A Study on the Trade-off between Energy Forecasting Accuracy and Computational Complexity in Lumped Parameter Building Energy Models

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Abstract: The development of urban scale cost-optimal retrofit decision making requires the development of simplified building energy models which provide satisfactory energy prediction accuracy while remaining tractable when implemented at scale. Lumped parameter building energy models are computationally efficient representations of building thermal performance. The current paper introduces a user-led iterative model reduction methodology which identifies potential trade-offs between model complexity (thus computational requirements) and energy estimation accuracy. Model complexity is progressively reduced using an energy performance criterion prior to model trimming. The methodology is applied to a building energy model of a mixed-use building, which is developed in the EnergyPlus Building Energy Model Simulation (BEMS) environment. The energy performance of the building is evaluated using a linear energy minimisation problem. The proposed methodology shows a potential reduction by half of the model complexity is possible, while retaining annual energy estimation errors below 10% for the target building.

Keywords: reduced order modelling, model order reduction, model calibration, energy estimation

INTRODUCTION

Urban Building Energy Modelling (UBEM) has been described by Cerezo Davila, Reinhart, and Bemis (2016) as a nascent field which is expected to provide urban planners, local administration and energy providers with the tools to analyse energy consumption at an aggregated urban level. Building energy models developed in Building Energy Model Simulation environments (BEMS) are often at the core of UBEMS. BEMS models provide realistic estimations of energy consumption at the building level and are often used for certification purposes (e.g., LEED). However, the aggregated use of BEMS can prove to be computationally demanding for complex problems such as the energy calibration of an urban building sample stock, as mentioned in Cerezo Davila, Reinhart, and Bemis (2016). Cost-optimal retrofit decision-making tools based on BEMS models rely on the iterative coupling of the BEMS model and a heuristic optimisation algorithms as shown by Ferdyn-Grygierek and Grygierek (2017), García Kerdan, Raslan, and Ruyssevelt (2016) and Penna et al. (2015). A retrofit planning approach requiring the iterative numerical interaction between BEMS models and heuristic optimisation algorithms is, in many cases, computationally prohibitive at the UBEM scale (e.g., 1,000 BEMS models were used in Sokol, Cerezo Davila, and Reinhart (2017)). An alternative to BEMS modelling consists of the development of numerically efficient simplified building energy models representative of the residential stock under study (Heidarinejad et al. (2017)). Simplified building energy models dramatically reduce the required computation time for energy simulations at scale as demonstrated by Kim et al. (2014). Furthermore, linear simplified building models, unlike BEMS, can be co-optimised with multi-energy systems models as demonstrated by Good et al. (2015).

Lumped parameter building energy models (also known as RC thermal networks) are simplified building energy models whose parameters possess a semi-physical meaning associated with the thermal performance of building elements (Vana et al. (2013)). The Ensemble Calibration method, introduced in Andrade-Cabrera et al. (2017), exploits this semi-physical interpretation of lumped parameter building models to simultaneously calibrate a cluster of retrofitted building models. Each model represents a retrofit configuration of the original BEMS model. Reducing the complexity of the retrofitted models using automated model reduction methods such as balanced truncation (as shown in Ma, Qin, and Salsbury (2012)) lead to loss of model structure (Deng et al. (2014)). The current paper introduces a user-led methodology which reduces model complexity while retaining the model structure. Insight is provided on the complexities associated with achieving both conflicting goals (computational tractability vs. energy estimation accuracy) by using a case study representative of mixed-use buildings.
**BACKGROUND**

**Lumped parameter building models**

Lumped parameter building energy models are an alternative representation of conductive and convective heat transfer through building elements. This approach is based on the electrical analogy method introduced by Robertson and Gross (1958). Under this modelling framework, electric resistances and capacitances represent analogous thermal resistance and capacitance of material layers. The resulting multi-nodal model is further simplified by lumping parameters together, hence the name of the method. The heat balance at a thermal node $n$ is modelled as a first order differential equation:

$$
C_n \frac{dT_n}{dt} = \sum_{i \in I} \frac{T_i - T_n}{R_i} + Q_n
$$

where $R_i$ is the thermal resistance between elements $i$ and $n$, $C_n$ is the thermal capacitance of the node, $T_n$ represents the node temperature and $Q_n$ models the heat fluxes applied to the node. The set $I$ includes all nodes connected to node $n$. Depending on the number of elements in the model topology (i.e., the model structure), a lumped parameter model is denoted as a $xRyC$ model, $x$ being the number of thermal resistances and $y$ the number of capacitances, respectively. The number of capacitances/thermal nodes is known as the order of the model.

One approach in the development of lumped parameter models consists of using the lowest possible topology that provides acceptable performance. The lowest possible complexity model (1R1C) has been successfully used for control purposes (via model adaptation) by Fux et al. (2014). However, Gouda, Danaher, and Underwood (2002) showed using that external heat flux impulse tests yielded steady-state errors when a 1R1C model is used. ISO 13790 (ISO (2008)) describes a 5R1C dynamic model for hourly energy model calculation which is often adopted as a reference of a low complexity model. Likewise, Guideline VDI 6007 (VDI (2012)), using a second order model, accounts for thermal mass as well as air mass. The latter standard was enhanced by Lauster et al. (2014) to develop a low-complexity model for urban simulation purposes. While computationally efficient, low-order models potentially do not capture transient thermal response adequately, as evidenced by Vivian et al. (2017). Thus, these models may not be suitable for applications where the thermal flexibility of the building envelope needs to be evaluated.

Another approach in the development of lumped parameter building energy models consists of increasing or reducing model order until a performance criterion is reached. Berthou et al. (2014) used regression techniques to verify the improvement in performance between structures. It was found that four model topologies (4R2C, 6R2C, 6R3C and 7R3C) had similar annual energy estimation error performances. Thus, increasing model complexity did not led to improved performance. Hazyuk, Ghiaus, and Penhouet (2012) reduced the complexity of an 8R4C model using system analysis of the transfer functions between heat inputs and room temperature. It was determined that a node representing heat exchanges with the ground was not necessary and can be seen as a disturbance. The resulting 5R2C network was identified using a least squares method. Furthermore, higher complexity topologies can be assembled, if building elements are assembled as low-order networks (e.g., 2R1C), as shown in De Rosa et al. (2014). Such models are more representative of the thermal flexibility potential of the building envelope. However, such models may be too complex for their implementation at scale and thus model reduction mechanisms are required.

**Ensemble Calibration**

Ensemble Calibration is an automated calibration methodology which enables researchers to obtain lumped parameter building models for a number of desired retrofit configurations of a residential dwelling modelled in BEMS. This modelling framework is of interest for the integrated analysis of building and grid models post-retrofit. The passive thermal mass of the building envelope is a potentially untapped source of thermal and electrical flexibility (De Coninck and Helsen (2016)).

Due to space constraints the method is not fully explained in this section and the reader is directed to Andrade-Cabrera et al. (2017) for a more detailed explanation. The key principle is shown in Figure 1. A lumped parameter model can be calibrated with optimal calibration parameters $\rho_0 = [R_0, C_0]$. The model thermal response is given by $T_{r,0}$. After retrofit measures are installed in the building envelope (e.g., external wall insulation), the room temperature post retrofit $T_{r,\Delta}$ will increase. The Ensemble Calibration method finds the variation in parameters $\Delta \rho$ which compensates for the variation in thermal performance using an automated calibration method.

![Figure 1](image-url). Variation in parameters associated with retrofit installation in a residential dwelling envelope.
The variation in parameter $\Delta_p$ can be inferred from the semi-physical modelling. For example, if external wall insulation is progressively added to the building envelope, then it would be expected that a parameter modelling external wall resistance would increase as the insulation layer thickness increases. This well-known theoretical relationship is described in Gouda, Danaher, and Underwood (2002). The theoretical expectation corresponds to a linear growth proportional to the thermal conductance and the thickness of the material. However, such relationships are exclusive to one dimensional heat transfer equations. It has been shown in Andrade-Cabrera et al. (2017) that this parametric growth can be modelled as an exponential function of the insulation layer thickness, where the calibration method compensates for non-linear heat transfers neglected in the one dimensional framework as well as the reduction of cumulative modelling errors.

During the development of this framework, the key modelling concern consisted of obtaining an accurate lumped parameter building model such that the thermal variation $T_{r,\Delta}$ was well captured. This approach implied the development of high-order models. At the integration stage, and mindful of the need to consider model diversity at a national scale perspective, model reduction approaches might be necessary. Automated model reduction methodologies previously used in building models such as balanced model truncation (described in Antoulas and Sorensen (2001)) preserve the dynamic characteristics of the building model as a system (e.g., stability, frequency). However, the model would potentially lose a clear semi-physical interpretation of the model parameters for posterior use. The aim of the current paper is to develop a mechanism that enables the retention of the semi-physical parameter interpretation in a manner such that the Ensemble Calibration methodology can still be applied after model reduction is performed.

**METHODOLOGY**

**Overview**

The methodology overview is shown in Figure 2. First, an energy model of the target building is developed in a BEMS environment such as EnergyPlus, TRNSYS, ESP-r, etc. Then, a lumped parameter building model representative of the target building is designed and calibrated until a calibration accuracy target is reached. The annual energy performance of the lumped parameter building energy model is calculated using a linear optimisation program. The energy performance of the EnergyPlus model and the lumped parameter building energy model are compared in order to validate the proposed model topology.

The complexity of the lumped parameter building model is progressively reduced by removing the nodes adjacent to the room node $T_r$. Only one node is removed at a given time. The exception of this rule is the attic node, which requires to remove both the attic and roof nodes. Every time a node is removed, the resulting sub-model is re-calibrated and the energy performance is evaluated and compared against the original lumped parameter building energy model (denoted full model). The node trimming corresponding to the smallest energy estimation error with respect to the full model is selected.

![Figure 2. Methodology overview](image)

For example, during the first iteration, four nodes adjacent to $T_r$ are identified. A trimmed model (i.e., a model where the selected node is omitted) is created for each node. The four sub-models are re-calibrated and their energy performance is compared with respect to the full model. The node with the smallest estimation error is discarded and the process iterates until the standard model topologies 3R2C (described in Tindale (1993)) and 1R1C (shown in Park et al. (2011)) are reached. The reasoning behind this model trimming strategy is that, the sub-model with the highest energy accuracy among its peers, is more representative of the full model and thus the best topology solution at the level of complexity being evaluated.
Building Energy Model Development

BEM modelling

The target building corresponds to a mixed-use detached building located near Stuttgart, Germany. This building is currently monitored in the framework of a European Commission supported H2020 research project Sim4Blocks, which is concerned with the implementation of demand response in building clusters (HFT 2018). The building features triple glazed windows \( u_{\text{win}}=0.77 \text{ [W/m}^2\text{K]} \), \( g_{\text{win}}=0.58 \text{ [-]} \), autoclaved aerated concrete and brickwork walls \( U_{\text{wall}}=0.21 \text{ [W/m}^2\text{K]} \), insulated reinforced concrete floors \( U_{\text{floor}}=0.19 \text{ [W/m}^2\text{K]} \) and an insulated roof \( U_{\text{roof}}=0.14 \text{ [W/m}^2\text{K]} \) with a pitch angle of 15°. The building envelope is modelled as airtight (0.3 ACH). Internal gains and ventilation loads were omitted at this stage. Weather conditions correspond to the Stuttgart IWEC weather file provided in ASHRAE (2001).

Figure 3 shows the BEM model developed in the EnergyPlus modelling environment. The three-storey detached building has a living floor area of 311 m². The volume of the building envelope is 975 m³. The average floor height is 2.6 m². Glazing is predominant in the north and south facades (36.5 m² and 28.6 m², respectively) while the East and West facades feature limited glazing (5.3 m² and 13 m², respectively).

![Figure 3. Isometric view of the BEM model developed](image)

Lumped Parameter Building Energy Model

Figure 4 shows the proposed heterogeneous lumped parameter building model topology associated with this building. The model was originally introduced in Andrade-Cabrera et al. (2016). The multi-zone building is approximated as a two-zone dwelling by using the average room temperature \( T_r \) of the three heated zones. Node \( T_{\text{amb}} \) represents the dry-bulb outdoor temperature. Nodes \( C_{\text{wt}} \) and \( C_{\text{w2}} \) model the outer and inner leaves of the external walls. The solar gains due to solar radiation on the walls, \( Q_{s,\text{wall}} \) are applied directly to node \( C_{\text{wt}} \). \( C_a \) represents the capacitance of the room air mass with room temperature \( T_r \). Node \( C_{\text{int}} \) captures the thermal mass of the internal partitions and other slow dynamics. The window solar heat gain \( Q_{s,\text{win}} \) and the heating power input \( Q_{\text{heat}} \) are split between \( C_a \) and \( C_{\text{int}} \) via the splitting fractions \( f_1 \) and \( f_2 \). Node \( C_{\text{cell}} \) models the ceiling between the room node \( C_r \) and the attic node \( C_{\text{attic}} \). The solar gains due to incident solar radiation on the roof surfaces, \( Q_{s,\text{roof}} \), are applied to the roof node \( C_{\text{roo}f} \). Finally, a ground node \( (C_{\text{gnd}}) \) is added to model the heat transfer between the conditioned volume and the foundations. The temperature \( T_{\text{floor}} \) models the temperature under the conditioned volume (boundary condition ‘Ground’ in EnergyPlus).

The model was calibrated using the procedure outlined in Andrade-Cabrera et al. (2016). In the current paper, the Root Mean Square Error (RMSE) will be considered as the key performance metric for model accuracy. This metric computes the error between the EnergyPlus synthetic room temperature time series (denoted \( T_{\text{EP}} \)) and the estimated room temperature obtained using a lumped parameter building energy model (denoted \( T_{\text{RC}} \)), over a horizon length \( NH \), using the relationship:

\[
RMSE = \sqrt{\frac{\sum_{k=1}^{NH} (T_{EP,k} - T_{RC,k})^2}{NH}}
\]  

An RMSE value below 0.5 K has been defined in the literature as an acceptable calibration tolerance value as described in He, Zhang, and Kusiak (2014).

Energy Estimation using Linear Optimisation

One of the problems identified in the early stages of the current study consisted of determining the trimming decision rationale given the small differences in calibration metrics (i.e., RMSE) post-trimming. The small magnitude of those variations (sometimes below 0.01 K) did not provide enough confidence for trimming decision making. One alternative consists in the use of energy performance metrics. The logic here is that energy system modellers should not concern themselves with model calibration.
accuracy metrics, but instead with energy estimation accuracy, which is a common metric amongst building and energy systems modellers.

In the current paper, the estimated annual energy consumption $\text{Energy}_{\text{Est}}$ is defined as the annual sum of the heating input $Q_{\text{heat},k}$

$$\text{Energy}_{\text{Est}} = \sum_{k=1}^{NH} Q_{\text{heat},k} \Delta t$$

where $NH$ is the horizon length and $\Delta t$ is the sampling period (15 mins.). The estimated annual heating energy consumption is transformed into MWh to allow for easier comparison of the obtained energy estimation results. Using this metric, it is therefore possible to compare the energy performance of the full lumped parameter building model (displayed in Figure 4) and any other reduced order model obtained after reducing one (or multiple) nodes.

The energy of a given lumped parameter building model is estimated using a linear optimisation problem which minimises the cost function $J(Q_{\text{heat}})$. In the current paper, the problem of interest is the minimization of the required energy consumed:

$$\min / (Q_{\text{heat}}) = \sum_{k=1}^{NH} Q_{\text{heat}}$$

The heating input $Q_{\text{heat}}$ is related to the room temperature $T_{RC,k}$ via the building model dynamics as follows:

$$x_{k+1} = Fx_k + GQ_{\text{heat},k} + Hd_k$$

$$T_{RC,k} = Zx_k$$

where $F$, $G$ and $H$ are system matrices that describe the building model, $x_k$ is the building model state (which is usually composed of room and wall temperatures), $d_k$ is the exogenous disturbance vector (e.g., weather variables such as $T_{\text{amb}}$) and $Z$ is a matrix which extracts the room temperature $T_{r,k}$ from the building model state. The room temperature is constrained by the comfort band:

$$SP_{\text{Low},k} \leq T_{r,k} \leq SP_{\text{High},k}$$

where $SP_{\text{Low},k}$ and $SP_{\text{High},k}$ are the upper and lower bounds of the comfort band. Finally, the heating input $Q_{\text{heat}}$ is constrained by the relationship

$$0 \leq Q_{\text{heat}} \leq Q_{\text{heat,max}}$$

where $Q_{\text{heat,max}}$ is the maximum heating rate. The proposed scheme calculates the accuracy of energy estimation capabilities in a linear optimisation framework, which is compatible with multi-energy systems modelling. In the case of buildings and grid co-optimisation, the heating input $Q_{\text{heat}}$ represents the heat delivered by an electrified heating system. All computations were carried out using a Dell Precision T1700 desktop computer equipped with a processor Intel i7-4790 with a frequency of 3.6 GHz and 8 GB RAM. The optimisation problems were written using the Yalmip interface (Lofberg (2004)) and the Gurobi optimisation solver (Wotao (2011)).
RESULTS

Full Model

Table 1 describes the key metrics of the calibrated lumped parameter building model described in Figure 4 (i.e., the full model before node reduction). The estimated annual energy consumption, \( E_{\text{Est}} \) (Equation 2), was calculated using the cost function, model and constraints described in Equations 3 - 7. For reference purposes, the EnergyPlus model results in an annual energy consumption of 12.29 MWh/year (heating load only). Hence the full lumped parameter building model showed an annual energy overestimation of 7.16% with respect to EnergyPlus.

<table>
<thead>
<tr>
<th>Node</th>
<th>RMSE</th>
<th>( E_{\text{Est}} )</th>
<th>Error(_{\text{Est}})</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Model</td>
<td>0.36</td>
<td>13.17</td>
<td>7.16</td>
<td>61.56</td>
</tr>
</tbody>
</table>

The key reason behind the discrepancy consists of the nature of the operations of the building models. The lumped parameter building energy models, obtained in Andrade-Cabrera et al. (2016), were conceived for their linear co-optimisation with energy systems (e.g., power grid models). Thus, low-level controllers were not developed, as the linear optimisation algorithm would identify the optimal heating load based on the optimisation objective (e.g., cost minimization) and constraints (e.g., reaching the set-point during comfort hours). This approach enables the co-optimisation of the building and grid models and allows for the full exploitation of thermal mass flexibility and power systems flexibility. The linear optimisation problem must reach the set-point when required. Otherwise the linear optimisation problem is infeasible.

On the other hand, EnergyPlus estimates energy requirements by means of performing heat and mass balances using a predictive/corrective approach as described in US Department of Energy (2016). This means that at each time-step, EnergyPlus predicts the building internal state, and if a set-point needs to be met, then the HVAC systems are operated (i.e., their system equations are solved) to meet the load. Depending on plant sizing, and given the thermal response of the air mass and the building envelope, it is possible that the EnergyPlus model may not be able to reach the set-point at each time-step. This issue is visualized in Figure 5. The optimised full model reaches the set-point when required (9AM) by pre-heating the thermal mass, whereas the EnergyPlus models do not meet the set-point even though a sizing factor of two was used.

![Figure 5. Temperature response of the optimised lumped parameter building model compared with EnergyPlus](image)

Evidently, both approaches are not comparable and unsurprisingly the energy estimates obtained using optimised lumped parameter building models are likely to exceed the EnergyPlus estimates, particularly if linear optimisation with hard constraints are considered (i.e., forcing the lumped parameter building model to reach the set-point). One alternative is the use of soft constraints, as carried out by Váňa et al. (2014). Under such an approach, the constraint violations are calculated as part of the penalty function by means of using weighting functions. This approach is not scalable for Ensemble Calibration, as the weighting functions would need to be re-tuned for each retrofit configuration and/or energy systems operations scenario (e.g., thermostatic set-points or flexible electricity load dispatch variation due to renewable power availability).

Model Reduction

Table 2 describes the performance metrics of the first iteration, whereby all nodes adjacent to the thermal zone \( T_r \) are reduced and the respective models are calibrated and their annual energy consumption is estimated. The nodes considered at this stage are the second external wall node \( C_{w2} \), the floor node \( C_{\text{floor}} \) and the internal mass (IM) node \( C_{\text{int}} \). It is not possible to directly remove the ceiling node \( C_{\text{ceil}} \) as this will result in a mixture of the conditioned and unconditioned spaces. Instead, the attic node \( C_{\text{att}} \) and the roof node \( C_{\text{roof}} \) are considered. In doing so, the attic temperature will be deemed to be similar to the outdoor temperature (which is likely to be the case for winter conditions). The inner wall node \( C_{\text{floor}} \) is trimmed for this iteration, as it results in an energy estimation error of 8.14% with respect to the EnergyPlus energy estimate.
Interestingly, the energy estimation errors, for the models where the floor and attic nodes were removed, will result in higher errors than the full model, even though the $RSME$ calibration accuracy metric is the same. Hence the reticence of using calibration metrics, such as $RSME$, as trimming decision variables. The computational decrement is already significant at this stage of model reduction (30% reduction with respect to the full model).

Table 3 shows the performance metrics for the second iteration (i.e., reduced models where $C_{floor}$ has already been trimmed beforehand). The attic node $C_{att}$ was considered for model reduction given that it has the smallest energy accuracy forecast error (9.28%). This selection implies that two nodes (roof node $C_{roof}$ and attic node $C_{att}$) are simultaneously removed. However, the computational run of this trimmed model (24.53 s) is higher than the computational run of the trimmed model when the wall node is removed (20.79 s), even though two nodes are removed ($C_{att}$ and $C_{roof}$) in the former scenario, whereas only one node ($C_{w2}$) is trimmed in the latter.

Table 4 describes the results of the third iteration after the internal wall node $C_{w2}$, the roof node $C_{roof}$ and the attic node $C_{att}$ have been removed. From a model calibration point of view, the internal mass node represents the largest error as the removal of the heating input splitting fractions $f_1$ and $f_2$ significantly alters the performance of the model. In this iteration, the ceiling node $C_{ceil}$ is discarded as it represents the lowest energy estimation error. Computationally, all models result in a negligible improvement with respect to the previous step, as shown in Table 3. Thus, this is a potential trade-off point for the selected target building for the current study and associated boundary conditions.

Table 5 describes the results of the fourth iteration after all the above mentioned nodes were discarded. In this case, it can be observed that the external wall node $C_{w1}$ represents the lowest energy estimation error. This means that, for the purposes of the current case study, a low-complexity model of a third order, featuring external wall nodes $C_{w1}$ and $C_{w2}$, as well as the room node $C_{r}$ and internal mass $C_{int}$, will result in an energy error estimation of 12%, while being 93% faster with respect to the full model. While the model can be deemed inaccurate for some applications (e.g., investment model), it is possible that modellers devising systems at scale might consider these trade-offs as suitable for their needs.

Table 6 describes the performance of the low order models 3R2C and 1R1C. Note that the $RSME$ of the 1R1C model is lower than the one shown for the 3R2C model. Yet, in an energy sense, the 3R2C model presents a lower energy estimation error. Hence, the importance of using energy metrics in lieu of calibration metrics for model reduction purposes. The 1R1C results in a topology with a potentially acceptable underestimation of -15.38% on an annual basis, while using only 1.4% of the computational time (0.92 s) of the full model (63.12 s). However, as it will be explained in the next section, this result may not translate to other scenarios.

<table>
<thead>
<tr>
<th>Node</th>
<th>$RMSE$ [K]</th>
<th>$Energy_{Est}$ [MWh/yr]</th>
<th>$Error_{Est}$ [%]</th>
<th>$Time$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall</td>
<td>0.37</td>
<td>13.54</td>
<td>10.17</td>
<td>45.85</td>
</tr>
<tr>
<td>Floor</td>
<td>0.36</td>
<td>13.29</td>
<td>8.14</td>
<td>44.65</td>
</tr>
<tr>
<td>Attic</td>
<td>0.36</td>
<td>13.45</td>
<td>9.44</td>
<td>29.67</td>
</tr>
<tr>
<td>IM</td>
<td>0.71</td>
<td>14.56</td>
<td>18.47</td>
<td>45.61</td>
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Table 3: Performance metrics (Second Iteration)

<table>
<thead>
<tr>
<th>Node</th>
<th>$RMSE$ [K]</th>
<th>$Energy_{Est}$ [MWh/yr]</th>
<th>$Error_{Est}$ [%]</th>
<th>$Time$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall</td>
<td>0.37</td>
<td>16.22</td>
<td>31.98</td>
<td>20.79</td>
</tr>
<tr>
<td>Attic</td>
<td>0.37</td>
<td>13.43</td>
<td>9.28</td>
<td>24.53</td>
</tr>
<tr>
<td>IM</td>
<td>0.58</td>
<td>16.23</td>
<td>32.06</td>
<td>31.98</td>
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</table>

Table 4: Performance metrics (Third Iteration)

<table>
<thead>
<tr>
<th>Node</th>
<th>$RMSE$ [K]</th>
<th>$Energy_{Est}$ [MWh/yr]</th>
<th>$Error_{Est}$ [%]</th>
<th>$Time$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor</td>
<td>0.38</td>
<td>19.03</td>
<td>54.84</td>
<td>24.09</td>
</tr>
<tr>
<td>IM</td>
<td>0.77</td>
<td>16.31</td>
<td>32.71</td>
<td>23.95</td>
</tr>
<tr>
<td>Ceiling</td>
<td>0.37</td>
<td>13.28</td>
<td>8.06</td>
<td>21.77</td>
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Table 5: Performance metrics (Fourth Iteration)

<table>
<thead>
<tr>
<th>Node</th>
<th>$RMSE$ [K]</th>
<th>$Energy_{Est}$ [MWh/yr]</th>
<th>$Error_{Est}$ [%]</th>
<th>$Time$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM</td>
<td>1.03</td>
<td>13.78</td>
<td>12.12</td>
<td>3.82</td>
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<tr>
<td>Wall</td>
<td>0.60</td>
<td>14.08</td>
<td>14.56</td>
<td>4.97</td>
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</table>

Table 6: Performance metrics (Fourth Iteration)
Table 6: Performance metrics (Standard models)

<table>
<thead>
<tr>
<th>Node</th>
<th>RMSE [K]</th>
<th>Energy\textsubscript{Est} [MWh/yr]</th>
<th>Error\textsubscript{Est} [%]</th>
<th>Time [s]</th>
</tr>
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<tbody>
<tr>
<td>3R2C</td>
<td>1.38</td>
<td>13.87</td>
<td>12.86</td>
<td>1.35</td>
</tr>
<tr>
<td>1R1C</td>
<td>1.24</td>
<td>14.18</td>
<td>15.38</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Figure 6 summarises the results of the model reduction methodology, where the reduction of computational time and estimation error Error\textsubscript{Est} as function of model complexity, are shown. A number \(n\) of nodes/thermal capacitances is denoted by \(nC\). Only the selected models prior to trimming have been represented. It can be inferred from Figure 6, that for this particular topology, a potential trade-off between computational complexity and energy prediction accuracy corresponds to a model topology with four nodes (4C) or three nodes (3C).

Note that the methods, being not equivalent (BEMS model run vs. linear optimisation), the computational performance is not comparable. For reference purposes, the EnergyPlus model ran in 148 sec (single core processing), whereas the full model executed in 61 seconds (all eight processor cores used). However, a single model evaluation of the full model using an on/off controller was completed in 0.18 seconds.

**Comparison with Balanced Truncation**

It is worth mentioning that automated model reduction methodologies (e.g., automated model reduction techniques using state-space model reduction tool in MATLAB Mathworks (2016)) resulted in a model with 4 nodes. The order of the resulting model is comparable with the result in Figure 6, where it was shown that a 4\textsuperscript{th} order model resulted in the best computational trade-off. The model reduction scheme keeps the frequency characteristics of the original model with respect to most of the inputs. However, the frequency response of the system with respect to the roof solar radiation \(Q_{s,\text{roof}}\) was not satisfactory. This implies that the balanced truncation algorithm is unable to capture the impact from the roof solar radiation when considered as an input. In the proposed scheme the effect of the solar radiation is discarded by means of cancelling the unnecessary inputs after model reduction. The model structure is lost as the values of the system matrices of the reduced model (i.e., the continuous-time analogous of the discrete-time system matrices shown in Equation 4) are significantly different when compared to the original model (i.e., the full model, shown in Figure 3).

The balanced truncation methodology may result in conservative models that restrict researchers from potential trade-offs on model complexity and accuracy. Not to mention the loss of semi-physical meaning in the resulting state-space model, which is the key motivation for the current paper. For example, building energy performance given potential alterations in the building fabric (e.g., external wall retrofits, as in Andrade-Cabrera et al. (2017)), can rapidly be associated with external wall node using the reduced models identified via the proposed methodology, whereas such a relationship is not straightforward (or has not been proven yet) when automated model reduction methodologies are applied.

![Trade-off between computational complexity and model accuracy](image-url)
CONCLUSIONS

The current paper described the results of a model reduction procedure, motivated by the interest to reduce the computational complexity of a lumped parameter building energy model, using an iterative approach in lieu of existing automated model reduction methods. The key advantage in the current paper is that the proposed methodology retains the structure of the lumped parameter model, thus enabling the use of the trimmed lumped parameter building model for other applications, whereby the semi-physical interpretation of the model topology is paramount. On the other hand, alternative model reduction methods (such as balanced truncation) and automated modelling frameworks (e.g., linear regression) have the potential to outperform the user-led heuristic model reduction approach as described in the current paper.

A potential advantage of the proposed methodology consists of the insights obtained at each intermediate iteration. Automated model reduction methodologies overlook computational/accuracy trade-offs that might be acceptable to both building energy modellers and energy systems modellers. Such compromises can only be evaluated after the models have been co-optimised. Hence the added value of the user-led iterative model reduction methodology hereby introduced.

The analysis introduced in the current research is relevant only to the target building, under the specified operational conditions (including lack of ventilation and occupant models). Further work is required on the stochastic testing of the framework by means of using different exogenous conditions (e.g., different weather conditions) and different comfort periods. In essence, this is a prior step towards a fully automated model reduction tool suitable of providing multi-energy systems researchers with insights in computational trade-offs during the modelling stage. Future work will focus on implementing the proposed methodology in an Ensemble Calibration framework, in order to understand the effect of model reduction in retrofit-decision making in an integrated building to grid environment.

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