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Multi-Group Multicast Beamformer Design for MIMO-OFDM Transmission

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***Abstract*—We study the problem of designing multicast precoders for multiple groups with the objective of minimizing total transmit power under certain guaranteed quality-of-service (QoS) requirements. To avail both spatial and frequency diversity, we consider a multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) system. The problem of interest is in fact a nonconvex quadratically constrained quadratic program (QCQP) for which the prevailing semidefinite relaxation (SDR) technique is inefficient for at least two reasons. At first, the relaxed problem cannot be equivalently reformulated as a semidefinite programming (SDP). Secondly, even if the relaxed problem is solved, the so-called randomization procedure should be used to generate a high quality feasible solution to the original QCQP. However, such a randomization procedure is difficult in the considered system model. To overcome these shortcomings, we adopt successive convex approximation (SCA) framework in this paper to find beamformers directly. The proposed method not only avoids the randomization procedure mentioned above but also requires lower computational complexity compared to the SDR approach. Numerical experiments are carried out to demonstrate the effectiveness of the proposed algorithm.**

I. Introduction

The demand for wireless multicasting is increasing due to the huge popularity of various multimedia applications providing streaming video and audio services. Due to the immense data requirements from on-demand content delivery services, current wireless standards such as the 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE) provide dedicated sub-frames to deliver such media contents in addition to the regular unicast transmissions [1]. Specifically, to provide both multicast and unicast services over cellular networks, evolved Multimedia Broadcast Multicast Service (MBMS) is specified in the LTE standard. For providing better radio efficiency, two modes of operation for the MBMS, namely, single-cell and multi-cell transmission are envisaged in the LTE. In the single base station (BS) mode, the resource allocation for multicast transmission is decided by a single BS with no coordination between neighboring BSs, but with the feedback from multicast users is available to perform adaptive modulation and precoding. However, in a multi-cell scenario, multiple BSs transmit the same physical signal at the same time in a coordinated manner to minimize the inter-cell interference while improving the signal quality.

Physical layer multicasting for either single or multiple groups has been studied extensively from a signal processing perspective [2]–[5]. The main challenges that has been addressed in the context of multicasting problem are the multicast resource allocation and the precoder design for various user groups. In both problems, the knowledge of channel state information (CSI) has been assumed at the BSs. In order to utilize the wireless resources efficiently, multicast precoders are designed by considering either minimizing the total transmit power or maximizing the minimum achievable rate among the multiplexed multicast groups, where the minimum rate is measured by the weakest link, *i.e.*, the user with the minimum rate. However, the design of transmit precoders in turn depends on the selection of users for various multicast groups which has drawn significant attention in the literature.

Multicast scheduling based on proportional fairness has been considered extensively and also applied in practice to provide fairness among multicast groups. In [6], the scheduler was designed to jointly optimize the proportional fair utility of both unicast and multicast users over multiple BSs. In [7], the problem of resource allocation was modeled as an optimization problem with power and sub-carrier allocation as control variables. Moreover, the objective was to maximize the sum throughput of all multicast group transmissions over multiple sub-carriers provided by orthogonal frequency division multiplexing (OFDM).

Upon allocating users to different multicast groups by a scheduler, efficient utilization of available space-frequency resources provided by multiple-input multiple-output (MIMO)-OFDM is achieved by properly designing multicast precoders. Due to the orthogonal sub-carriers in OFDM, multicast precoders has been usually designed over spatial dimensions only. The transmit precoder design for single multicast group with perfect knowledge of CSI at the BS was introduced in [2]. Due to the nonconvex nature of the optimization problem, the precoders were designed using semidefinite relaxation (SDR) and the resulting problem is solved by semidefinite programming (SDP) in [2]. Instead of finding a precoder vector, say, $\mathbf{m}$, the SDR technique defines a hermitian matrix $\mathbf{M} = \mathbf{m}\mathbf{m}^{\mathrm{H}}$ and solves the original problem as a SDP with $\mathbf{M}$. If the solution obtained for $\mathbf{M}$ is not rank-one, [8] proposed a randomization search to determine an efficient rank-one multicast beamformers.

An extension of multicast precoder design for multiple groups was first studied in [3] with the objective of minimizing the total transmit power while ensuring certain quality-of-service (QoS) requirements for all users. A closely related problem, which is maximizing the minimum signal-to-interference-plus-noise ratio (SINR) among all users was considered in [3] and [4]. In both designs, the precoders were designed by the SDR method. A similar problem with the same design objective was considered for uniform linear array (ULA) in [9]. As an alternative to the SDR method, [10] employed successive convex approximation (SCA) to solve the multicast precoder design for a single group. The main advantage of the proposed solution in [10] is that it eliminates the search for an efficient rank-one vector through a randomization procedure when the obtained solution to the relaxed SDP problem has higher rank.

In this paper, we extend the multicast resource allocation problem in [7] to a multi-user MIMO-OFDM scenario, thereby exploiting both space and frequency resources at the same time. In the MIMO-OFDM context, we address the problem of designing transmit precoders for multiple multicast groups to provide guaranteed QoS in the form of minimum rate. Due to the presence of multiple sub-channels in the problem, SDR method proposed in [3], [4] cannot be used directly. Therefore, we adopt an alternative approach by employing SCA principle to solve the design problem, which is inspired by the superior performance of the SCA based method in [10].

II. System Model and Problem Formulation

A. System Model

We consider a single-cell multi-user MIMO system with N_T transmit antennas transmitting N_G independent multicast data streams to K single-antenna receivers over N OFDM sub-channels (or sub-carriers). Each user belongs to one of the N_G multicast groups, where the users in each group receive a common data stream. Let $\mathcal{V} = \{1, 2, \ldots, N_G\}$ denote the set of all multicast groups present in the system and $\mathcal{O} = \{1, 2, \ldots, N\}$ be the set of all OFDM sub-channels considered in the model. The set of all users associated with multicast group g is denoted by $\mathcal{U}_g$. Now, by using the above notations, the received symbol $y_{k,n}$ on the nth sub-channel for user k belonging to multicast group g is given by

$$y_{k,n} = \mathbf{h}_{k,n}\mathbf{m}_{g,n}d_{g,n} + \sum_{g' \in \mathcal{V}\setminus\{g\}} \mathbf{h}_{k,n}\mathbf{m}_{g',n}d_{g',n} + n_{k,n} \quad (1)$$

where $\mathbf{h}_{k,n} \in \mathbb{C}^{1\times N_T}$ denotes the channel between user k and serving BS on the nth sub-channel. The transmit data symbol $d_{g,n}$ corresponding to each group g is normalized as $\mathbb{E}[|d_{g,n}|^2] = 1$, and $n_{k,n}$ is the additive complex Gaussian noise drawn from $\mathcal{CN}(0, N_0)$. The transmit precoders $\mathbf{m}_{g,n} \in \mathbb{C}^{N_T\times 1}$ are designed to guarantee certain QoS requirements associated with each multicast group. Let $\Gamma_{k,n}(\mathbf{m})$ be the SINR seen by user k, belonging to multicast group g, on the nth sub-channel as

$$\Gamma_{k,n}(\{\mathbf{m}\}) = \frac{|\mathbf{h}_{k,n}\mathbf{m}_{g,n}|^2}{N_0 + \sum_{g'\in\mathcal{V}\setminus\{g\}} |\mathbf{h}_{k,n}\mathbf{m}_{g',n}|^2} \quad (2)$$

where $\{\mathbf{m}\} \triangleq \{\mathbf{m}_{g,n}\}, \forall g \in \mathcal{V}, \forall n \in \mathcal{O}$ denotes the collection of all transmit precoders and $\Gamma_{k,n}(\{\mathbf{m}\})$ is a function of all transmit precoders as shown in (2). However, for simplicity, we refer the SINR of user k on the nth sub-channel as $\Gamma_{k,n}$ instead of $\Gamma_{k,n}(\{\mathbf{m}\})$ in the forthcoming discussions.

B. Problem Formulation

Let us consider the problem of minimizing the total transmit power required to achieve certain guaranteed minimum rate associated with each multicast group. In order to do so, we formulate the multicast precoder design as an optimization problem as

$$\mathcal{P}_1: \quad \underset{\{\mathbf{m}\}}{\text{minimize}} \quad \sum_{g\in\mathcal{V}}\sum_{n=1}^{N} \|\mathbf{m}_{g,n}\|^2 \quad (3a)$$

$$\text{subject to} \quad \sum_{n=1}^{N} \log(1+\Gamma_{k,n}) \geq \bar{r}_k, \ \forall k \quad (3b)$$

where $\bar{r}_k$ in (3b) corresponds to the minimum rate requirement for user k. The problem $\mathcal{P}_1$ is nonconvex due to the SINR expression (2) in the formulation. Therefore, in order to solve the nonconvex problem efficiently, we relax the SINR expression in (2) by a series of convex subproblems that can be solved efficiently.

III. Proposed Solution

In this section, we propose a solution for multicast precoder design for $\mathcal{P}_1$ introduced in Section II-B. Before proceeding further, let us discuss some drawbacks in extending the SDR approach proposed in [2], [3], [8] for multiple sub-channels.

The SDR technique is a powerful signal processing method that has been widely used in wireless communications. In particular, it has been used in demodulating higher order constellations and for the physical layer beamformer designs for single and multiple groups. Unfortunately, the SDR method is not applicable to the multi-carrier system considered in this paper. To understand this problem, let us introduce a positive semidefinite matrix $\mathbf{M}_{g,n} = \mathbf{m}_{g,n}\mathbf{m}_{g,n}^{\mathrm{H}}$ as an optimization variable together with the constraint that $\mathrm{rank}(\mathbf{M}_{g,n}) = 1$ to ensure the rank-one solution. Now, by using $\mathbf{M}_{g,n}$, we can express the SINR $\Gamma_{k,n}$ in (2) as

$$\Gamma_{k,n} = \frac{\mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g,n}\right)}{N_0 + \sum_{g'\in\mathcal{V}\setminus\{g\}} \mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g',n}\right)} \quad (4)$$

where $\mathbf{H}_{k,n} = \mathbf{h}_{k,n}\mathbf{h}_{k,n}^{\mathrm{H}}$ denotes the outer product of the channel seen by user k on the nth sub-channel. Using (4), an equivalent formulation for $\mathcal{P}_1$ is given as

$$\underset{\{\mathbf{M}\}}{\text{minimize}} \quad \sum_{g\in\mathcal{V}}\sum_{n=1}^{N} \mathrm{tr}\left(\mathbf{M}_{g,n}\right) \quad (5a)$$

$$\text{subject to} \quad \mathrm{rank}\left(\mathbf{M}_{g,n}\right) = 1, \ \forall g \in \mathcal{V}, \forall n \in \mathcal{O} \quad (5b)$$

$$\sum_{n=1}^{N} \log\left(1+\Gamma_{k,n}\right) \geq \bar{r}_k. \tag{5c}$$

The constraint (5c) ensures the minimum guaranteed QoS requirement of all users in the system. In case of $N=1$, *i.e.*, for a single sub-channel model, (5) can be reformulated as a SDP, which can be solved efficiently [3], [4]. Even though (5) can be computed for a single sub-channel case, we may still require to extract a rank-one solution if (5) yields a result with rank greater than one. In case of multiple multicast groups, the randomization method is carried out by solving a linear program for each random rank-one precoders. Therefore, as the number of sub-channels and the multicast groups grows, the complexity of (5) increases quickly.

A. Successive Convex Approximation Method

Due to the overall complexity involved in solving $\mathcal{P}_1$ by adopting the SDR approach, we propose an alternative method for solving $\mathcal{P}_1$ by employing the SCA technique without any need for the randomization search. In order to do so, we relax the SINR expression in (2) by the following constraints as

$$\gamma_{k,n} \leq \frac{|\mathbf{h}_{k,n}\mathbf{m}_{g,n}|^2}{b_{k,n}} \tag{6a}$$

$$b_{k,n} \geq N_0 + \textstyle\sum_{r\in\mathcal{V}\setminus\{g\}} |\mathbf{h}_{k,n}\mathbf{m}_{r,n}|^2. \tag{6b}$$

The relaxation in (6a) is an under-estimator for $\gamma_{k,n}$, since the newly introduced variable $b_{k,n}$ in (6b) is an over-estimator for the total interference seen by user k from the other multicast transmissions on the nth sub-channel. However, even after relaxing the SINR expression in (2) by the inequality in (6), the problem is still nonconvex due to the constraint (6a). Therefore, to solve the problem efficiently, we adopt the SCA technique discussed in [11], where the nonconvex set is relaxed by a sequence of convex subsets and solved iteratively until convergence. The relaxed subproblem for (3) is obtained by linearizing (6a) around an operating point as $\{\mathbf{m}^{(i)}\} = \{\mathbf{m}_{g,n}^{(i)}\}, \forall g \in \mathcal{V}, \forall n \in \mathcal{O}$, where $\{\mathbf{m}^{(i)}\}$ is a solution obtained from the $(i-1)$th SCA iteration.

Let us now consider an equivalent representation of the SINR constraint (6a) of user k on sub-channel n as

$$\gamma_{k,n} \leq \frac{p_{k,n}^2 + q_{k,n}^2}{b_{k,n}} \tag{7}$$

where $p_{k,n} = \Re\{\mathbf{h}_{k,n}\mathbf{m}_{g,n}\}$ and $q_{k,n} = \Im\{\mathbf{h}_{k,n}\mathbf{m}_{g,n}\}$ are scalars. The fractional term in (7) is of *convex-over-linear* form, and thus can be bounded from below by a linear first order Taylor approximation as

$$\begin{aligned}\mathcal{L}_{k,n}^{(i)} &\triangleq \frac{2}{b_{k,n}^{(i)}} \left\{p_{k,n}^{(i)}\left(p_{k,n} - q_{k,n}^{(i)}\right) + q_k^{(i)}\left(q_k - q_k^{(i)}\right)\right\} \\ &+ \frac{p_{k,n}^{2(i)} + q_{k,n}^{2(i)}}{b_{k,n}^{(i)}}\left(1 - \frac{b_{k,n} - b_{k,n}^{(i)}}{b_{k,n}^{(i)}}\right) \leq \frac{p_{k,n}^2 + q_{k,n}^2}{b_{k,n}}\end{aligned} \tag{8}$$

where $p_{k,n}^{(i)} = \Re\{\mathbf{h}_{k,n}\mathbf{m}_{g,n}^{(i)}\}$ and $q_{k,n}^{(i)} = \Im\{\mathbf{h}_{k,n}\mathbf{m}_{g,n}^{(i)}\}$ are fixed operating points over which the linearization is performed. Therefore, by using (8) instead of (6a) in (3), the convex subproblem for the ith SCA iteration is given as

$$\underset{\{\mathbf{m}\},\{\boldsymbol{b}\},\{\boldsymbol{\gamma}\}}{\text{minimize}} \quad \sum_{g\in\mathcal{V}}\sum_{n=1}^{N} \|\mathbf{m}_{g,n}\|^2 \tag{9a}$$

$$\text{subject to} \quad \sum_{n=1}^{N} \log(1+\gamma_{k,n}) \geq \bar{r}_k,\ \forall k \tag{9b}$$

$$\mathcal{L}_{k,n}^{(i)} \geq \gamma_{k,n},\ \text{and (6b)},\ \forall k, \forall n \tag{9c}$$

which can be solved using the existing solvers [12]–[14]. The QoS constraint in (9b) is equivalent to the geometric mean as

$$\prod_{n=1}^{N} (1+\gamma_{k,n}) \geq \exp(\bar{r}_k) \tag{10}$$

which can be casted as a second-order cone (SOC) constraint, thereby leading to a second-order cone programming (SOCP) formulation for (9) with reduced complexity. The fixed operating points $\{\mathbf{m}_{g,n}^{(i)}\}$ and $\{b_{k,n}^{(i)}\}$ are updated with the solution of convex subproblem (9) in the $(i-1)$th SCA iteration. Now, by updating the operating point recursively, (9) is solved until convergence to obtain transmit precoders to guarantee the minimum target rate corresponding to all users in the system. The convergence analysis of the SCA based iterative scheme follows the discussions presented in [15], [16].

B. Choice of Initialization Points

Finding an initial operating point $\{\mathbf{m}_g^{(0)}\}$ to start the iterative SCA procedure is not a trivial problem. For the case of single group multicasting, initialization can be easily done as shown in [10], *i.e.*, by merely scaling the beamformers until all the QoS constraints are satisfied. However, to find a feasible starting point to initialize (9), we first solve a associated convex problem by relaxing the SINR constraint as

$$\underset{\{\mathbf{m}\},\{\boldsymbol{b}\},\{\boldsymbol{\gamma}\},\tilde{R}}{\text{minimize}} \quad \sum_{g\in\mathcal{V}}\sum_{n=1}^{N} \|\mathbf{m}_{g,n}\|^2 + \delta\tilde{R} \tag{11a}$$

$$\text{subject to} \quad \exp\left(\bar{r}_k\right) - \prod_{n=1}^{N}(1+\gamma_{k,n}) \leq \tilde{R},\ \forall k \tag{11b}$$

$$\mathcal{L}_{k,n}^{(i)} \geq \gamma_{k,n},\ \text{and (6b)},\ \forall k, \forall n \tag{11c}$$

where δ is a regularization term and $\tilde{R}$ denotes the maximum difference between target and actual rate among all users in the system. Note that problem (11) is feasible for any randomly initialized multicast precoders, since the minimum rate requirements are relaxed by a slack variable $\tilde{R}$ in (11).

If δ is chosen to be infinite, then (11) reduces to a feasibility check problem. However, if δ is finite but sufficiently large, then (11) solves for multicast precoders with the objective of minimizing total transmit power and the slackness term $\tilde{R}$ in each iteration. Once $\tilde{R} < 0$, the above iterative procedure can be terminated and the precoders obtained at the termination step can be used as an initial feasible point for problem (9). Algorithm 1 outlines the procedure to solve (9).

Algorithm 1 Iterative algorithm for multicast precoder design

Require: Initialize $i = 1$, $\{\mathbf{m}^{(0)}\}$ randomly
Evaluate $\{\mathbf{b}^{(0)}\}$ with (6b) using $\{\mathbf{m}^{(0)}\}$ and $\mathbf{h}_{k,n}, \forall k, n$
Require: Fix δ large enough to ensure feasibility
repeat
  solve (11) for optimal $\mathbf{m}_*^{(i)}$ and $\mathbf{b}_*^{(i)}$
  update $\mathbf{m}^{(i+1)} = \mathbf{m}_*^{(i)}$ and $\mathbf{b}^{(i+1)} = \mathbf{b}_*^{(i)}$
  $i = i + 1$
until $\tilde{R} < 0$
let $\mathbf{m}_*$ and $\mathbf{b}_*$ be a solution of problem (11) when $\tilde{R} < 0$
Ensure: $j = 1$ and set $\mathbf{m}^{(0)} = \mathbf{m}_*$ and $\mathbf{b}^{(0)} = \mathbf{b}_*$
repeat
  solve (9) for optimal $\mathbf{m}_*^{(j)}$ and $\mathbf{b}_*^{(j)}$
  update $\mathbf{m}^{(j+1)} = \mathbf{m}_*^{(j)}$ and $\mathbf{b}^{(j+1)} = \mathbf{b}_*^{(j)}$
  $j = j + 1$
until convergence

C. Extension to Semidefinite Relaxation

In order to find a solution for (5), we propose a method by employing the SCA technique for the SDP problem. To obtain a tractable solution, we proceed by relaxing the SINR expression in (4) for user k on the nth sub-channel using two new optimization variables as

$$\gamma_{k,n} \leq \frac{\mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g,n}\right)}{b_{k,n}} \tag{12a}$$

$$b_{k,n} \geq N_0 + \textstyle\sum_{g' \in \mathcal{V}\setminus\{g\}} \mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g',n}\right) \tag{12b}$$

where $\gamma_{k,n}$ is an under-estimator for $\Gamma_{k,n}$ in (2). Let $\{\gamma\}$ be the collection of all $\gamma_{k,n}$, $\forall k$ and $\forall n \in \mathcal{O}$. The SINR constraint in (12a) can be equivalently written as

$$\mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g,n}\right) \geq \gamma_{k,n}\, b_{k,n} \tag{13a}$$

$$\mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g,n}\right) \geq \frac{1}{4}\left[(\gamma_{k,n} + b_{k,n})^2 - (\gamma_{k,n} - b_{k,n})^2\right]. \tag{13b}$$

The constraint defined by (13b) is nonconvex due to the presence of difference of convex (DC) term in the r.h.s of (13b). Therefore, we use the SCA method to handle the DC term in (13b) by the first order Taylor approximation around the SCA operating point $\gamma_{k,n}^{(i)}$ and $b_{k,n}^{(i)}$ in the ith iteration as

$$\begin{aligned}\mathcal{D}_{k,n}^{(i)} \triangleq \frac{1}{4}\Big[&(\gamma_{k,n} + b_{k,n})^2 - 3\,(\gamma_{k,n}^{(i)} - b_{k,n}^{(i)})^2 \\ &+ 2\,(\gamma_{k,n}^{(i)} - b_{k,n}^{(i)})(\gamma_{k,n} - b_{k,n})\Big]\end{aligned} \tag{14}$$

where the approximate linear expression $\mathcal{D}_{k,n}^{(i)}$ satisfies

$$\mathcal{D}_{k,n}^{(i)} \geq \frac{1}{4}\Big((\gamma_{k,n} + b_{k,n})^2 - (\gamma_{k,n} - b_{k,n})^2\Big). \tag{15}$$

Now, by using the linear approximation in (14) and by dropping the rank-one constraint (5b), the SDP-SCA subproblem for SCA step i is given as

$$\underset{\{\mathbf{M}\},\{\mathbf{b}\},\{\gamma\}}{\text{minimize}} \quad \sum_{g \in \mathcal{V}} \sum_{n=1}^{N} \mathrm{tr}\left(\mathbf{M}_{g,n}\right) \tag{16a}$$

$$\text{subject to} \quad \mathrm{tr}\left(\mathbf{H}_{k,n}\mathbf{M}_{g,n}\right) \geq \mathcal{D}_{k,n}^{(i)} \tag{16b}$$

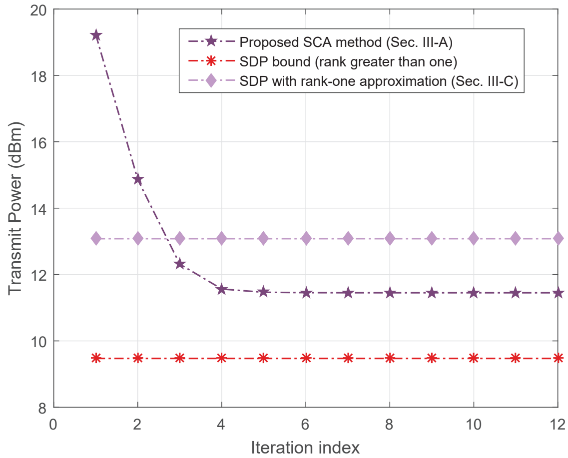

Fig. 1. Convergence behavior of the proposed methods in terms of the total transmit power for $\{N_T, N, K, N_G, \bar{r}\} = \{8, 1, 20, 2, 2\,\text{bits}\}$.

$$\sum_{n=1}^{N} \log\left(1 + \gamma_{k,n}\right) \geq \bar{r}_k, \forall k \in \mathcal{U} \tag{16c}$$

$$\mathbf{M}_{g,n} \succeq 0 \text{ and (12)}, \forall g \in \mathcal{V}, \forall n \in \mathcal{O}. \tag{16d}$$

However, upon the convergence of (16) as $i \to \infty$, if any of the resulting precoder matrices $\mathbf{M}_{g,n}$, $\forall g, \forall n$ has $\mathrm{rank}(\mathbf{M}_{g,n}) > 1$, then we must find a feasible rank-one precoders for $\mathcal{P}_1$ by using the randomization procedure mentioned in [2], [8]. It is performed by fixing the target SINR for each sub-channel with the respective $\gamma_{k,n}$ obtained upon the convergence of (5).

IV. Numerical Simulations

In this section, we compare the performance of proposed precoder designs with the existing solutions in [3], [9]. The path loss seen by all users is fixed to $0\,\mathrm{dB}$ and the channels are drawn from $\mathbf{h}_{k,n} \sim \mathcal{CN}(0, 1)$. The noise variance is fixed as $N_0 = 1$ in all simulation.

A. Multiple Groups with Single Sub-Channel

Fig. 1 demonstrates the convergence behavior of the proposed SCA based precoder design for a single sub-channel scenario. The comparison is made between the SDR method in [3], [9] and the SCA based design. Since the final precoders obtained by the SDR approach is not rank-one, randomization search has been performed over 500 realizations to find rank-one precoders with minimum transmit power. In addition, Fig. 1 also includes the SDP bound, *i.e.*, the total transmit power required to achieve the desired rate of 2 bits using precoders with rank larger than one. In Fig. 1, the SDP performance is represented as a constant throughout the SCA iterations.

B. Multiple Groups with Multiple Sub-Channels

Fig. 2 illustrates the convergence behavior of the proposed algorithms for a system with $N = 2$ sub-channels accommodating $N_G = 3$ multicast groups with 12 users each. The minimum guaranteed rate requirement is fixed as 5 bits and the total number of transmit antennas is set to be $N = 8$. The

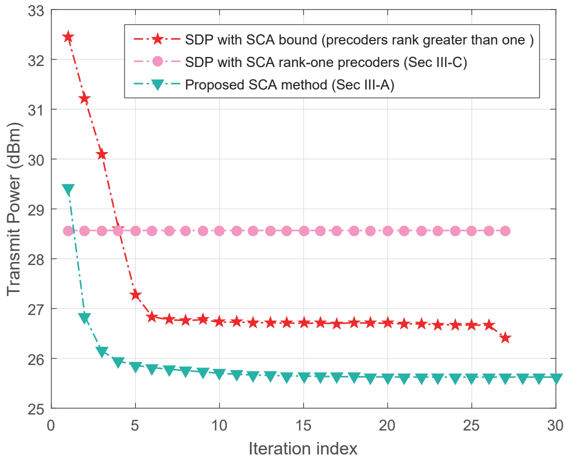

Fig. 2. Convergence behavior of the proposed methods with respect to the total transmit power for $\{N_T, N, K, N_G, \bar{r}\} = \{8, 2, 36, 3, 5\,\text{bits}\}$

comparison is made between the SDP-SCA method and the SCA based approach. As can be seen from Fig. 2, the total transmit power required to serve $N_G = 3$ multicast groups by the SCA scheme is $\approx 3\,\text{dB}$ less than the rank-one SDP along with the SCA method. It is due to the nonconvex nature of the problem (5) for multiple sub-channels, the SDP-SCA method is inferior to the SCA formulation in (9).

C. ULA Model

In this subsection, we consider a ULA with $N_T = 8$ antenna elements serving $N_G = 3$ multicast groups, each consisting of $|\mathcal{U}_g| = 10$ users, $\forall g \in \mathcal{V}$. The minimum guaranteed rate requirement for each user is fixed as 1.587 bits, which corresponds to the SINR $3\,\text{dB}$ when the number of sub-channels $N = 1$. The multicast groups are located at $0°$, $120°$ and $270°$, respectively. The individual users in each multicast group are separated within by $2°$. A line-of-sight (LoS) channel is assumed as $[1\ \exp\{j\theta_k\}\ \ldots\ \exp\{j(N_T - 1)\theta_k\}]$, where θ_k is the angular location of user k w.r.t the broadside of antenna array and the gain is normalized to unity.

Fig. 3 compares the radiation pattern of the proposed and existing algorithms with $N = 1$ and $N = 2$ sub-channels for the ULA model. The radiation pattern shows that both the SDP and the SCA methods ensure the SINR of $3\,\text{dB}$ to all users in both multicast groups. However, when the number of sub-channels is $N = 2$, both the SDP-SCA method and the SCA design require $\approx 1.6\,\text{dB}$ less than the respective schemes with $N = 1$. For certain scenarios, the precoders obtained by the SDP method has rank greater than one. In those cases, we extract a feasible rank-one precoder by searching over 500 random combinations. In the case of $N = 2$, the total power is obtained by taking the sum of individual powers on each sub-channel while plotting Fig. 3.

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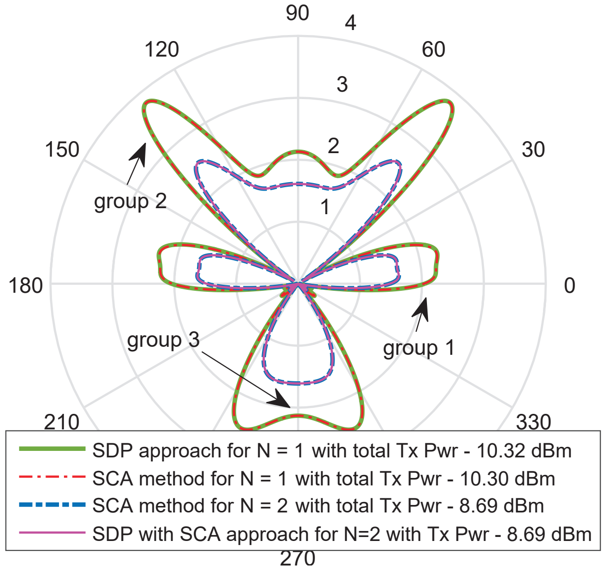

Fig. 3. Radiation pattern for $\{N_T, K, N_G, \bar{r}\} = \{8, 30, 3, 1.587\,\text{bits}\}$ ULA model with multicast groups located at $0°$, $120°$ and $270°$.

V. Conclusions

We discussed the problem of designing multicast precoders for multiple groups with the objective of minimizing total transmit power under certain guaranteed quality-of-service requirements. The system model considered for the current study consists of a single-cell multiple-input multiple-output transmission with orthogonal frequency division multiplexing. We modeled the multicast precoder design as a joint optimization problem over space and frequency dimensions. Due to the nonconvex nature of the proposed formulation, we employed successive convex approximation (SCA) and solved the resulting convex subproblems iteratively. The performances of proposed methods were demonstrated by the numerical simulations. Moreover, we also compared the results of existing solutions based on semidefinite relaxation with the proposed scheme, which is based on the SCA technique.

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