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Analysing mathematics teacher learning in lesson study - a proposed theoretical framework

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The purpose of this paper is to analyse the development of teacher knowledge for mathematics through teacher participation in lesson study. Analysis is undertaken utilising an extended framework which combines both the theoretical frameworks of Mathematical Knowledge for Teaching (Ball et al., 2008) and Levels of Teacher Activity (Margolinas et al., 2005). The proposed framework is situated as a tool to detail and analyse the use and evolution of mathematics teacher knowledge in planning, conducting, and reflecting on research lessons in a lesson study cycle in a primary-school case study in Switzerland.

Keywords: Teacher Collaboration, Lesson Study, Professional Development, Mathematical Knowledge for Teaching

Lesson study and models of knowledge for teaching

Lesson study is a collaborative model of professional development which supports teacher learning (Huang & Shimizu (Eds.), 2016). Originating in Japan, this model has grown in international popularity over the past two decades, particularly in the field of mathematics education, and much research has detailed evidence of mathematics teacher learning through lesson study (e.g. Lewis et al., 2009; Murata et al., 2012; Ni Shuilleabhain, 2016).

Lesson study provides teachers with opportunity to contextualize representations of their classroom activities, while also making their implicit knowledge and practices explicit through their conversations within the group (Fujii, 2016). Each lesson study cycle consists of a number of steps where teachers begin by studying the curriculum and deciding on a research theme, planning a research lesson according to that theme, conducting and observing the live research lesson, and reflecting on student learning within the lesson (see Fig. 1) (Lewis 2016; Lewis et al., 2009)

With increased international educational research on lesson study, there have been calls to deepen the knowledge base of the development of teacher knowledge within this model in order to provide a solid theoretical foundation for its use in teacher education (Clivaz, 2015; Miyakawa & Winsløw, 2009). In this paper, we hope to contribute to the literature on professional development for mathematics teachers by analysing the mathematical knowledge utilized and developed by teachers in their participation in lesson study, utilizing our proposed theoretical framework.

The two authors of this paper, in their analysis of teacher knowledge and learning in lesson study, seek to deliberately build on previous existing frameworks of teacher knowledge: Mathematical Knowledge for Teaching (Ball et al., 2008) and the Levels of Teacher Activity (Margolinas et al., 2005). Through analysis utilizing a combination of these frameworks (Prediger et al., 2008), we will detail features of the mathematical knowledge for teaching utilized by teachers in their participation in lesson study and will also track the evolution and progression of this knowledge.

MKT/ levels in LS: towards a coordinated model

In this paper, we propose a framework which was developed based on data generated in two case study sites - with eight participating primary (grade 3-4) teachers in Switzerland and five lower secondary (middle school, grade 7) teachers in the Republic of Ireland. Analysis began by utilizing the Mathematical Knowledge for Teaching framework (Ball et al. 2008) to investigate the contributions by teachers in a lesson study cycle. However, we found that this model did not fully incorporate all the elements of teacher knowledge included in the lesson study cycle, particularly in capturing the educational values and conceptions of teaching and delineating between the layers of planning sequenced content of instruction while also attending to students' thinking during the lesson. At this point in the analysis, Margolinas et al.'s (2005) Levels of Teacher Activity was identified as a framework which could encapsulate these elements of teachers' knowledge. Building on qualitative data generated through audio/video recordings of teacher conversations during lesson study meetings, teacher notes from lesson study meetings, researcher field notes, and selected samples of student work from research lessons, we present an extended model of the categorization of knowledge required for the teaching of mathematics. Our first example of analysis presented is from the Swiss case study and future work will present further analysis from the Irish case study data.

Mathematical Knowledge for Teaching

In their ground-breaking work in 2008, Ball, Thames and Phelps addressed the concepts of content and pedagogical content knowledge in their model of Mathematical Knowledge for Teaching (MKT - see upper part of Figure 1). In this paper, they identified domains of Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) used in teaching, which further defined the knowledge and skills required of mathematics teachers in relation to student learning and to mathematics content.

Research on these different categories of MKT has demonstrated direct links between teacher knowledge and high-level teaching practices (Clivaz, 2014; Hill, Ball, & Schilling, 2008) and with subsequent student learning outcomes (Hill, 2010).

Incorporating this model of MKT with teacher learning in lesson study, research has shown that Knowledge of Content and Students and Knowledge of Content and Teaching (features of PCK as defined by Ball et al. (2008)) are important elements of teacher knowledge utilized in lesson study cycles (Leavy, 2015; Ni Shuilleabhain, 2015b; Tepylo & Moss, 2011). However, considering the multitude of teacher knowledge and practices incorporated within each lesson study cycle in planning, conducting, and reflecting on a mathematics lesson, this model may not capture all the decisions, actions, practices, and skills required of mathematics teachers participating in lesson study.

Levels of teacher activity and MKT

To describe teacher activity, both in and outside of the classroom, Margolinas developed a model of the mathematics teacher's milieu based on Brousseau (1997). This model was designed to take into account the complexity of teachers' actions and to capture the broad range of activities contained in teaching and learning (Margolinas et al., 2005, p. 207).

+3	*Values and conceptions about learning and teaching*
+2	*The global didactic project*
+1	*The local didactic project*
0	*Didactic action*
- 1	*Observation of pupils' activity*

Table 1: Levels of a teacher's activity (Margolinas et al., 2005, p. 207)

At every level of environment (or milieu) the teacher must consider all that is occurring at the current level as well as those levels that are directly above and below. These multidimensional tensions relate to a non-linear interpretation of teacher's work (Margolinas et al., 2005, p. 208). In addition to commonplace professional opportunities where teachers speak about their beliefs and experiences on general educational concepts or about teaching and learning mathematics (level +3), about teaching and learning of a particular mathematical subject (level +2), or about the lesson they are preparing (level +1), during the phases of planning and reflection in LS teachers also have opportunity to discuss their classroom activities (level 0) or observations of student activity from a lesson (level -1).

This activity model was used by Clivaz (2014) and aligned with the MKT model in order to capture the movement of didactical situations, beyond the possible static characterisation which may be interpreted in the MKT model (Ball et al., 2008, p. 403). The combination of these frameworks allowed teacher knowledge to not only be analysed in terms of mathematical knowledge for teaching, but also mathematical knowledge in teaching (Rowland & Ruthven, 2011). Similarly, Ni Shuilleabhain (2015a) used the MKT model to analyse teacher learning in lesson study, but combined this with the idea of the 'student lens' (as suggested by Fernandez, Cannon, & Chokshi, 2003, p. 180), in proposing an additional layer of the model put forward by Ball et al. (2008). This concept of a 'critical student lens' incorporated the PCK a teacher utilises in seeing mathematics "through the eyes of their students" (Fernandez et al., 2003, p. 179).

When aligned with Margolinas et al.'s (2005) model, this layer of teacher knowledge relates partly to the -1 'Observation of pupils' activity' which can be anticipated and interpreted, but extends this observation to thinking of the mathematical content from the students' perspective. In our proposed framework, we therefore see this view of the mathematics through the eyes of the student as a layer below the observation of a students' work and include a new level of -2 level relevant to teacher knowledge titled the "critical student lens" (see Figure 1).

Proposed Theoretical Framework

Explicitly combining these two approaches to analyse the knowledge utilized by mathematics teachers during lesson study, the authors here present a new theoretical framework (see Figure 1). This framework attempts to capture the knowledge required of mathematics teachers, in the broad and complex range of teaching and learning activities, and represents teacher knowledge and activities incorporated during each phase of a lesson study cycle (see Lewis & Hurd, 2006, p. 4).

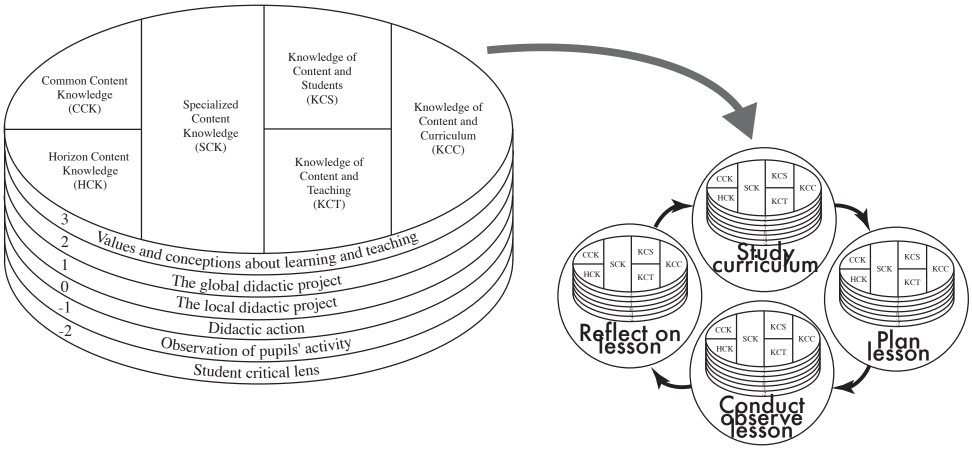

Figure 1: MKT and levels of teacher activity at lesson study phases

We first utilize the model to categorize the knowledge (MKT and levels of activity) appearing during the lesson study cycle. The knowledge about a particular mathematical topic will then be tracked over each phase of lesson study and the relations between the occurrence of this knowledge examined. At this stage, 'knowledge' is considered as collective (e.g. Ni Shuilleabhain, 2016).

Analysis

In this paper, data generated though video recordings of the Swiss case study are analysed utilising the proposed framework. Eight primary generalist teachers, new to lesson study, and two facilitators (one specialist in teaching and learning and the other a specialist in mathematics didactic (first author of this paper)) participated in the research which occurred over two academic years. Four cycles of lesson study were undertaken in this time, with a meeting held on average every two weeks during the school year (Clivaz, 2016). Each of these 37 meetings (about 90' each) were videotaped and transcribed and form the base of the analysis utilizing the framework outlined above (Figure 1) and incorporating defined features of KCS and KCT as utilized in lesson study (Ni Shuilleabhain, 2015b).

We present analysis of the first lesson study cycle where teachers chose to focus on the topic of integers and place value. The main reason for choosing this subject was the difficulty students had with whole numbers. In the first session, teachers discussed a particular difficulty their students had with counting through to new groups in base 10:

Océane: The counting through to the next ten.

Caroline: But each time they have to count through to (tens, hundreds, …)

Stéphane (facilitator): What's happening with counting through to the next ten?

Caroline: It's… that we have no more to write here! We have to use the digits which already exist. So, we count through to come back to one… In fact… Yes, it is the abacus, in fact, we need to move by one each time we arrive at a nine at the end. We need to move by one.

Océane: We exchange one packet of ten.

In this passage, during the *study curriculum* phase, teachers are at level of the *global didactic project* (+3) and this unpacking of mathematical knowledge is a Specialised Content Knowledge (SCK) i.e. the mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students (Ball et al., 2008, p. 399). At this stage, the place aspect of number system was predominant in teachers' discourse and, when the value aspect appeared, it was linked with the value. To further address this knowledge, the facilitators suggested working on students' actual mistakes. Teachers and facilitators proposed mistakes like:

5 hundreds + 12 tens + 3 units = 515

This work prompted teachers to do the task as if they themselves were students. At some moments during the activity teachers even spoke like students - placing them at the level of *student critical lens* (-2). This allowed the teachers to go deeper into potential difficulties for students and by further studying curriculum materials (referred to as *kyozai kenkyu* by Takahashi & McDougal, 2016), teachers had opportunity to clarify this aspect for the research lesson. This passage is situated at the same phase, level and type of MKT as the previous excerpt above.

Anne (facilitator): […] It's a particular type of exchange since it's in the place value system. So, we can distinguish the two dimensions: the dimension of the place and the dimension of the decimal value which is revealed in the exchanges.

Stéphane: In fact, I prefer to talk about grouping/ungrouping instead of exchanging.

Océane: Oh, I see!

Following these two excerpts, we will briefly summarize the work undertaken by these teachers planning the second research lesson and focus on this phase for analysis. The group chose a task in the form of a board game involving the exchange of "1 hundred", "1 ten" and "1 unit" cards. Following a planning exploration of the task, this research lesson was taught by one of the group and, during the post lesson discussion, teachers agreed that the task should be modified to allow students practice the exchange of values and relate these to aspects of the number system. This revised lesson was taught by another member of the group to a different group of students.

At the beginning of the game a student, Julie, arrived on the square "give 35". She had three cards of "1 unit", three cards of "1 ten" and four cards of "1 hundred". In order to get three cards of "1 ten" and two cards of "1 unit", Julie wanted to exchange two "1 hundred" cards. The teacher, Edith, wanted to explain to Julie that two "1 hundred" cards were worth more than these three cards of "1 ten" and two cards of "1 unit".

Edith: So, two hundreds - that's how many?

Julie: Two hundred.

Edith: That's two hundreds. If you tell me: "I want three tens and two units." Three tens, how many is that?

Julie: Thirty.

Edith: You told me three tens makes thirty. And what about two units?

Julie: Two.

Edith: If you put the thirty and the two together? How many is that?

Julie: Thirty-two.

Edith: So you swap two-hundred for thirty-two! You're very generous!

In this passage situated during the *conduct lesson* phase, at level 0 (*didactic action*), the teacher converted all cards into numbers to compare them, instead of doing direct exchanges. Julie followed the teacher without expressing her own way of reasoning (which can be observed in another passage and demonstrates a 'direct exchange' way of thinking). In this case, we categorize the MKT in two ways. First as a KCS, where Edith did not notice or interpret Julie's mathematical thinking or strategies. Second as a SCK, related to the unpacking of mathematical knowledge, as detailed in the following excerpt.

During her dialogue with the class, Edith had to explain that one hundred is the same as ten tens. Here, again, her argument is to convert to units - which requires students to already understand place value. This argument can be summarized as follows:

1 hundred = 100 units

and 10 tens = 100 units

therefore, 1 hundred = 10 tens

In the final lesson plan, the group of teachers reflected on this strategy and argued against it:

> "Often exchanges are not really carried out and we go through the number. For example, when asked to exchange 12 hundreds into tens, many students (and adults) will go through the number 1200, namely 1200 units, to say that that 1200 is 120 tens, without being able to make a direct exchange from hundred to tens. Teachers also often explain this exchange in this way. In this case, we are in a type of vicious circle, since it means that it is necessary to have understood number system to understand the grouping/ungrouping in the place value system."

This episode appears in our data in the research lesson (*conduct and observe lesson* phase, level 0), in the notes of the observing teachers (*conduct and observe lesson* phase, level 0), in the post lesson discussion (*reflect on lesson* phase, level 0) and, in the above extract, in the lesson plan (*reflect on lesson* phase, level +2) where observations and analysis of the group were generalized and decontextualized from the particular lesson to the level of a global didactic project. In each case the knowledge represents a typical SCK.

The final example of this knowledge was found at the end of the *reflect on the lesson* phase. After discussing the lesson and the mathematical difficulty of directly converting hundreds into tens, Valentine (a teacher with over 30 years of teaching experience) realized she had observed a similar difficulty her own students in this topic, outside of the lesson study group. As a result of their collaborative reflection conversations, she began to realize that her students' errors were likely due to her use of only one strategy in teaching this topic:

Valentine: But, I've got a question. For example, in nine-hundred-sixty-three - how many tens are there? Ninety-six. But my students, they learned a trick - they write the number 963 and just go to the tens digit and write what is left: 96. I'm convinced they just use this trick. I probably didn't know how to explain that to them! Myself… I always convert in money! You will have nine hundred and sixty three one-franc coins. If you need to only have ten-francs notes… then you will have ninety-six ten-francs notes.

Although this observation was not directly related to observations during the research lesson, we still categorize it as level -1 since Valentine put herself in the position of observing her students converting 963 into tens. This conversation incorporates teacher KCS in interpreting students' responses and is situated at level -1 (*observation of pupils' activity*).

Utilising our proposed framework and building on our analysis of teachers' collective conversations, we can detail the types and levels of knowledge incorporated by mathematic teachers in their participation of lesson study. Utilising this framework provides us with opportunity to track the knowledge included in the planning and reflection of mathematics research lesson over various phases of lesson study, therefore describing the evolution of teacher learning in this model of professional development.

Conclusion

Categorising all the knowledge required of those who teach mathematics is complex. Ball and her colleagues (2008) began identifying the specifics of knowledge required of mathematics teachers in decision-making, actions, and practices in their framework of Mathematical Knowledge for Teaching (MKT). Similarly, Margolinas and colleagues (2005) proposed a model delineating the multitude of actions and skills required of teachers during varied stages of teaching, from the over-arching pedagogical values underpinning a lesson to the didactic action within the classroom.

This paper proposes an extended theoretical framework of mathematics teacher learning in lesson study combining the existing frameworks of Mathematical Knowledge for Teaching (Ball et al., 2008) with Levels of Teacher Activity (Margolinas et al., 2005). In this paper the proposed framework is situated as a tool used to detail and analyse the use and evolution of mathematics teacher knowledge in planning, conducting, and reflecting on research lessons. Based on case study data generated through mathematics teachers' participation in lesson study, we have analysed teachers' qualitative conversations and considered the evolution of mathematics teacher knowledge over a cycle of lesson study. Analysis to date has demonstrated that in planning and reflecting on research lessons, teacher knowledge of various forms (e.g. SCK and KCS (Ball et al., 2008)) and across varying levels of activity (Margolinas et al., 2005) are incorporated in these separate phases of lesson study.

We hope this model will contribute to the literature on professional development of mathematics teachers and may serve to underpin further evidence of teacher learning in lesson study.

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