tbody>
</table>

Figure B.9: The density functions for static test yield strength proposed by (Mirza 1979a).

Figure B.10: The density functions for mill test ultimate strength proposed by (Mirza 1979a).
B2.4.2 Distributions for static strength

It is proposed in (Mirza 1979a) that the static ultimate strength of Grade 40 and Grade 60 reinforcing steel, referred to the nominal area of cross section, be represented by the beta distributions in equation B.25 with the values in Table B.14. The density functions are shown in Figure B.11.

It is suggested in (JCSS 2001) that the ultimate strength be represented by a normal distribution with a standard deviation of 40 MPa. Comparison of Figure B.11 and Figure B.12 shows that the values proposed by (JCSS 2001) indicate less uncertainty in the distributions.

Table B.14: Coefficients for equation 24, ultimate static test strength

<table>
<thead>
<tr>
<th>Grade</th>
<th>Mean</th>
<th>COV</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>475</td>
<td>0.107</td>
<td>2.646</td>
<td>2.21</td>
<td>3.82</td>
<td>351</td>
<td>662</td>
<td>310</td>
<td>MPa</td>
</tr>
<tr>
<td>60</td>
<td>719</td>
<td>0.092</td>
<td>4.922</td>
<td>2.02</td>
<td>6.95</td>
<td>579</td>
<td>1089</td>
<td>510</td>
<td>MPa</td>
</tr>
</tbody>
</table>

Figure B.11: The density functions for static ultimate strength proposed by (Mirza 1979a)

Figure B.12: The density functions for ultimate strength proposed by (JCSS 2001) - $N$(mean, 40)
B2.5 Variations in area of cross section bar

The actual areas of reinforcing bars tend to deviate from the nominal areas due to the rolling process. Table B.15 shows various density functions determined by various researchers to represent $A_m/A_n$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Distribution</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Allan 1972)</td>
<td>normal</td>
<td>0.988</td>
<td>0.0240</td>
<td>Range: 3rd to 99th percentile</td>
</tr>
<tr>
<td>(Allan 1972, Wiss 1973 and American Society for testing materials 1972)</td>
<td>normal</td>
<td>0.989</td>
<td>0.0203</td>
<td>Range: 1st to 99th percentile</td>
</tr>
<tr>
<td>(Allan 1972)</td>
<td>normal</td>
<td>0.970</td>
<td></td>
<td>Truncated at 0.94</td>
</tr>
<tr>
<td>(Mirza 1979a)</td>
<td>normal</td>
<td>0.990</td>
<td>0.0238</td>
<td></td>
</tr>
<tr>
<td>(JCSS 2001)</td>
<td>normal</td>
<td>1.000</td>
<td>0.0200</td>
<td></td>
</tr>
</tbody>
</table>

B2.6 Modulus of Elasticity

It is found in (Mirza 1979a) that the same probabilistic models can be used to represent the modulus of elasticity of Grade 40 and 60 reinforcing steel (Table B.16). In (CEB-FIP 1991) it is assumed to be deterministic.

<table>
<thead>
<tr>
<th>Source</th>
<th>Distribution</th>
<th>$\mu$</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CEB-FIP 1991)</td>
<td>deterministic</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(Mirza 1979a)</td>
<td>normal</td>
<td>201000 MPa</td>
<td>0.033</td>
</tr>
</tbody>
</table>

B2.7 Coefficients of correlation

In (JCSS 2001) the coefficients of correlation between reinforcement area, yield stress, ultimate strength are given as shown in Table B.17.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Bar area</th>
<th>Yield Stress</th>
<th>Ultimate strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar area (mm$^2$)</td>
<td>1.00</td>
<td>0.50</td>
<td>0.35</td>
</tr>
<tr>
<td>Yield Stress (MPa)</td>
<td>0.50</td>
<td>1.00</td>
<td>0.85</td>
</tr>
<tr>
<td>Ultimate strength (MPa)</td>
<td>0.35</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Appendix C  Practical example: Safety assessment of a viaduct

The following appendix presents a possible bridge assessment procedure when full probability analysis is not required or necessary.

C1  General

Using the reliability assessment procedures, described in Chapter 7 and Chapter 8, the reliability levels are most often expressed by:

- probability-based formats (Level C reliability approach), such as reliability index $\beta$, or
- partial safety factor formats (Level B reliability approach), such as rating factor $RF$.

The reliability index $\beta$ (section 8.1.3) is usually presented in the normal or log-normal form:

$$\beta = \frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad \text{or} \quad \beta = \frac{\ln \frac{\bar{R}}{\bar{S}}}{\sqrt{V_R^2 + V_S^2}}$$  \hspace{1cm} (C.1)

where:

$\bar{R} = \text{mean value of the actual carrying capacity of the critical structural element;}$

$\bar{S} = \text{mean value of the total load effect in the section;}$

$\sigma_R, \sigma_S = \text{standard deviations of resistance and total load effect;}$

$V_R, V_S = \text{coefficients of variation of resistance and of total load effect.}$

This approach requires modelling the loads effects, strength, geometry and the model uncertainties when defining the stochastic variables. Level C reliability approach requires specific knowledge on reliability modelling.

Easier to apply are the Level B reliability formats (section 7.2.2). For example, a partial safety factor format level calculation of safety can be expressed by the rating factor, $RF$. This is obtained from the ultimate limit state formula:

$$RF = \frac{\Phi \times R_d - \gamma_G \times G_n}{\gamma_Q \times Q_n}$$  \hspace{1cm} (C.2)

where:

$R_d = \text{design resistance of the section, calculated by the load factor design formula}$

$\Phi = \text{capacity reduction factor or resistance factor}$

$G_n, Q_n = \text{dead load and traffic load effects in the section}$

$\gamma_G, \gamma_Q = \text{partial safety factors of dead and traffic load effects.}$
Uncertainties in equations C.1 and C.2 are accounted for by standard deviations (coefficients of variation) and partial safety factors respectively. The following subchapters illustrate how information obtained during an in-depth inspection of the structure (carrying capacity) and the load effects reduce the uncertainties applied in safety calculations.

### C1.1 Evaluation of carrying capacity based on bridge inspection results

Carrying capacity of existing deteriorated structures is usually difficult to estimate. One possible way of assessing it is to multiply the design resistance by a capacity reduction factor which is based on (Znidarič and Moses 1997):

- information about the bridge condition, collected during the regular or in-depth inspections,
- quality of maintenance (will the faults be promptly repaired or can they accelerate the deterioration process),
- redundancy of the structure (does the structure have several alternative means of supporting the load or, in the worst case, does the structure have several failure mechanisms in case of collapse of one of its components).

One possible way of calculating the effective carrying capacity of deteriorated concrete bridge members is to multiply the design resistance, $R_d$, by the capacity reduction factor, $\Phi$:

$$
\Phi = B_R \times e^{-\alpha_R \times \beta_c \times \alpha_R} \times V_R
$$

where:

- $B_R$ = bias, the ratio of the actual to the designed mean resistance of the critical member section,
- $\alpha_R$ = deterioration factor, based on condition of the structural element (Table C.1),
- $V_R$ = coefficient of variation of member resistance recognising reliability of the test data,
- $\beta_c$ = target value of the safety index as a function of the expected service life.

Figure C.1 illustrates importance of data to reduce uncertainties in bridge assessment. The procedure for selecting the capacity reduction factor, $\Phi$, which accounts for parameters described above, was developed for the bridge assessment recommendations in Slovenia. It clearly differentiates the uncertainties of condition assessments, either done with the help of inspection (right side of the

<table>
<thead>
<tr>
<th>Class</th>
<th>Inspected condition</th>
<th>Necessary intervention</th>
<th>Condition value $R_c$</th>
<th>Deterioration factor $\alpha_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Very good</td>
<td>No maintenance/repair work required</td>
<td>&lt;5</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>Good</td>
<td>Regular maintenance work needed</td>
<td>3 to 10</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>Satisfactory</td>
<td>Intensified maintenance/repair work needed within 6 years</td>
<td>7 to 15</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>Tolerable</td>
<td>Substantial repair work needed within 3 years</td>
<td>12 to 25</td>
<td>0.6</td>
</tr>
<tr>
<td>5</td>
<td>Inadequate</td>
<td>Immediate posting and repair required</td>
<td>22 to 35</td>
<td>0.7</td>
</tr>
<tr>
<td>6</td>
<td>Critical</td>
<td>Immediate closing, then repair (strengthening) required</td>
<td>$&gt;30$</td>
<td>0.8</td>
</tr>
</tbody>
</table>
figure) or without it. The latter case is ‘penalised’ by specifying a capacity reduction factor, \( \Phi \), of 0.85 (0.65, if severely deteriorated). On the other hand, the calculation which is based on bridge inspection, gives values of \( \Phi \) in the range of 0.56 to 0.93, multiplied by the bias \( B_R \) (Chapter 5). This generally gives values of \( \Phi \) considerably higher than if they were calculated without bridge inspections data.

\[
\Phi = \Phi \pm 0 \\
\Phi = \Phi - 0.1 \\
\Phi = \Phi - 0.2 \\
\Phi = \Phi + 0.1 \\
\Phi = \Phi + 0.05 \\
\Phi = \Phi + 0 \\
\Phi = \Phi - 0.15 \\
\Phi > 0.95 \\
\Phi \leq 0.95
\]

\[\Phi = B_R \times e^{-\alpha_R \times \beta_c \times V_R}\]

**Figure C.1:** Selection of the capacity reduction factor, \( \Phi \)

### C1.2 Loading

To reduce uncertainties about the traffic loading, procedures from Chapter 4 can be used. These include weigh-in-motion measurements, as proposed by Moses (1987) in his traffic load model:

\[
Q = a \times W_{95} \times H \times m \times I \times g
\]  

(C.4)
Practical example: Safety assessment of a viaduct

where:

$Q =$ predicted (expected) maximum load effect;

$a =$ deterministic value relating the load effect to a reference loading scheme;

$W_{95} =$ characteristic vehicle weight, defined as 95th percentile of the weight probability function;

$H =$ headway factor, describing the simulated multiple presence of vehicles on the bridge;

$m =$ factor, reflecting the variations of load effects of random heavy vehicles, compared to the standard, reference vehicle;

$I =$ coefficient of impact;

$g =$ girder (lateral) distribution factor.

All parameters except $a$ are random variables and are evaluated from the WIM data. While very basic statistical calculations are applied to determine $W_{95}$, $m$, $I$ and $g$, a simulation method is used to obtain the headway factor $H$.

For very short span bridges (10 m and less), the weights of axle groups (tandems and tridems) are important. For two-lane bridges with spans in the 20 to 30 m range, the meeting or passing event of two trucks generally governs (Grave 2001). At the upper end of this range, the simultaneous occurrence of three heavily loaded trucks can be critical but is a relatively rare event. For 40 m spans on the other hand, these 3-truck events are relatively frequent and can have a significant influence on the calculated characteristic value. For spans in this range and greater, depending on the load effect being considered, congested or jammed traffic conditions start to govern the design (O’Connor 2001).

Simulation of a 2-vehicle event is known as convolution method. With the supposition of independent traffic in both lanes, the probability of such event is expressed as a product of probability functions of gross weights in individual lanes. WIM systems provide the necessary data, including gross weight and axle load histograms, headway histograms ( spacings between vehicles), classification data, average speed and length of the vehicles. In addition, bridge WIM systems provide information about the impact factors, about distribution of loads under traffic and about real values of strain due to traffic loading.

C1.3 Partial safety factors

Uncertainty in reliability calculations is taken into account through the load or safety factors. Factors used for design are not appropriate for assessment of existing structures as they are related to much higher uncertainties during the entire life span of the structure. In many cases assessment is done for shorter periods, e.g., until the next detailed inspection of the structure, and thus the design partial safety factors would be too conservative.

The dead load factor, $\gamma_G$, can be for example selected based on the available information about the structure. One possible way of selecting $\gamma_G$ can be to:

- Use the design or other high values only when approximate methods are used for dead load assessments. On Level 1, dead load effects can be calculated from formulas which are based on statistical evaluation of dead loads of some typical structures in a country, or on rough estimates of the dead loads of some old structures when drawings are unavailable.
- Multiply the design value by 0.9 if dead load effects are calculated based on the design data and drawings.
Multiply the design value by 0.8 if dead load effects are calculated based on measurements of cross sections, on taking specimens of material to obtain composition and weight of individual layers (e.g. depth of concrete deck and asphalt layers).

Another example of selection of dead load partial safety factors is given in Table C.2.

The selection of the traffic load factor $\gamma_Q$ is affected by:

1. Uncertainty of the traffic information used. The important factors are:
   - methodology used (e.g. rough estimates, rating loading schemes, traffic counting, weigh-in-motion data),
   - level of the overloaded vehicles,
   - quality of enforcement, etc.
2. Available information on load transfer from traffic to structure. This includes:
   - knowledge about the impact factors,
   - distribution of loads,
   - real influence lines, etc.
3. Expected service life of the bridge or period for which the assessment is valid. With longer periods, for which the assessment was done, the probability that the traffic conditions (e.g. types and volume of vehicles, permitted loadings) will become more severe, increases.
4. Traffic density. The denser the traffic, the higher the possibility of achieving exceptional load cases.

Žnidarič and Moses (1997) proposed a procedure in Figure C.2 which varies the $\gamma_Q$ value from 1.4 to 2.2. These values are compared to the $\gamma_Q = 1.8$ which is used in Slovenia for the design of new bridges. $\gamma_Q$ can be higher than the design value, because the effect of loading schemes, which are calculated from the site-specific conditions, can be much lighter than the design load models.

C1.4 Ravbarkomanda viaduct example

The importance of the uncertainties involved and quality of data is illustrated by the heavily deteriorated Ravbarkomanda viaduct. The bridge was constructed in the early 1970's and is composed of 17 prestressed 36.3 m long simple spans on the very busy European 5th Transport Corridor on the road section between Ljubljana, Slovenia, and Trieste, Italy. While the more damaged of the two parallel structures was being repaired, the bridge owner wanted to decrease traffic jams on the other structure by introducing a third traffic lane in the middle of the bridge (Figure C.3). The in-depth inspection showed that 4 of 11 tendons of one of the beams were completely corroded and could not contribute to the carrying capacity of the section. Therefore, the design resistance moment of this beam was reduced by the number of corroded tendons to $M_R = 16,402 \text{ kNm}$. A high coefficient of variation, $V_{R} = 15\%$, was selected. According to Figure C.1, a capacity reduction factor $\Phi = 0.87$ was applied for a redundant structure with good maintenance and deterioration factor $\alpha_R = 0.7$. A maximum moment due to dead load, $M_G = 7,034 \text{ kNm}$, with $V_G = 8\%$, was based on the project data and on the in-depth inspection information.

<table>
<thead>
<tr>
<th>Structural data</th>
<th>$\gamma_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>obtained by an in-depth inspection</td>
<td>1.2</td>
</tr>
<tr>
<td>based on design information (drawings)</td>
<td>1.4</td>
</tr>
<tr>
<td>obtained by another simplified procedure</td>
<td>1.6</td>
</tr>
</tbody>
</table>
#### Practical example: Safety assessment of a viaduct

**Figure C.2:** Selection of the live load factor $\gamma_Q$

**Figure C.3:** Proposed traffic regime on the Ravbarkomanda viaduct

### C1.4.1 Traffic load modelling

Seven days of WIM measurements were done on the Ravbarkomanda viaduct. Figure C.4 presents the gross weight histograms of 8,544 heavy vehicles in both directions, which were used to calculate the probability functions of traffic loading in both lanes. Using the convolution method, a maxi-
mum expected gross weight of 1,159 kN was obtained for a period of 5 years (Figure C.5). Such a short period was used as the evaluated structure was planned for reconstruction immediately after the completion of the parallel viaduct.

For the Ravbarkomanda viaduct, the extreme loading could be achieved only with longer and heavier trucks that had more than 3 axles. Therefore, the coefficient $a$ was determined by comparing the weight of a 40-tonne 5-axle reference vehicle with the corresponding bending moment on a 36.3 m long simply supported span:

$$a = \frac{M_{SLS}}{W_{SLS}} = \frac{3,055 \text{ kNm}}{420 \text{ kN}} = 7.27 \text{ m}$$

All other parameters from equation C.4 were evaluated from the WIM data and are summarised in Table C.3. The same 5-axle semi-trailer was used as the reference truck in the calculation of the $m$ factor.

![Gross weight histograms](image1.png)

**Figure C.4:** Gross weight histograms

![Expected maximum gross weight of a 2-vehicle event](image2.png)

**Figure C.5:** Expected maximum gross weight of a 2-vehicle event
Table C.3: Parameters evaluated from the WIM measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean value</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>W&lt;sub&gt;95&lt;/sub&gt;×H</td>
<td>1,159 kN</td>
<td>5.7 %</td>
</tr>
<tr>
<td>I</td>
<td>1.15</td>
<td>4.6 %</td>
</tr>
<tr>
<td>g</td>
<td>0.25</td>
<td>8.4 %</td>
</tr>
<tr>
<td>m</td>
<td>0.923</td>
<td>8.4 %</td>
</tr>
</tbody>
</table>

Hence, the maximum expected bending moment in the first beam due to the truck loading was:

\[ M_{Q,T} = 7.27 \times 1.159 \times 0.923 \times 1.15 \times 0.25 = 2,236 \text{kNm} \]

with a coefficient of variation:

\[ V_Q = \sqrt{\frac{V^2_{W_{95}}}{W_{95}} + V^2_I + V^2_g} = \sqrt{5.7^2 + 8.4^2 + 8.4^2 + 4.6^2} = 14.0\% \]

A uniform loading of 5 kN/m² was added in the middle lane to simulate car traffic to give total moment in the critical beam due to traffic, \( M_Q = 2,349 \text{kNm} \).

Selection of the load factors \( \gamma_G \) and \( \gamma_Q \) was based on Table C.2 and Figure C.2:

- \( \gamma_G = 1.2 \times 1.15 \approx 1.4 \)
- \( \gamma_Q = 1.4 \times 1.15 \approx 1.65 \)

Both factors were multiplied by 1.15 to preserve the same ratio between partial safety factors of prestressed and reinforced concrete structures, as prescribed in the relevant design code.

C1.4.2 Calculation of safety

As a result, the following values of the rating factor \( RF \) and safety index \( \beta \) were obtained:

\[ RF = \frac{0.87 \times 16,402 - 7,034 \times 1.4}{2,349 \times 1.65} = 1.14 \geq 1.0 \]

\[ \beta = \frac{\ln \left( \frac{16,402}{7,034 + 2,349} \right)}{\sqrt{0.15^2 + (0.08^2 + 0.13^2)}} = 2.54 \geq 2.5 \]

Both values were close to the limits, yet sufficient to permit the additional lane of car traffic in the middle of the bridge. In spite of this, to avoid any unexpected events, regular monthly inspections of the critically damaged beams were prescribed.

Additional analysis showed that any other more conservative assessment procedure would result in \( RF \) and \( \beta \) falling below the acceptable limit values 1.00 and 2.50 respectively.
Appendix D  A case study in bridge assessment

Weigh-in-motion measurements on a reinforced concrete bridge

D1  Introduction

The objective of Working Groups 4 and 5 is to recommend numerical procedures for the assessment of highway structures such as bridges and retaining walls. For illustration purposes, a Case Study was conducted. A suitable bridge was identified in Vienna which was the subject of this Case Study.

As part of the Case Study, site-specific traffic data was gathered to investigate the influence of using such data on the reliability rating of an existing structure. Weigh-in-motion (WIM) data was recorded on the structure using the Slovenian Bridge-WIM system called SiWIM. This data can then be employed to create a site-specific traffic load model for the bridge which can be used in a reliability assessment of the structure. The use of site-specific WIM data in the load models provides a considerably more accurate indication of the reliability index than where a generic load model is employed.

D2  Description of the structure

The bridge where the WIM measurements were performed is located in one of the outer districts of Vienna and is part of National Road No. B224. The whole structure consists of two individual bridges, each carrying two lanes in one direction. One bridge was built in 1953, the other in 1961. For both bridges the traffic volume per day is approximately 62 000 vehicles with about 2 500 lorries. The WIM measurements were performed on the 1953 bridge which is shown in Figure D.1.

A view of the bridge from below shows two of the four main girders as well as the transverse diaphragm beam at midspan and a water pipe. A hole in the diaphragm girder is clearly visible. This hole was cut into the girder after construction to carry a pipe which no longer exists. In the process of making the hole, some of the shear reinforcement of the girder was cut through.

Figure D.1: Investigated bridge in Vienna
D2.1 Technical information

The bridge is a single span grillage system made of reinforced concrete with four main longitudinal beams dividing the structure into three bays. Transverse diaphragm beams are located at midspan and at the supports. The bridge length and breadth are 14.32 m and 10.20 m respectively. The main girders have a depth of 1.13 m and a width of 0.50 m. The diaphragm beam at midspan is 0.90 m deep and 0.42 m wide. The beams carry the reinforced concrete deck which is 0.20 m thick. The elevation and cross-section are shown in Figure D.4.

D2.2 Results of bridge inspection and condition of the structure

Ahead of the WIM measurements a survey and inspection of the structure was performed to assess its current condition. The general condition of the structure could be classified as good even though girder D showed some cracks and voids in a distance of about 0.50 m from the supports (Figure D.5). However, these cracks are in the concrete cover only and are due to the fact that in this area the concrete cover had to be replaced. It seems that the shotcrete used for the additional concrete cover did not bond very well to the existing structural concrete in this part of the girder. To check whether the reinforcement was corroded in this area a small part of the shotcrete cover was removed. As shown in Figure D.5, right, the reinforcement was found to be corroded but the cross-sectional area of the steel was not decreased. To avoid further damage, this area should be re-
habilitated. The other parts of the structure did not show any such problems and were in good condition. No signs of corrosion of the reinforcement were visible from the outside.

As part of the case study, actual concrete covers were measured as well as reinforcement locations and diameters with a re-bar locator. It was found that the reinforcement was in accordance with the structural drawings and the concrete covers matched those given in the calculations. The concrete covers were higher than stated in the technical reports in the areas where shotcrete had been applied.

Figure D.4: Elevation and cross-section of the bridge
D3 Description of the weigh-in-motion system

D3.1 Background information - principle of the measurements

WIM were traditionally used to collect truck and axle load data for statistical purposes and also for the design of new and assessment of existing structures, for traffic studies, bridge code calibration, bridge monitoring etc. (BRIME 1999). Most of the WIM systems currently in use are based on weighing detectors which are embedded into the pavement and which measure wheel or axle pressures as the vehicles pass over them (WAVE 2002). As an alternative to pavement WIM, measurement concepts have been developed where the bridge as a whole is used as scales to weigh vehicles in motion. These so-called Bridge WIM (B-WIM) systems have many advantages but have been until today used only in a few countries around the world (WAVE 2001).

Figure D.6 shows the principle of Bridge WIM instrumentation with the SiWIM system which was used in this case study. To perform the measurements, certain members of the structure are instrumented and strains are measured to provide information about the structural behaviour under the moving vehicle. Information about velocity and axle spacings and vehicle type is determined either by axle detectors or by additional strain sensors under the bridge. The latter type of installation, known as FAD (Free-of-Axle Detector) is particularly useful as it does not interfere with the traffic. Strains are recorded during the whole vehicle pass over the structure and, in addition to being used to determine weights, provide useful information on the influence of dynamic effects due to vehicle-bridge interaction.

D3.2 Installation of the System

The WIM measurements took place in Vienna between the 10th and 14th of June 2002. On the first day the system was installed on the bridge. During the time of installation, which all took place in one day, the bridge was instrumented with strain gauges on the main beam soffits to record the strain signals of vehicles passing over the structure. The upper surface of the bridge deck was fitted with pneumatic axle detectors. Each lane of the bridge had to be closed to traffic for a short period of time (about 15 minutes) whiles the axle detectors were fixed to the road surface. The short closure times and the fact that one lane was always open for traffic were a significant advantage as the traffic volume on the bridge is quite high.
Figure D.7 shows the pneumatic axle detectors. Two detectors were used to provide velocity of each axle and thus the dimensions, velocity and type of the vehicle. A third pneumatic detector was installed for research purposes to allow for an evaluation of transverse location of vehicles on the bridge. The detectors were connected to two pneumatic converters which changed the air pressure into a voltage spike for transmission to the central data acquisition unit (Figure D.9, left).

![Bridge WIM instrumentation](image)

**Figure D.6:** Bridge WIM instrumentation

![Pneumatic axle detectors on the road surface](image)

**Figure D.7:** Pneumatic axle detectors on the road surface

To measure the strains in the structure, strain transducers were used. Two transducers were fixed to each of the four main girders at midspan (Figure D.8). Thus, a total of eight transducers were used which were connected to the central data acquisition unit (Figure D.9, left).

**D3.3 Calibration of the System**

After the installation, the system had to be calibrated. Trucks of known weight are used to calibrate all bridge WIM systems to take into account site dynamics and local traffic conditions. The system was calibrated with a three axle calibration truck. The static axle loads (87.5 kN, 89.5 kN and 7.85 kN), the total weight of the vehicle (246.5 kN) as well as the axle distances (3.85 m and 1.40 m) were determined in advance.
The calibration truck crossed the bridge 10 times in each lane with no other traffic present at a velocity of about 30-35 km/h (Figure D. 12 left). For experimental purposes, further runs with heavy vehicles were performed in the presence of traffic in the adjacent lane (Figure D. 12 right).

Once the calibration procedure was over, the actual data collection began and ran continuously for the duration of the whole experiment. The permanent traffic data collection required ongoing inspection to ensure the system was working correctly. In the process, the pneumatic tubes which were fixed to the road surface were additionally protected by tape to reduce damage by the heavy traffic.

During the data acquisition process, a preliminary processing of the traffic data was performed i.e., conversion of strain signals into vehicle weights and spacings. This preliminary analysis allowed a visual inspection to ensure that the system was performing correctly.

The aim was to record and collect data as continuously as possible. Thus, the uninterrupted measurements lasted for a total of four days. The actual measurements started on Tuesday, June 11th, 2002 at noon, immediately after the system calibration and ended at noon on Friday, the 14th of June, 2002. The instrumentation was removed from the structure on Friday afternoon.

Figure D.8: Strain transducers on main girders

Figure D.9: Data acquisition unit (left) and pneumatic converter (right)
D3.4 Measured Data

The data was recorded at a frequency of 512 Hz. Strains were stored for all time steps when a vehicle was present on the bridge. In principle all vehicles were recorded, but only heavy vehicles were used for further analysis as maximum loading of the bridge is of interest.

Figure D.11 shows the raw data for a three-axle truck (left), which crossed the first pneumatic axle load detector on the bridge on June 6th, 2002 at 14:48:28, followed by a four-axle truck (right). Axle detector 1, drawn in navy blue, clearly shows the three axles of the vehicle. Axle detector 2 (in pink) shows the corresponding signal from the second detector. From the X-axis, the time taken to cross the bridge can be directly determined. In the present case the three-axle truck needed approximately one second to cross. One second after the three-axle truck crossed the bridge, a four-axle truck arrived.

With the measured data from the sensors and the SiWIM software package, the velocity of the vehicles can be determined easily. Axle loads and gross weights were calculated using similar principles as described by Moses (1979).
D4  Data Evaluation and Results of Measurements

While initial checks on the data were performed on site using a notebook computer, most analysis took place afterwards on a desktop. For classification, the vehicles crossing the bridge were divided into 36 different vehicle classes. The traffic intensities in one day for heavy vehicles only are shown in Figure D.12.

![Traffic intensities for heavy vehicles](image)

**Figure D.12:** Traffic intensities for heavy vehicles

D4.1  Overloading

Overloads were determined according to the legal weight limits in Austria. As far as axle loads are concerned the law states that axle loads must not exceed 10,000 kg and 11,500 kg for the steering and driving axle, respectively. In Figure D.13 the percentage of overloaded axles which were measured on both lanes during the whole measurements period is presented. It can be seen that 3.38% of axles were overloaded.

D4.2  Distribution of traffic over the day

The distribution of traffic over the day was based on the seven vehicle categories but, for illustration purposes, these were simplified to “light” and “heavy” vehicles. In Figure D.14 one typical distribution is shown. The relative percentages are based on the total number of vehicles per class.

![Overloaded axles - Lanes 1 and 2](image)

**Figure D.13:** Overloaded axles
Based on the traffic data gathered by the Bridge Weigh-in-motion system, a probabilistic traffic load model can be developed. This site-specific traffic load model for the bridge can be used in a reliability assessment of the structure providing a more accurate indication of the reliability index than a load model based on a code of practice.

D5.1 Traffic simulation

Simulations were performed using programs developed at University College Dublin (Caprani 2002b, Grave 2001) to determine the characteristic values of the mid-span bending moment of a simply supported two-lane bridge. For such a structure it is clear that the free flow scenario in two lanes will govern the extreme (O’Connor 1999). No dynamic amplification was applied to the calculated load effects at this stage. However in free traffic some dynamic factor should be applied to allow for dynamic interaction between the vehicle and the bridge. The factor will increase the free traffic characteristic load effect values.

D5.2 Simulation from WIM data

WIM systems can provide a complete picture of the random variables governing traffic flow, i.e., vehicle gross weights, axle loads, spacing, speed, headway etc.. Therefore, if sufficient data is available, it is possible to calculate bridge load effects directly from WIM data and this will be more representative than results obtained from artificially generated traffic files (O’Connor and O’Brien 1999). However, when the data available is limited, it is necessary to fit it to theoretical frequency distributions and use these to simulate sufficient load effects to determine the orientation of the Extreme Value distributions. The latter approach was adopted for this Case Study.

D5.2.1 Monte Carlo Simulation

Monte Carlo (MC) simulation is the process by which vehicles are randomly generated using known or assumed statistical distributions for vehicle gross weights and axle loads, speed, axle spacings etc. within the assumed vehicle classes. The vehicle classification system adopted for this study is illus-
trated in Table D.1 (Bailey 1996) with the relative frequencies of each class, as recorded at the Vienna site illustrated in Figure D.15. In total, twelve vehicle classes are adopted, demonstrating the varying vehicle forms for a given number of axles.

Axle groups (double and triple axles) are defined as a set of successive axles with a spacing of less than 2 m (O’Connor et al 1998). In the process of generation of axle loads and spacing, correlations between the vehicles’ gross weight and the governing axle or axle group loads is identified. Subsequent correlation between axles is employed to determine individual axle loads.

The relationships found for the A113 class are illustrated in Figure D.17 which in the leftmost diagram demonstrates strong correlation between the gross weights and the governing axle group loads. Thus, for a known GVW, the governing axle group load may be estimated quite accurately using MC generation. Similarly, correlation between the principal axle group (W3) and second axle (W2) may be used to estimate its weight etc..

Table D.1: Vehicle Classification System

<table>
<thead>
<tr>
<th>2-Axle</th>
<th>3-Axle</th>
<th>4-Axle</th>
<th>5-Axle</th>
<th>6-Axle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11</td>
<td>A12</td>
<td>A22</td>
<td>A113</td>
<td>A123</td>
</tr>
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<td>A111</td>
<td>A112</td>
<td>A122</td>
<td>A1212</td>
<td></td>
</tr>
<tr>
<td>A11-11</td>
<td>A11-12</td>
<td>A12-11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure D.15: Truck proportion by Lane
Figure D.16: Gross Weight distribution – Class A113

Figure D.17: Correlation of axle loads of vehicle class A113 (W1 = steer axle, W2 = drive axle, W3 = tridem)
It is observed that no statistically significant correlation exists between the gross weight or the axle loads and the axle spacing, as illustrated in Figure D.18. In other words, the axle spacings within a given class are relatively independent of vehicle gross weight or axle load. Inter-vehicle distances (headways) are modelled using an exponential distribution (O’Connor 2001). Data showed that the velocity in each lane is well fitted by a normal distribution and therefore it is modelled as such.

The simulations were run using an 'experimental' influence line, namely an influence line that the SiWIM system generated directly from the measured response to the calibration truck. In Figure D.19 this normalised measured influence line is compared to a theoretical influence line for a simply supported span.

**Figure D.18:** GVW versus axle spacing

**Figure D.19:** Influence lines
D5.3 Prediction of Extremes

Prediction of the extremes can be calculated from Rice’s extrapolation or from the Extreme Value Type III (Weibull) or Type I (Gumbel) distributions. In the latter case it can include a suitability test of the distribution to the mathematical model (O’Connor and O’Brien 1999b).

A period of 50 days, representing 10 working weeks, was simulated in each of the five runs carried out due to the low truck flow rate of the site. The simulation results plotted on Gumbel and Weibull probability papers, along with the extreme value distributions (EVD’s) are shown in Figure D.20. While neither distribution is ideal it is concluded that the Weibull distribution is more appropriate given the degree of convexity in the tail of the data when plotted on Gumbel probability paper. Characteristic values of load effect are predicted from extrapolation of the Weibull distribution. It is to be noted that the absolute values of the results can only be taken as approximate as this is a preliminary analysis of the site with limited data. The results are intended to be indicative of a process for assessment.

Figure D.21 presents a comparison of the characteristic extreme values for the midspan moment load effect compared with deterministic values predicted during the original assessment of the site.
structure. The values are compared for a range of return periods, with a 5% fractile. The probabilistic results are on average 38% lower than the deterministic results. When dynamic amplification factors are applied to the results this difference is 20%. Although a limited amount of recorded data was available for simulation, it is apparent that significant savings could be made in rehabilitation of this structure if probabilistic rather than deterministic methods are employed in assessment of characteristic traffic load effects.

Table D.2: Mean extrapolated results from five full simulations

<table>
<thead>
<tr>
<th>Return Period (years)</th>
<th>Measured Influence Line – Gumbel</th>
<th>Measured Influence Line – Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>3236</td>
<td>2543</td>
</tr>
<tr>
<td>500</td>
<td>3127</td>
<td>2492</td>
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<td>1</td>
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</tr>
</tbody>
</table>

D6 Conclusions

The example demonstrated how probabilistic load modelling could be applied using traffic data supplied by weigh-in-motion technology. The use of a site-specific traffic load model rather than a design load model has confirmed a reduction in the characteristic extremes to which the bridge may be subject and this clearly has an important role in the reliability analysis of the critical limit state identified. It is clear from the comparison of influence lines that there are significant differences between simple theoretical calculations and the actual line as found experimentally. Clearly, in any assessment it would be prudent to measure the response of the structure using WIM; even sophisticated finite element modelling does not guarantee a realistic prediction of response to load. It is clear that the use of WIM and appropriate simulation and statistical techniques yield valuable data upon which reliable decisions can be made.

D7 Acknowledgement

The authors gratefully acknowledge the financial support provided by the European Commission and the contribution of the many involved in collecting the WIM data.
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