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# Homogenization of a Composite, Multi-Girder Bridge Deck as an Equivalent Orthotropic Plate

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*Abstract*— Most bridge decks are orthotropic, because of the orthotropic nature of their component parts (e.g. isotropic slabs, grillages, T-beam bridge decks, multi-beam bridge decks, multi-cell boxbeam bridge, and slabs stiffened with ribs of box section). Thus, the orthotropic plate theory plays an important role in the static and dynamic analysis of bridges. For example, a multicellular Fiber Reinforced Polymer (FRP) composite bridge deck can be modeled as an orthotropic plate with equivalent stiffnesses that account for the size, shape, and constituent materials of the cellular deck. Thus, the complexity of material anisotropy of the panels and orthotropic structure of the deck system can be reduced to an equivalent orthotropic plate with global elastic properties in two orthogonal directions – parallel and transverse to the longitudinal axis of the deck cell.

This paper investigates a homogenization of composite, orthotropic, three-span, multi-girder bridge to explore the concept of the volumetric and mass fractions of a reinforced composite material. This homogenization takes into account all properties of this composite structure (deck slab, girders and diaphragms). From those, all the equivalent orthotropic plate properties were obtained. The work is highly relevant with respect to evaluating the dynamic interaction between bridges and vehicles.

*Keywords*— *composite materials, homogenization; multi-girder bridge deck; orthotropic plate; equivalent properties* 

## 1. Introduction

In the past few years, the Fiber Reinforced Polymer (FRP) composite materials have gained popularity in a wide range of applications and have been experimentally implemented in bridge structures. FRP deck research began in 1983 in the U.S. when the U.S. Department of Transportation initiated a research project titled "Transfer of Composite Technology to Design and Construction of Bridges". However, the results from these studies were scattered and unevaluated [Zureick, 1995]. In 1992, the FHWA initiated an investigation with the objectives of compiling and reviewing the literature pertaining to FRP bridge decks research studies. This review was published in 1995 [Zureick, 1995]. Since then, a number of papers and reports concerning research in FRP bridge decks have been published [Karbhari 1997, Lopez-Anido 1997, GangaRao 1999, Hays 2000, Qiao 2000, Zhou, 2001]. Today, the development of FRP decks has expanded beyond the academia and into actual practice. However, only a few studies have looked at multiple vehicles atop a multi-lane, FRP, bridge deck. In 1985, Bakht and Jaeger published a book entitled: Bridge Analysis Simplified. In 1995, Humar and Kashif simplified a slab-type bridge as a single span orthotropic plate. To date, there have been three main methods approaches to dynamic analysis of bridge decks: finite element, finite strip, and orthotropic plate theory methods. The last is applicable to vibration analysis bridge decks that are slabs, composite, orthotropic, right, curved and simply supported.

In this paper, a simplified method is presented to determine the equivalent orthotropic plate properties of a composite three-girder bridge deck system such as its modulus of elasticity, Poisson's ratio, flexural, and torsional stiffness, etc. This method is based on the concept of the volumetric and mass fractions of a reinforced composite material to evaluating the dynamic interaction between bridge and moving truck.

#### 2. Brief description of the proposed method

Arguably the two principal characteristics of primary importance in FRP bridge deck applications are its stiffness and strength. The stiffness of an FRP deck is the ability of the deck to resist changes in shape when a load is applied. The deck strength is the deck's ability to resist permanent deflection from applied loads (static, dynamic and environmental loads etc.). The components of the bridge above the bearings are referred to as superstructure. A number of terms commonly used to describe a bridge's superstructure are its deck slab, girders, diaphragms, and pavement [see Wacker and Smith, 2001 for detailed definitions].

In this study, a previously published bridge was modeled [Zhu-Law 2002]. It consisted of a symmetric, composite, three-span bridge deck with outer span lengths of 24 m and an inner one of 30 m. The bridge deck comprised a concrete deck slab, 5 I-steel girders, and 14 steel diaphragms (Fig.1). All relevant characteristics are summarized in Table 1.



Fig.1. Continuous three-span multi-girder bridge deck.

Concrete Deck slab:			
Length	78 m		
Width	13.715 m		
Thickness	0.2 m		
Young's moduli	$E_{\rm x} = 4.17 \times 10^{10} \text{ N/m}^2$ , $E_{\rm y} = 2.97 \times 10^{10} \text{ N/m}^2$		
Mass density	3000 kg/ m <sup>3</sup>		
Poisson's ratio	$v_{xy} = 0.3$		
Steel girders:			
Number	5		
Distance between to adjacent girders	2.743 m		
Web height	1.49 m		
Web thickness	0.0111 m		
Flange width	0.405 m		
Flange thickness	0.018 m		
Mass density	$7850 \text{ kg/m}^3$		
Steel diaphragms:			
Number	14		
Distance between to adjacent diaphragms	6 m		
Cross-sectional area	$15.48 \times 10^{-4} \text{ m}^2$		
Mass density	7850 kg/ m <sup>3</sup>		
Moments of inertia	$I_y = 0.71 \times 10^{-6} \text{ m}^4$ , $I_z = 2 \times 10^{-6} \text{ m}^4$ , $J = 1.2 \times 10^{-7} \text{m}^4$		

**Table 1.** Parameters of the three-span multi-girder bridge deck

The applied method involves the composite three-girder bridge deck system that will be eventually represented by an equivalent orthotropic plate (Fig.2), with the same length and width of that of the composite bridge deck. Zhu and Law (2002) used the theory of thin orthotropic plates to model the bridge deck, whereas in the second numerical example, the structure is a composite (deck slab + diaphragms + girders). In the reference case used herein(Zhu and Law, 2002), the equivalent orthotropic plate data were not explicitly provided. From those, a homogenization of this composite structure was made to explore the concept of the volumetric and mass fractions of a reinforced composite material [Chateauminois 2000]. This homogenization takes into account all properties of this composite structure (deck slab, girders and diaphragms).



Fig.2. Homogenization of a composite three-girder bridge deck as an equivalent orthotropic plate

The mass  $(M_b)$  and the volume  $(V_b)$  of concrete deck slab respectively are:

$$M_{b} = \rho_{b} lbh = 3000 \times 78 \times 13.715 \times 0.2 = 641862 \ kg$$
$$V_{b} = \frac{M_{b}}{2} = 213.954 \ m^{3}$$

The section, mass, and volume of each girder respectively are:

 $\rho_{h}$ 

 $S_r = 0.405 \times 0.018 \times 2 + 0.0111 \times 1.49 = 0.03113 m^2$ 

$$M_r = \rho_r S_r l = 7850 \times 0.03113 \times 78 = 19063.287 \, kg ;$$
  
$$V_r = \frac{M_r}{\rho_r} = \frac{641862}{7850} = 2.4284 \, m^3$$

The mass and the volume of each diaphragm respectively are:

$$\begin{split} M_e &= \rho_e S_e b_e = 7850 \times 0.001548 \times 10.972 = 133.329 \, kg \\ V_e &= \frac{M_e}{\rho_e} = \frac{133.329}{7850} = 0.01698 \, m^3 \end{split}$$

Now we calculate the volume and mass fractions of a reinforced composite material, respectively [Chat. 2000]:

$$v_b = \frac{V_b}{V_c} = 0.9453$$
;  $v_a = \frac{V_a}{V_c} = 0.0547$ 

The term  $V_c = V_b + 5V_r + 14V_e$  is the volume of the composite, and  $V_a = 5V_r + 14V_e$  is the volume of the steel

The thickness and mass density of the equivalent orthotropic plate (composite) are, respectively:

$$h_c = \frac{M_c}{\rho_c lb} = 0.21157 \ m \ ; \qquad \rho_c = \rho_b v_b + \rho_a v_a = 3265.295 \ kg \ / \ m^3$$

With  $M_c = M_b + 5M_r + 14M_e$  as the composite mass.

The rigidities of the equivalent orthotropic plate can be calculated according to Bakht and Jaeger (1985):

$$D_x = 2.415 \times 10^9 Nm; \quad D_y = 2.1807 \times 10^7 Nm; \quad D_{xy} = 1.1424 \times 10^8 Nm$$

Poisson's ratio, Young's moduli and shear modulus of of the equivalent orthotropic plate are, respectively:

$$\begin{split} v_{xy} &= 0.3; \quad v_{yx} = \frac{D_y}{D_x} v_{xy} = 0.0027; \quad E_x = \frac{12(1 - v_{xy}v_{yx})D_x}{h_c^3} = 3.0576 \times 10^{12} \ N/m^2; \\ E_y &= E_x \frac{D_y}{D_x} = 2.7607 \times 10^{10} \ N/m^2; \qquad G_{xy} = \frac{12D_{xy}}{h_c^3} = 1.4475 \times 10^{11} \ N/m^2 \end{split}$$

To validate the homogenization of the simplified method presented in this paper, an example is presented in which the frequencies and mode shapes of an equivalent orthotropic three span plate are obtained based on the approach published previously in [Rezaiguia and Laefer 2009] (using the modal method and average integration) as those presented in reference [Zhu and Law 2002] based on the Rayleigh-Ritz method and those calculated by ANSYS software based on the finite element method. To obtain frequencies from ANSYS, first all characteristics of the equivalent orthotropic plate were introduced, and then the bridge deck was modeled with 28080 shell63 type elements with 4 nodes and 6 degrees of freedom per node. The results showed excellent agreement for all frequencies compared with those obtained by ANSYS (no error exceeding 2%) and are described in detail in reference [Rezaiguia and Laefer 2009].

		Natural frequencies [Hz]			Er	ror [%]
Mode	Order	Rez. and Lae.	ANGVG	Zhu and	Rez. and Lae.	Rez. and Lae./Zhu
shapes	of freq.	(2009)	ANSIS	Law (2002)	/ANSYS	and Law
1	1.1	4.13	4.13	4.13	0.00	0.00
2	1.2	5.45	5.45	4.70	0.00	13.76
3	2.1	6.30	6.30	6.31	0.00	-0.16
4	2.2	7.59	7.58	6.86	0.00	9.62
5	3.1	7.75	7.76	7.76	-0.13	-0.13
6	3.2	8.77	8.79	8.20	-0.23	6.50
7	1.3	9.08	9.00		0.88	
8	2.3	11.26	11.23		0.27	
9	3.3	11.97	12.01		-0.33	
10	1.4	15.07	14.88		1.26	

 Table 2. Comparison of natural frequencies of the orthotropic three-span bridge decks

Fig. 3 shows the first four mode shapes of the three-span bridge deck obtained by the proposed approach (compiled in FORTRAN with the results plotted in MATLAB). Those obtained from the FEM work in ANSYS are shown in fig. 4. Excellent agreement between the mode shapes is seen.

Saleh, I would suggest reorganizing figures 3 and 4 so that the images in 3 are on the left side and the corresponding ones in 4 on the right side. This will make comparisons for the reader much easier.







**Fig.3.** The first four mode shapes of the three-span bridge deck obtained through the present approach. Modes: (a) 1,  $f_1 = 4.13$  Hz; (b) 2,  $f_2 = 5.45$  Hz; (c) 3,  $f_3 = 6.30$  Hz; (d) 4,  $f_4 = 7.59$  Hz





**Fig.4.** The first four mode shapes of the three-span bridge deck obtained through ANSYS. Modes: (a) 1,  $f_1 = 4.129$  Hz; (b) 2,  $f_2 = 5.446$  Hz; (c) 3,  $f_3 = 6.301$  Hz; (d) 4,  $f_4 = 7.582$  Hz

### Conclusion

The objective of this study wass to determine the equivalent properties of a superstructure of an orthotropic three-span multi-girder bridge to evaluate the dynamic interaction between the bridge and a moving truck. This homogenization was based on the concept of the volumetric and mass fractions of a reinforced composite material. The approach was validated through a numerical computation in ANSYS that matched previously published results.

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