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Visualizing Voltage Relationships Using The Unity Row Summation and Real Valued Properties of the F_{LG} Matrix

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Abstract

By manipulating the bus admittance matrix of a power system, a useful submatrix, F_{LG} , can be derived. This matrix identifies, for every load bus, the set of generators that establish its no-load voltage, and the varying degree of their influence. The first contribution of the present work is to rigorously prove two observed properties of the F_{LG} matrix; that it is substantially real-valued, and that its rows sum close to one. Six test systems are used in this work to validate these properties. With this proof in hand, this work also introduces a new conception of voltage profile monitoring in power systems, by explicitly mapping the relationships between load and generator voltages. This new visualization makes it easier to identify how influential each generator is in establishing the network's voltage profile. Poorly supported load buses, which may be vulnerable to voltage deviations, are clearly identified. This new visualization framework is suitable for pedagogy, research, and control room applications.

Keywords: Laplacian matrix; power flow; admittance matrix; voltage profile; visualization

1 Introduction

Electrical power system calculations rely heavily on the bus admittance matrix, Y_{bus} , which is the Laplacian matrix of the network under study, weighted by the complex-valued admittance of each branch [1]. In various circumstances it is useful to partition the Y_{bus} into submatrices, to separately quantify the connectivity between and among the load and generation nodes in the network. Manipulation of these submatrices gives the F_{LG} matrix, introduced by [2], which offers useful insights on how voltage profiles are established in a power system. Various authors [3], [4], [5] have observed that in practice the elements of F_{LG} are real-valued and its rows sum close to one: the first contribution of the present paper is to explain and prove these properties.

The F_{LG} matrix is principally used in assessing voltage security in power systems. Two factors are essential in this ambit: what reactive power sources determine the voltage at a load node, and how electrically proximate are those sources? A further contribution of the present work is to demonstrate that both the voltage dependencies between load and generator buses (using F_{LG}), and their electrical proximities (using Thevenin impedances),

can be simultaneously represented on a legible diagram by employing force-directed network layouts (e.g [6]). As no abstruse calculations are required, this visualization has applications for the real-time monitoring of a power systems voltage profile, as it can be readily updated as generator commitments change. The visualization also has expository and pedagogic value.

The matrix proofs are presented in Section 2. Validations of the proven properties are undertaken for six test system in Section 3. These properties are employed to visualize voltage relationships in Section 4.

2 Methodology

2.1 Y_{bus} partitioning

The partitioning of the Y_{bus} matrix into load and generation blocks was introduced by Kessel and Glavitsch in [2], and has subsequently been applied to a wide range of power engineering problems [4], [7], [8], [9], [10], [11], [12], [13], [14]. Recent work by Abdelkader et al. [15], [16], [17] offers a new perspective on these Y_{bus} partitions, demonstrating that they allow the explicit separation of currents in the network into load and generator induced components.

Currents (I) and voltages (V) in an electrical power system are related by the admittance matrix, Y_{bus} , which is generally constructed to have the properties of a weighted Laplacian matrix (disregarding shunt element modelling), see [18], [19]. It can usefully be partitioned as follows by reordering to group generator (G) and load (L) nodes separately:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = Y_{bus} \begin{bmatrix} V_G \\ V_L \end{bmatrix}. \quad (1)$$

Where $I_G, V_G \in \mathbb{C}^m$, $I_L, V_L \in \mathbb{C}^n$ and

$$Y_{bus} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix}.$$

Where $Y_{GG} \in \mathbb{C}^{m \times m}$, $Y_{LL} \in \mathbb{C}^{n \times n}$ and $Y_{LG} = Y_{GL}^T \in \mathbb{C}^{n \times m}$. With $()^T$ we denote the (non-conjugate) transposed tensor. The Y_{bus} matrix is square by definition. It has N rows and N columns, where N is the number of nodes in the power system. The n L buses plus the m G buses equals the total numbers of buses N . Although there may be buses in a power system that don't connect any loads or generators, but are just passive interconnection points, in this analysis they would be grouped with the load buses. Typically Y_{LL} will be larger than Y_{GG} , as load buses are typically more numerous than generator buses. The remaining sub-matrices will not generally be square, as typically $n \neq m$. Note that $Y_{LG} = Y_{GL}^T$. The justification for this assertion is the symmetry of the Y_{bus} matrix (it is generally constructed to have this property, which is only broken under such circumstances as including phase-shifting transformers in the model). The Y_{bus} matrix is complex-valued. The physical properties of electrical conductors imply that the imaginary part of each element is larger than the real part in most cases.

From (1):

$$I_G = Y_{GG}V_G + Y_{GL}V_L \quad (2)$$

and

$$I_L = Y_{LG}V_G + Y_{LL}V_L. \quad (3)$$

Rearranging (3):

$$V_L = Z_{LL}I_L - Z_{LL}Y_{LG}V_G. \quad (4)$$

Where

$$Z_{LL} = \begin{cases} Y_{LL}^{-1}, & \text{if } \det Y_{LL} \neq 0 \\ Y_{LL}^\dagger, & \text{if } \det(Y_{LL}) = 0 \end{cases}.$$

The matrix $Y_{LL}^\dagger$ is the Moore-Penrose Pseudoinverse of Y_{LL} , calculated by the singular value decomposition of Y_{LL} , see [20], [21]. Substituting for V_L in (2):

$$I_G = (Y_{GG} - Y_{GL}Z_{LL}Y_{LG})V_G + Y_{GL}Z_{LL}I_L. \quad (5)$$

Equations (4) and (5) are typically represented in matrix form as follows:

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GGM} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix}. \quad (6)$$

Where:

$$Y_{GGM} = Y_{GG} - Y_{GL}Z_{LL}Y_{LG} \quad (7)$$

and

$$F_{LG} = -Z_{LL}Y_{LG} = -K_{GL}^T \quad (8)$$

Various works (e.g [3], [4], [5]) have noted, and, indeed, relied upon, the observation that the elements of F_{LG} are in practice real-valued and that its rows sum close to unity. These properties make intuitive sense, as from (6) we have that:

$$V_L = Z_{LL}I_L + F_{LG}V_G \quad (9)$$

This equation shows that load voltages can be calculated as the sum of two entirely separate vectors: the first describes load current induced effects ($Z_{LL}I_L$), which act to depress voltages, and the second vector ($F_{LG}V_G$) describes the no-load voltage each bus is exposed too. By matrix multiplication, we can see that each bus' no-load voltage is equal to the inner product of its corresponding row in F_{LG} and V_G , the vector of generator voltages. In this sense, we can interpret the entries in F_{LG} as showing the percentage participation that each generator has in establishing the no-load voltage at each bus [15] [8]. As each element of V_G is known to be close to one in magnitude, we may anticipate that the row sums of F_{LG} must also be close to unity, or else the resulting no-load voltages would be unrealistic.

2.2 Matrix proof

As will be validated subsequently, the ratio ($Re : Im$) of each entry of the Y_{bus} matrix will tend to be fairly homogeneous. Interestingly, such a property has physical consequences: [17] demonstrates how a strictly equal ($Re : Im$) ratio for every element in the Y_{bus} matrix serves to nullify one component of the network's power losses. The Y_{bus} matrix will generally have diagonal elements that are positive real and negative imaginary. Off-diagonals will be negative real and positive imaginary. Based on these observations we can provide the following proposition:

Proposition 2.1 Assume the Y_{bus} matrix as defined in (1). If the entries in each row have the real:imaginary ratio equal, then all entries of the F_{LG} matrix, defined in (8), are real numbers. In addition if the Y_{bus} matrix has diagonal elements that are non-negative real, non-positive imaginary and off diagonals are non-positive real, non-negative imaginary, then all entries of F_{LG} , are non-negative real numbers.

Proof. Let

$$Y_{LG} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}, \quad Y_{LL} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix}$$

and $a_{kj} = Re(a_{kj}) + iIm(a_{kj})$, $b_{kj} = Re(b_{kj}) + iIm(b_{kj})$. Then for each row $k = 1, 2, \dots, n$ of Y_{LG} , Y_{LL} we have

$$\frac{Im(a_{kj})}{Re(a_{kj})} = \frac{Im(b_{kj})}{Re(b_{kj})} = u_k \in \mathbb{R}, \quad k = 1, 2, \dots, n.$$

From (8) we have

$$Y_{LL}F_{LG} = -Y_{LG},$$

or, equivalently,

$$\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} F_{LG} = - \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix},$$

or, equivalently,

$$\begin{bmatrix} Re(b_{11})[1 + iu_1] & Re(b_{12})[1 + iu_1] & \dots & Re(b_{1n})[1 + iu_1] \\ Re(b_{21})[1 + iu_2] & Re(b_{22})[1 + iu_2] & \dots & Re(b_{2n})[1 + iu_2] \\ \vdots & \vdots & \ddots & \vdots \\ Re(b_{n1})[1 + iu_n] & Re(b_{n2})[1 + iu_n] & \dots & Re(b_{nn})[1 + iu_n] \end{bmatrix} F_{LG} =$$

$$- \begin{bmatrix} Re(a_{11})[1 + iu_1] & Re(a_{12})[1 + iu_1] & \dots & Re(a_{1m})[1 + iu_1] \\ Re(a_{21})[1 + iu_2] & Re(a_{22})[1 + iu_2] & \dots & Re(a_{2m})[1 + iu_2] \\ \vdots & \vdots & \ddots & \vdots \\ Re(a_{n1})[1 + iu_n] & Re(a_{n2})[1 + iu_n] & \dots & Re(a_{nm})[1 + iu_n] \end{bmatrix},$$

or, equivalently,

$$\begin{aligned} & diag \{ 1 + iu_1, 1 + iu_2, \dots, 1 + iu_n \} \begin{bmatrix} Re(b_{11}) & Re(b_{12}) & \dots & Re(b_{1n}) \\ Re(b_{21}) & Re(b_{22}) & \dots & Re(b_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ Re(b_{n1}) & Re(b_{n2}) & \dots & Re(b_{nn}) \end{bmatrix} F_{LG} = \\ & -diag \{ 1 + iu_1, 1 + iu_2, \dots, 1 + iu_n \} \begin{bmatrix} Re(a_{11}) & Re(a_{12}) & \dots & Re(a_{1m}) \\ Re(a_{21}) & Re(a_{22}) & \dots & Re(a_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ Re(a_{n1}) & Re(a_{n2}) & \dots & Re(a_{nm}) \end{bmatrix}. \end{aligned}$$

or, equivalently, since $det(diag \{ 1 + iu_1, 1 + iu_2, \dots, 1 + iu_n \}) \neq 0$

$$Re(Y_{LL})F_{LG} = -Re(Y_{LG}).$$

This is a linear system where the known matrices have only real entries. Hence, any solution for F_{LG} lies inside of $\mathbb{R}^{n \times m}$, i.e. all entries of F_{LG} are real numbers. In addition since the Y_{bus} matrix is assumed to have diagonal elements that are non-negative real & non-positive imaginary and off diagonals are non-positive real & non-negative imaginary, the matrix $Re(Y_{LG})$ has all its entries non-positive, i.e. the matrix $Re(Y_{LG})$ has all its entries non-negative and the matrix $Re(Y_{LL})$ has its diagonal elements non-negative and its off diagonals non-positive, i.e. the inverse (or pseudo inverse) of $Re(Y_{LL})$ has all its entries non-negative, see [22]. Hence the matrix F_{LG} will have all its elements non-negative since it is given from the product of two matrices, the inverse (or pseudo inverse) of $Re(Y_{LL})$ and the matrix $Re(Y_{LG})$. The proof is completed.

If we use a short line model of the system and so neglect shunt elements, then each row of the Y_{bus} matrix sums to zero. If we include shunts, then each row sums close to zero. We can state the following Theorem.

Theorem 2.1. Assume the Y_{bus} matrix as defined in (1) and the matrix F_{LG} , as defined in (8). Then the row sum of F_{LG} is one if the rows of Y_{bus} sum to zero and $det(Y_{LL}) \neq 0$ and is close to one if the rows of Y_{bus} sum close to zero or sum to zero and $det(Y_{LL}) = 0$.

Proof. Let for system (1)

$$Y_{LG} = [a_{ij}]_{i=1,2,\dots,n}^{j=1,2,\dots,m} \quad (10)$$

and

$$Y_{LL} = [b_{ij}]_{i=1,2,\dots,n}^{j=1,2,\dots,n}. \quad (11)$$

Where

$$[a_{ij}]_{i=1,2,\dots,n}^{j=1,2,\dots,m} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}, \quad [b_{ij}]_{i=1,2,\dots,n}^{j=1,2,\dots,n} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix}.$$

We assume that each row of Y_{bus} sums to zero, i.e. $\forall$ row $i = 1, 2, \dots, n$ of the matrices (10), (11) we have $\sum_{j=1}^m a_{ij} + \sum_{j=1}^n b_{ij} = 0$, or equivalently

$$\sum_{j=1}^m a_{ij} = -\sum_{j=1}^n b_{ij}, \quad \forall i = 1, 2, \dots, n. \quad (12)$$

From (8) we have

$$Y_{LL} F_{LG} = -Y_{LG}, \quad (13)$$

Let

$$F_{LG} = [c_{ij}]_{i=1,2,\dots,n}^{j=1,2,\dots,m}. \quad (14)$$

Then $\forall i = 1, 2, \dots, n$ by substituting (10), (11), (14) into (13) and by calculating every component in row i , we get

$$\begin{aligned} -a_{i1} &= \sum_{k=1}^n b_{ik} c_{k1} \\ -a_{i2} &= \sum_{k=1}^n b_{ik} c_{k2} \\ &\vdots \\ -a_{im} &= \sum_{k=1}^n b_{ik} c_{km}. \end{aligned}$$

By taking the sum of the above equalities, $\forall i = 1, 2, \dots, n$ we arrive at

$$-[a_{i1} + a_{i2} + \dots + a_{im}] = \sum_{k=1}^n b_{ik} c_{k1} + \sum_{k=1}^n b_{ik} c_{k2} + \dots + \sum_{k=1}^n b_{ik} c_{km},$$

or, equivalently,

$$-\sum_{j=1}^m a_{ij} = \sum_{j=1}^m \sum_{k=1}^n b_{ik} c_{kj}.$$

By using (12) on the above expression we get

$$\sum_{j=1}^n b_{ij} = \sum_{j=1}^m \sum_{k=1}^n b_{ik} c_{kj},$$

or, equivalently,

$$\sum_{k=1}^n b_{ik} = \sum_{j=1}^m \sum_{k=1}^n b_{ik} c_{kj},$$

or, equivalently,

$$b_{i1} + b_{i2} + \dots + b_{in} = \sum_{j=1}^m [b_{i1} c_{1j} + b_{i2} c_{2j} + \dots + b_{in} c_{nj}],$$

or equivalently,

$$\begin{aligned} b_{i1} + b_{i2} + \cdots + b_{in} = & b_{i1}c_{11} + b_{i2}c_{21} + \cdots + b_{in}c_{n1} + \\ & b_{i1}c_{12} + b_{i2}c_{22} + \cdots + b_{in}c_{n2} + \\ & \vdots \\ & b_{i1}c_{1m} + b_{i2}c_{2m} + \cdots + b_{in}c_{nm}, \end{aligned}$$

or, equivalently,

$$\begin{aligned} b_{i1} + b_{i2} + \cdots + b_{in} = & b_{i1}(c_{11} + c_{12} + \cdots + c_{1m}) + \\ & b_{i2}(c_{21} + c_{22} + \cdots + c_{2m}) + \\ & \vdots \\ & b_{in}(c_{n1} + c_{n2} + \cdots + c_{nm}), \end{aligned}$$

or, equivalently,

$$b_{i1} + b_{i2} + \cdots + b_{in} = b_{i1}\left(\sum_{j=1}^m c_{1j}\right) + b_{i2}\left(\sum_{j=1}^m c_{2j}\right) + \cdots + b_{in}\left(\sum_{j=1}^m c_{nj}\right),$$

or, equivalently,

$$b_{i1}\left[\left(\sum_{j=1}^m c_{1j}\right) - 1\right] + b_{i2}\left[\left(\sum_{j=1}^m c_{2j}\right) - 1\right] + \cdots + b_{in}\left[\left(\sum_{j=1}^m c_{nj}\right) - 1\right] = 0. \quad (15)$$

Let $\forall i = 1, 2, \dots, n$

$$q_i = \left(\sum_{j=1}^m c_{ij}\right) - 1. \quad (16)$$

Then by replacing (16) into (15), $\forall i = 1, 2, \dots, n$ we have

$$b_{i1}q_1 + b_{i2}q_2 + \cdots + b_{in}q_n = 0,$$

or, equivalently,

$$\begin{bmatrix} b_{i1} & b_{i2} & \cdots & b_{in} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = 0, \quad \forall i = 1, 2, \dots, n.$$

Since the above expression holds $\forall i = 1, 2, \dots, n$ we have equivalently

$$\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

By setting $q = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}$, $0_{n,1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ and by using (11) we have

$$Y_{LL} \cdot q = 0_{n,1}.$$

From the above expression if $\det(Y_{LL}) \neq 0$, then $q = 0_{n,1}$, or, equivalently, $q_i = 0$, $\forall i = 1, 2, \dots, n$, i.e. by using (16)

$$\sum_{j=1}^m c_{ij} = 1. \quad (17)$$

If $\det(Y_{LL}) = 0$, then $q \cong 0_{n,1}$ (by using the pseudo inverse of Y_{LL} via the SVD method), or, equivalently, $q_i \cong 0$, $\forall i = 1, 2, \dots, n$, i.e. by using (16)

$$\sum_{j=1}^m c_{ij} \cong 1. \quad (18)$$

Thus, from (17) and (18) every row of F_{LG} will sum to 1 if $\det(Y_{LL}) \neq 0$ and will sum closely to 1 if $\det(Y_{LL}) = 0$. If the rows of Y_{bus} sum close to zero, then with similar steps we arrive at (18). The proof is completed.

3 Numerical Matrix Examples

In this section six test systems are used to validate the assumptions made in the foregoing proofs. These well-known test systems [23–26] range in size from 30 to 300 buses, and represent various medium scale empiric networks.

3.1 Consistency of Y_{bus} Real:Imaginary Ratios

It is clear that the ratios between the real and imaginary components of the Y_{bus} elements are quite consistent for each of the six test systems shown in figure 1. The degree of linear correlation, R^2 is noted in each case, with the regression line shown also. The scatter plots make clear that most Y_{bus} elements are clustered quite closely around this line. This ratio is closely related to the X/R ratios of the conductors in the system, and this ratio is known to be fairly consistent, and typically dominated by the reactance component. This empiric investigation shows that the ratio $Re(Y_{ij}) : Im(Y_{ij})$ is quite homogeneous, and so supports the assumption stated in Proposition 2.1.

3.2 Empiric F_{LG} Properties

Having validated a key assumption of the presented proof in the previous subsection, in this subsection we seek to validate the conclusions drawn about the nature of the F_{LG} matrix.

Figure 1: The relationship between the real and imaginary components of the non-zero elements of the Y_{bus} matrix for six well-known test systems

From figure 2, it is clear that the elements of the F_{LG} matrix are almost purely real, with only negligible imaginary components visible in all cases. This empirically verifies the proven real-valued nature of this matrix.

Similarly, in figure 3, we verify the unity row summation property of F_{LG} . This figure disregards the imaginary components of F_{LG} as these have been shown to be negligible. In most cases, it is clear that row summations are equal, or close to, one. However, there are some deviations from this, most notably for the 118 bus system, where numerous row summations are in the region of 1.2. These deviations can be anticipated as real systems do not fully conform to the proof's assumptions, having lines with heterogeneous X/R ratios, and shunt-connected equipment that mean that Y_{bus} row sums are not always equal to zero. However, figure 3 does still verify that the row summations of F_{LG} will be reasonably close to one across various power systems. This property is sufficient for engineering applications, as it allows the F_{LG} matrix to be conceptualized as showing the approximate percentage participation each generator has in establishing the no-load voltage at a particular bus.

Figure 2: The relationship between the real and imaginary components of the elements of F_{LG} for six well-known test systems

4 Visualizing Voltage Relationships

The F_{LG} matrix explicitly describes the functional relationships between generator and load voltages. In this section, these relationships are presented as a complex network, to give a tangible new mapping which clearly shows how controllable generator settings affect load voltages.

There are several motivations for this new visualization. Firstly, for pedagogic reasons: rather than presenting bus voltages as the inscrutable output of an iterative loadflow solver, this visualization allows the role generators play in establishing load voltages to be more easily grasped by those new to power system analysis. Secondly, there may be an operational value to monitoring such relationships in real-time, to allow control room operators to see which generators would be most effective at alleviating voltage breaches. Finally, this visualization highlight those load buses that are electrically remote from their voltage-establishing generators, which may have implications for their voltage stiffness.

Figure 3: For each load bus, this figure shows the row summation of $Re(F_{LG})$, over G , for six test systems

4.1 Layout Approach

Within complex network analysis, force-directed layout algorithms, such as [6], are often used to diagrammatically represent inter-node connectivity, and tend to aesthetically reveal structures of interest [27]. Such algorithms have a physical inspiration: nodes act to repel each other, but are drawn together by the spring-like forces of the edges that connect them. Where these algorithms have previously been applied to power systems [28], branch admittance has been used to weight the strengths of these spring forces.

To meaningfully gauge an electrical distance measure between load and generator buses, which will be connected through a complex meshing of lines, the present visualization uses the total effective impedances between load and generator buses, Z_{thv} , as an edge weighting. In this way, the visualization identifies which generators support the voltage at each load bus, and simultaneously indicates the magnitude of the impedance through which they must transmit reactive power to achieve this.

Note that only the two structural matrices, F_{LG} and Z_{thv} , are depicted in the present visualization, for clarity's sake. In an operational context, it may also be useful to overlay other relevant quantities such as voltage magnitudes and spot loadings also.

4.2 Calculating Effective Impedances

This visualization uses the Klein resistance distance [29] to precisely quantify the effective impedance, Z_{thev} , between any pair of buses i and j . This approach uses simple arithmetic on the entries of Y_{bus} matrix's inverse, the Z_{bus} :

$$Z_{ij}^{thev} = Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji} \quad (19)$$

This distance measures captures the inherently meshed and parallel nature of transmission systems (c.f some recent contributions on voltage stability [30] [31] which have naïvely used the shortest-path between loads and their topologically nearest generator to gauge electrical distances)

The novel visualization uses a subset of the available inter-node impedances: just those between a load node and those generators that establish that node's voltage, per F_{LG} . Note that matrix inversion is not essential here; spectral/eigen techniques can also find the effective impedance between two points [32].

4.3 Visualization Example: IEEE RTS96 Two Area System

The diagram of figure 4 is a traditional single line representation of the test system. Here, bus numbers are coloured to separately show the **load** and **generator** nodes. Various clusters of load buses can readily be identified. These loads will have their voltages determined exclusively by the generators that encircle them. This intuition is made concrete by the F_{LG} matrix, as shown in figure 5.

The edge thicknesses in figure 5 show that, for instance, load **124** is principally dependent on generator **115** for its voltage determination. Some buses are not represented in figure 5; generators like **122** do not appear as they have no substantial influence on any load voltages. By contrast, generator **113** affects the voltage of many load nodes; this diversity of generator influence could be conceptualized as a novel measure of electrical centrality (c.f [33]).

Edge length in figure 5 is related to the effective admittance between nodes. So, for example, it is clear that load **112** must import reactive power through a substantial impedance from its supporting generators, **123** and **113**. This may tax their reactive power capabilities, and could be worrisome if either generator reaches a machine limit. Two well defined load clusters can be discerned in figure 5, at bottom-left and top-right. These clusterings imply substantial redundancy and electrical proximity between their constituent buses, suggesting that serving demand here will not unduly burden the power system. However, more peripheral buses do not appear so voltage secure: for instance, loads **224**, **203**, **112** and **212** do not have such a diversity of generators supporting their voltages, and they are electrically remote from such sources of reactive power.

Figure 4: The simple pseudo-geographical diagram of the IEEE RTS96 two area system

5 Conclusions

The partitioning of the Y_{bus} matrix has opened numerous fruitful avenues in power system analysis, and it is hoped that a clearer understanding of the matrix properties underpinning these partitions will support this strand of research. This work has proved two numerically observed properties of the F_{LG} matrix: future work may consider the matrix characteristics of the other sub-matrices derived by the Y_{bus} partitioning approach.

Force directed layouts have been demonstrated as being suitable for simultaneously portraying the voltage dependencies, and effective admittances, between loads and generators in a test power system. Such a visualization makes concrete the structural features underpinning several popular voltage stability metrics, and its simplicity means it can be readily implemented in real-time operational contexts.

Figure 5: Voltage relationships within the test system: F_{LG} elements over 10% are represented by thickness, with edge forces weighted by the effective admittances between loads and generator.

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