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Binary PSO Based Dynamic Multi-objective model for DG Planning under Uncertainty

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Abstract

This paper proposes a dynamic multi-objective model for integration of distributed generations in distribution networks. The proposed model optimizes three objectives, namely technical constraint dissatisfaction, costs and environmental emissions and simultaneously determines the optimal location, size and timing of investment for both DG units and network components. The uncertainties of electric load, electricity price and wind power generations are taken into account using scenario modeling. A scenario reduction technique is used to reduce the computational burden of the model. The Pareto optimal solutions of the problem are found using a binary PSO algorithm and finally a fuzzy satisfying method is applied to select the optimal solution considering the desires of the planner. The effectiveness of the proposed model is demonstrated by applying it to a realistic 201-node distribution network.

Index Terms

Distributed generation, PSO, Dynamic planning, scenario reduction, Pareto optimal front.

NOMENCLATURE

Constants

C_ℓ	Cost of reinforcement of feeder ℓ
C_{tr}	Cost of investment in transformer
d	Discount rate
$E_{grid/dg}$	Emission factor of the grid/dg

IC_{dg}, OC_{dg}	Investment and operating cost of a dg
T	Planning horizon
α	Rate of demand growth

Variables

$P_{i,t,dl,s}^{D}$	Active power demand in bus i, in year t, demand level dl and state s
$P_{t,dl,s}^{grid}$	Active power purchased from grid in year t, demand level dl and state s
$P_{i,t,dl,s}^{dg}$	Active power injected by a dg in bus i, in year t, demand level dl and state s
$S_{t,dl,s}^{grid}$	Apparent power imported from grid in year t, demand level dl and state s
$S_{i,t,dl,s}^{dg}$	Apparent power of dg installed in bus i, in year t, demand level dl and state s
$I_{\ell,t,dl,s}$	Current magnitude of ℓth feeder in year t, demand level dl and state s
v_{in}^{cut}	Cut-in speed of wind turbine
v_{out}^{cut}	Cut-out speed of wind turbine
$\mu_{i,t,dl,s}^{V}$	Degree of voltage constraint satisfaction for bus i, in year t, demand level dl and state s
$\mu_{i,t}^{V}$	Degree of voltage constraint satisfaction for bus i, in year t
μ_{t}^{V}	Degree of voltage constraint satisfaction for whole system in year t
$\mu_{\ell,t,dl,s}^{I}$	Degree of thermal constraint satisfaction for feeder ℓ in year t, demand level dl and state s
$\mu_{\ell,t}^{I}$	Degree of thermal constraint satisfaction for feeder ℓ in year t
μ_{t}^{I}	Degree of thermal constraint satisfaction for whole system in year t
$\mu_{t,dl,s}^{S^{grid}}$	Degree of thermal constraint satisfaction for substation in year t and state s
$\mu_{t}^{S^{grid}}$	Degree of overall thermal constraint satisfaction for substation in year t
$\mu^{f_k(X_n)}$	Degree of minimization satisfaction of kth objective function by solution X_n
$S_{i,t,dl,s}^{D}$	Demand in bus i, year t, demand level dl and state s
EP_s^{λ}	Electricity price in state s
FN_n	Front number to which n^{th} solution belongs
GD_n	Global Diversity of nth solution
γ_t^{ℓ}	Investment decision in feeder ℓ, in the year t
$\xi_{i,t}^{dg}$	Investment decision for DG technology dg in bus i, in the year t
ψ_t^{tr}	Investment decision in transformer, in the year t

d_ℓ	Length of feeder ℓ in km
LD_n^k	Local diversity of nth solution in k^{th} objective function
MD_k	Maximum difference between the values of kth objective function, regarding all solutions
$\overline{S}_{lim}^{dg}$	Maximum operating limit of a dg
$P_{i,t,dl,s}^{net}$	Net active power injected to bus i, in year t, demand level dl and state s
$Q_{i,t,dl,s}^{net}$	Net reactive power injected to bus i, in year t, demand level dl and state s
N_b	Number of buses in the network
N_p	Number of population
N_ℓ	Number of feeders in the network
N_o	Number of objective functions
N_J	Number of combined states
N_s	Number of reduced states
ρ	Peak price of energy purchased from the grid
$S_{i,peak}^D$	Peak demand in bus i, in state s
$\lambda_{dl,s}$	Percent of peak electricity price in state s and demand level dl
$D_{dl,s}$	Percent of peak electricity demand in state s and demand level dl
wp_s	Percent of rated capacity of installed wind turbine in state s
$P_{i,r}^w$	Rated power of installed wind turbine
$Q_{i,t,dl,s}^{dg}$	Reactive power injected by a dg in bus i, year t, demand level dl and state s
$Q_{i,t,dl,s}^D$	Reactive power demand in bus i, year t, demand level dl and state s
SF_n	Pseudo fitness of nth particle
$prob_s^l$	Probability of load state s
$prob_s^\lambda$	Probability of electricity price state s
$prob_s^w$	Probability of wind speed state s
$prob_s^c$	Probability of combined state s
v_{rated}	Rated speed of wind turbine
c	Scale factor of Rayleigh PDF of wind speed
TD_t	Technical dissatisfaction in year t
GC	Total cost paid to grid
DGIC	Total Investment cost of DG units

DGOC	Total operation cost of DG units
FC	Total cost of feeder reinforcement
SC	Total cost of transformer investment
$\overline{S}_{safe,t}^{tr}, \overline{S}_{crit,t}^{tr}$	Upper safe and critical values of operation thermal limits of substation feeding the network
$\overline{I}_{\ell}^{safe,t}, \overline{I}_{\ell}^{crit,t}$	Upper safe and critical values of operation thermal limits of feeders
$V_{safe}^{max}, V_{safe}^{min}$	Upper and lower safe operation limits of voltage
$V_{crit}^{max}, V_{crit}^{min}$	Upper and lower critical operation limits of voltage
$V_{i,t,dl,s}$	Voltage magnitude in bus i, year t, demand level dl and state s
$\delta_{i,t,dl,s}$	Voltage angle in bus i, year t, demand level dl and state s
v_s	Wind speed in state s

I. INTRODUCTION

A. Motivation and problem description

Distributed generation (DG) is an electric power source connected directly to the distribution network [1]. There are five major factors that motivate increasing the share of DG units in electricity generation: electricity market liberalization, development in DG technology, constraints on the construction of new transmission lines, reliability enhancement [2] and concerns about the environment [3]. DG may also offer distribution network operators (DNOs) more diverse, flexible, and secure options for managing their electricity systems to benefit customers [4]. A powerful tool for planners is needed to model the uncertainties of intermittent power generations of wind turbines and also electric load and price values. It should not only reduce the computational burden but also maintain the accuracy of computation. The motivation of this study is to provide such a tool.

B. Literature review

In recent years, many approaches have been proposed to solve the DG planning problem. The literature suggests a wide range of objectives, such as voltage stability improvement [5], risk aversion in load procurement [6], active loss reduction [7], [8], reactive loss reduction [9], reliability improvement, reducing the cost of energy required for serving the customers, increasing the incentives received by distribution network owners for using DGs, reducing the

cost of energy not supplied, injecting power into the grid at peak load and emission reduction [10]. These studies have considered a variety of technical issues including voltage profile [11], [12], thermal limits of conductors [13], substation capacity [14], three phase and single phase to ground short circuit [13], [15], and load modeling [9]. The reported models for DG planning can be divided into two major categories: static and dynamic models. In static models, investment decisions are implemented in the first year of the planning horizon [5], [9], [16]. In this category, the models are single or multi-objectives. The single-objective models are either originally single-objective [5], or multi-objective which are converted into a single-objective (using a benefit to cost ratio index or an additive utility function [9]); multi-objective models of this category are solved using Pareto optimality concept [17]. The dynamic models of the literature are those which determine the optimal investment decisions to be made at each year (or time segment) during the planning horizon [18]. These models consider the time value of money and are more efficient than static models. The planing models are summarized in Table I.

C. Contributions

In this paper, a dynamic multi-objective DG-planning problem is formulated and a two-stage algorithm is proposed to solve the problem. In the first stage, the set of Pareto optimal solutions is found using a novel binary PSO method, and in the second stage, the best solution is chosen using fuzzy satisfying techniques. The model aims to cover all three aspects of DG planning problem, i.e., siting, sizing and timing of investment simultaneously, in order to increase the technical, economical and environmental benefits accrued by a DNO. The contributions of this paper are three fold:

1) Multi-objective dynamic modeling of DG planning problem for simultaneous determination of timing, sizing and sitting, when costs, emissions and technical attributes of the proposed plans, and a variety of DG technologies are considered.

2) The uncertainties of input variables are taken into account.

3) Proposing a BPSO method for solving the aforementioned problem.

D. Paper organization

This paper is set out as follows: section II presents problem formulation, section III sets out the principles of multi-objective optimization and the proposed solution algorithm for solving

the problem. Simulation results are presented in section IV and finally, section V summarizes the findings of this work.

II. PROBLEM FORMULATION

The multi-objective DG planning problem is formulated in this section. The decision variable is the number of DG units from a specific technology, to be installed in bus i in year t, i.e., $\xi_{i,t}^{dg}$; Investment in feeders, i.e. γ_t^{ℓ}, or transformer, i.e. ψ_t^{tr}.

A. Assumptions

The following assumptions are employed in problem formulation:

- Connection of a DG unit to a bus is modeled as a negative PQ load with a fix power factor.
- DNO is authorized to invest in DG units.

B. Uncertainty modeling

The electricity price and electric load are both uncertain in deregulated environment but these parameters are specifically tied together. An increase/decrease in electric load will tend to increase/decrease in electricity price. Without loss of generality, the correlation between wind speed and load-price pattern are assumed to be independent [19]. If any correlation exists between load-price and wind pattern this can be easily considered in the proposed algorithm. The price and load duration curves are divided into N_{dl} levels in each year as shown in Fig. 1. The vertical axis in Fig. 1, shows the demand/price level factors (the ratio of load/price to the peak value of load/price in this level). The duration of each level is described by τ_{dl}. It is assumed that the demand/price level factors (λ_{dl}, D_{dl}) are normally distributed around their specified expected values as shown in Fig. 1. Each normal distribution is divided into 7 states and the probability of each state is specified in Fig. 1. Although the expected price and demand values are dependent but ,in each demand level, the variation of price and electric load around its expected value can be assumed to be independent. The electricity price, electric load and wind generation are modeled as follows:

1) Electricity price: The price of energy purchased from the grid is competitively determined in a liberalized market environment. Assuming a peak electricity price of ρ, the electricity price in demand level dl, and state s can be calculated as:

$$EP_{dl,s}^{\lambda} = \rho \, . \, \lambda_{dl,s} \tag{1}$$

2) Electric load: Assuming a peak load of $S_{i,peak}^{D}$ and a demand growth rate of α, the demand in bus i, in year t, demand level dl and state s can be calculated as:

$$S_{i,t,dl,s}^{D} = S_{i,peak}^{D} \, . \, D_{dl,s} \, . \, (1+\alpha)^{t} \tag{2}$$

3) Wind speed and wind turbine power generation: The generation schedule of a wind turbine highly depends on the wind speed in the site. There are various methods to model wind behavior. In this paper, the variation of wind speed, i.e. v, is modeled using a Rayleigh PDF and its characteristic function which relates the wind speed and the output of a wind turbine.

$$PDF(v) = (\frac{2v}{c^2}) \; exp[-(\frac{v}{c})^2] \tag{3}$$

where c is the scale factor of the Rayleigh PDF of wind speed in the zone under study.

The generated power of the wind turbine is determined using its characteristics as follows:

$$P_{i,t}^{w}(v) = \sum_{t=1}^{t} \xi_{i,t}^{dg} . \begin{cases} 0 & \text{if } v \leq v_{in}^{c} \text{ or } v \geq v_{out}^{c} \\ \frac{v - v_{in}^{c}}{v_{rated} - v_{in}^{c}} P_{i,r}^{w} & \text{if } v_{in}^{c} \leq v \leq v_{rated} \\ P_{i,r}^{w} & \text{else} \end{cases} \tag{4}$$

Where, $P_{i,r}^{w}$ is the rated power of wind turbine installed in bus i, P_{i}^{w} is the generated power of wind turbine in bus i, v_{out}^{c} is the cut out speed, v_{in}^{c} is the cut in speed and v_{rated} is the rated speed of the wind turbine. The speed-power curve of a typical wind turbine is depicted in Fig. 2. Using the technique described in [19], the PDF of wind speed is divided into several states. In each state, the probability of falling into this state is calculated as follows:

$$prob_{s}^{w} = \int_{V_{1,s}}^{V_{2,s}} (\frac{2v}{c^2}) \; exp[-(\frac{v}{c})^2] dv \tag{5}$$

$$v_{s} = \frac{V_{2,s} + V_{1,s}}{2}$$

The generated power of wind turbine is calculated using the v_{s}, as obtained in (5), and (4).

4) Combined states model: As it is already mentioned, the states of each demand level are independent (the correlation between load and price is already considered in their mean value of D_{dl} and λ_{dl}. In each demand level, the states are combined to construct the whole set of states as follows:

$$C(s) = load(s) \cdot price(s) \cdot wind(s) \tag{6}$$

$$Prob_s^c = prob_s^l \cdot prob_s^\lambda \cdot prob_s^w \tag{7}$$

where $Prob_s^c$ is the probability of each combined state.

C. Constraints

1) Power flow constraints: The flow equations that shall be satisfied for each configuration and states are:

$$P_{i,t,dl,s}^{net} = -P_{i,t,dl,s}^D + P_{i,t,dl,s}^{dg} \tag{8}$$

$$Q_{i,t,dl,s}^{net} = -Q_{i,t,dl,s}^D + Q_{i,t,dl,s}^{dg}$$

$$P_{i,t,dl,s}^{net} = V_{i,t,dl,s} \sum Y_{ij}^t V_{j,t,dl,s} cos(\delta_{i,t,dl,s} - \delta_{j,t,dl,s} - \theta_{ij})$$

$$Q_{i,t,dl,s}^{net} = V_{i,t,dl,s} \sum Y_{ij}^t V_{j,t,dl,s} sin(\delta_{i,t,dl,s} - \delta_{j,t,dl,s} - \theta_{ij})$$

2) Operating limits of DG units: Each DG should be operated considering its capacity limits, i.e.:

$$S_{i,t,dl,s}^{dg} \leq \sum_{\acute{t}=1}^{t} \xi_{i,t}^{dg} \cdot \overline{S}_{lim}^{dg} \tag{9}$$

3) Fuzzy technical satisfaction: The satisfaction of soft constraints can be modeled by fuzzy sets. The idea of fuzzifying the technical constraints was used by [20]. In the present work, this idea is extended to model the problem with different states for a dynamic planning problem. Fuzzy modeling is used to quantify the value of satisfaction of technical constraints of voltages and thermal limits of feeders and substation, as follow:

a) Voltage profile: The voltage magnitude of each bus should be kept between the safe operation limits. However, the DNO may ignore violation of these limits to some degree, in hope of achieving a better solution regarding other necessities [20]. The membership function of the voltage constraint satisfaction is represented by a trapezoidal fuzzy number [17].

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Observe that a voltage magnitude between the up and low safe operation limits, i.e., $V_{safe}^{min}, V_{safe}^{max}$ has a satisfactory value of 1. As the voltage exceeds these limits, the value of satisfaction decreases until it becomes zero after the critical voltage values, i.e., $V_{crit}^{min}, V_{crit}^{max}$. This function can be mathematically represented as:

$$\mu_{i,t,dl,s}^{V} = \begin{cases} \frac{V_{i,t,dl,s}-V_{crit}^{min}}{V_{safe}^{min}-V_{crit}^{min}} & V_{crit}^{min} \leq V_{i,t,dl,s} \leq V_{safe}^{min} \\ 1 & V_{safe}^{min} \leq V_{i,t,dl,s} \leq V_{safe}^{max} \\ \frac{V_{i,t,dl,s}-V_{crit}^{max}}{V_{safe}^{max}-V_{crit}^{max}} & V_{safe}^{max} \leq V_{i,t,dl,s} \leq V_{crit}^{max} \\ 0 & else \end{cases} \tag{10}$$

The values obtained from (10) show the condition of voltage constraint satisfaction for bus i in state s in year t. Since there are more than one state in a real system, the planner will have different satisfaction levels of voltage constraint for a given bus. To obtain an index which shows the condition of a given bus i in year t, it is proposed in this work to calculate the weighted average of satisfaction of voltage over the states, as follows:

$$\mu_{i,t}^{V} = \frac{1}{8760} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_s} prob_s^c \cdot \tau_{dl} \cdot \mu_{i,t,dl,s}^{V} \tag{11}$$

In (11), if the voltage of bus i does not fully satisfy the constraints in state s but the probability of this dissatisfaction is short, the satisfaction of this bus is not very degraded in the whole year t. The average value of $\mu_{i,t}^{V}$ over all buses of the network, can provide information about the overall voltage condition in year t as follows:

$$\mu_t^V = \frac{\sum_{i=1}^{N_b} \mu_{i,t}^V}{N_b} \tag{12}$$

b) Thermal limit of feeders and Substation: To maintain the security of the feeders and the substation, the flow of current/energy passing throw them should be kept below the feeders/substation capacity limit. This is incorporated here, in the form of a fuzzy membership function [17]. A strictly monotonically decreasing and continuous function is considered for this

limit, as follows:

$$\mu_{\ell,t,dl,s}^{I} = \tag{13}$$

$$\begin{cases} 1 & I_{\ell,t,dl,s} \leq \overline{I}_{\ell}^{safe,t} \\ \dfrac{I_{\ell,t,dl,s} - \overline{I}_{\ell}^{crit,t}}{\overline{I}_{\ell}^{safe,t} - \overline{I}_{\ell}^{crit,t}} & \overline{I}_{\ell}^{safe,t} \leq I_{\ell,t,s} \leq \overline{I}_{\ell}^{crit,t} \\ 0 & I_{\ell,t,s} \geq \overline{I}_{\ell}^{crit,t} \end{cases}$$

$$\overline{I}_{\ell}^{safe,t} = \overline{I}_{\ell} + Cap_{\ell} \cdot \sum_{t=1}^{t} \gamma_{t}^{\ell}$$

Similar to the voltage limit, an overall satisfaction value is considered for each feeder, as follows:

$$\mu_{\ell,t}^{I} = \frac{1}{8760} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_s} prob_s^c \cdot \tau_{dl} \cdot \mu_{\ell,t,dl,s}^{I} \tag{14}$$

An index is needed to provide information about the overall performance of the system regarding the thermal limits. The average value of $\mu_{\ell,t}^{I}$ over all feeders of the network can provide such information as follows:

$$\mu_t^{I} = \frac{\sum_{\ell=1}^{N_{\ell}} \mu_{\ell,t}^{I}}{N_{\ell}} \tag{15}$$

For substation capacity constraint, also, the same philosophy holds, as follows:

$$\mu_{t,dl,s}^{S^{grid}} = \tag{16}$$

$$\begin{cases} 1 & S_{t,dl,s}^{grid} \leq \overline{S}_{safe,t}^{tr} \\ \dfrac{S_{t,dl,s}^{grid} - \overline{S}_{crit,t}^{tr}}{\overline{S}_{safe,t}^{tr} - \overline{S}_{crit,t}^{tr}} & \overline{S}_{safe,t}^{tr} \leq S_{t,dl,s}^{grid} \leq \overline{S}_{crit,t}^{tr} \\ 0 & S_{t,dl,s}^{grid} \geq \overline{S}_{crit,t}^{tr} \end{cases}$$

$$\mu_t^{S^{grid}} = \frac{1}{8760} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_s} prob_s^c \cdot \tau_{dl} \cdot \mu_{t,dl,s}^{S^{grid}}$$

D. Objective functions

The proposed model minimizes three objective functions, namely, technical dissatisfaction, total costs and total emissions of the planning problem (see Appendix-I), as follows:

$$min \{OF_1, OF_2, OF_3\}$$

$$subject\ to:$$

$$(2) \rightarrow (16)$$

The objective functions are formulated next.

1) Technical dissatisfaction: The first objective function to be minimized is dissatisfaction of technical constraints. The technical dissatisfaction, denoted by TD_t, is defined as the maximum dissatisfaction of all technical constraints as follows:

$$TD_t = 1 - min\left\{\mu_t^V, \mu_t^I, \mu_t^{S^{grid}}\right\} \tag{17}$$

The objective function to be minimized is proposed here as the multiplication of maximum and average value of yearly technical dissatisfaction over planning horizon as:

$$OF_1 = \qquad w_{avg} \cdot \sum_{t=1}^{T} \frac{TD_t}{T} + \tag{18}$$

$$w_{sev} \cdot (1 - \min_{t,dl,s,\ell}[\mu_{t,dl,s}^{S^{grid}}, \mu_{i,t,dl,s}^{V}, \mu_{\ell,t,dl,s}^{I}])$$

By minimizing the OF_1, the algorithm tries to simultaneously improve the overall satisfaction of the network, represented by $\sum_{t=1}^{T} \frac{TD_t}{T}$, and the severity of technical dissatisfaction over the planning horizon, represented by the second term. In (18) the values of w_{sev} and w_{avg} are the weighting factor representing the importance of severity of technical dissatisfaction and the average dissatisfaction of technical constraints. If w_{sev} is chosen much more bigger than w_{avg}, then the algorithm tries to find solutions which fully satisfy the technical constraints. On the other hand, if w_{avg} is bigger than w_{sev}, then the technical satisfactions of the solutions are more relaxed.

2) Total costs: The second objective function, i.e., OF_2, to be minimized is the total costs which includes the cost of electricity purchased from the grid, the investment/operating costs of the DG units. The cost of energy procurement from the grid is calculated as:

$$TGC = \sum_{t=1}^{T}\sum_{dl=1}^{N_{dl}}\sum_{s=1}^{N_s} prob_s^c \cdot EP_{t,s}^{\lambda} \cdot P_{t,s}^{grid} \cdot \tau_{dl} \cdot \frac{1}{(1+d)^t} \tag{19}$$

Investment costs of the DG units can be calculated as:

$$DGIC = \sum_{t=1}^{T}\sum_{i=1}^{N_b}\sum_{dg} \xi_{i,t}^{dg} \cdot IC_{dg} \cdot \frac{1}{(1+d)^t} \tag{20}$$

The operating costs of the DG units can be calculated as:

$$DGOC = \sum_{t=1}^{T}\sum_{i=1}^{N_b}\sum_{dl=1}^{N_{dl}}\sum_{dg}\sum_{s=1}^{N_s} prob_s^c \cdot \tau_{dl} \cdot OC_{dg} \tag{21}$$

$$\cdot P_{i,t,s}^{dg} \cdot \frac{1}{(1+d)^t}$$

The reinforcement cost of the distribution network is the sum of all costs paid for investment and operation of new feeders and transformers. The total feeder reinforcement cost, i.e. FC, and substation reinforcement cost, i.e. SC, are calculated as follows:

$$FC = \sum_{\acute{t}=1}^{T} \sum_{\ell=1}^{N_\ell} C_\ell.d_\ell \, . \, \gamma_{\acute{t}}^{\ell} . \frac{1}{(1+d)^t} \tag{22}$$

$$SC = \sum_{\acute{t}=1}^{T} C_{tr} \, . \, \psi_{\acute{t}}^{tr} \, . \, \frac{1}{(1+d)^t}$$

Where, FC and SC are the total feeder and substation reinforcement cost, respectively. C_ℓ, C_{tr} are the cost of each feeder and transformer, respectively.

Thus, OF_2 is defined as:

$$OF_2 = DGIC + DGOC + TGC + FC + SC \tag{23}$$

3) Total emission: The third objective function, i.e., OF_3, is the total CO_2 produced by the DG units and the main grid. OF_3 can be formulated as:

$$OF_3 = \sum_{t=1}^{T} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_s} prob_s^c \, . \quad \tau_{dl}.[E_{grid} \, . \, P_{t,s}^{grid} + \tag{24}$$

$$\sum_{i=1}^{N_b} \sum_{dg} E_{dg} \, . \, P_{i,t,s}^{dg}]$$

III. PROPOSED SOLUTION ALGORITHM

To solve the dynamic multi-objective DG planning problem formulated in section II, a two-stage algorithm is proposed in this section.

A. Proposed Binary PSO

The PSO algorithm is a population-based search technique proposed first by Kennedy and Eberhart in 1995 [21]. The basic idea of PSO is that each particle uses the swarm's best experience as well as its own best experience in finding food. The PSO algorithm starts with a population of particles with random positions in the search space. Each particle is a solution of the problem and has a fitness value. The fitness is evaluated and is to be optimized. A velocity is defined which directs each particle's position and gets updated in each iteration. Particles gradually move toward the optima due to their best position they have ever experienced and the best solution which group has experienced. The velocity of a particle is updated due to three

factors: the past velocity of the particle, the best position particle has experienced so far and the best position the entire swarm has experienced so far. In each iteration, every particle modifies its direction by its updated velocity affected by the three factors mentioned above. Mathematically the modification process may be expressed as follows:

$$X_p^{iter+1} = X_p^{iter} + V_p^{iter} \tag{25}$$

$$V_p^{iter+1} = V_p^{iter} + Rand_1 . (X_{p,Best}^{iter} - X_p^{iter})$$

$$+Rand_2 . (X_{g,Best}^{iter} - X_p^{iter})$$

$$p = 1, 2, \cdots, N_p$$

X_p^{iter} is a particle which represents a potential solution to the optimization problem. In multi-dimensional problems like DG planning problem, each particle, X_p^{iter} is a vector containing the decision variables and V_p^{iter} is the velocity of particle p, respectively. $X_{p,Best}^{iter}$ is the best personal position of particle p has had up to now. Similarly $X_{g,Best}^{iter}$ is the best global position which the entire particles have had up to now. The concept of direction modification in PSO implies that the direction of particle p is influenced by its present velocity, its best position up to now $X_{p,Best}^{iter}$, and the best position of the whole particles up to now $X_{g,Best}^{iter}$. In the context of multi-objective optimization (see Appendix-I for more details), it is needed that the population be directed toward the Pareto optimal front considering two important aspects: getting closer to Pareto optimal front and maintaining the diversity among the solutions [22]. To do so, a pseudo fitness value is assigned to each solution, referred to as affinity factor SF_n, as follows:

$$SF_n = w_1 . FN_n^{-1} + w_2 . GD_n \tag{26}$$

The first term in (26) guides the population toward the Pareto optimal front since the solutions which belong to lower fronts get higher affinity (fitness). The second term insures the diversity among the solutions. In order to calculate the global diversity of the nth solution, i.e. GD_n, a local diversity factor, i.e. LD_n^k, is defined for each objective function [22]. For every objective function k, the solutions are sorted and the difference between the maximum and minimum values is calculated as:

$$MD_k = \max_n(f_k(X_n)) - \min_n(f_k(X_n)) \tag{27}$$

$$n = 1, \cdots, N_P$$

Since the solutions are sorted, the first and the last ones are the maximum and minimum, respectively. The local diversity of each of the other solutions is its average distance to its neighbors, as follows:

$$LD_n^k = \tag{28}$$
$$\frac{|f_k(X_n) - f_k(X_{n+1})| + |f_k(X_n) - f_k(X_{n-1})|}{2MD_k}$$

For the first and the last solutions, local diversity can be calculated as:

$$LD_{N_p}^k = LD_1^k = max(LD_n^k) \tag{29}$$

The global diversity factor for each solution is then calculated as the average of its local diversities as follows:

$$GD_n = \sum_{k=1}^{N_o} \frac{LD_n^k}{N_o} \tag{30}$$

In initial iterations, a small number of solutions belong to the first Pareto front, so getting closer to Pareto optimal front is more important than maintaining the diversity among them. It is necessary to enable the algorithm in distinguishing between the solutions in different Pareto fronts, w_1 and w_2 in (26) are adaptively selected which guarantees that the solution belonging to a lower Pareto front has a bigger affinity factor than a solution belonging to an upper front level (w_1 is bigger than w_2 in the initial iterations) and when most of the solutions are in the pareo optimal front, w_2 is chosen bigger than w_2 to maintain the diversify among the solutions.

B. The Proposed two-stage solution algorithm

The solution algorithm proposed here consists of two stages. In the first stage, the solutions which form the Pareto optimal front are found and in the second stage, the best solution is selected considering the planner's preferences. Both stages are described as follows:

1) Stage I (finding the Pareto optimal front): The PSO algorithm proposed in section III-A is used to find the Pareto optimal front. To do so, each particle is a vector containing the decision variables coded in binary format (to show the investment decisions) investment decision of DG units, the bus on which a DG units is to be installed, the year of investment and their generated power and for all available DG technologies. Steps of the first stage of the solution algorithm are as follows:

step. 1. Generate an initial random particles.

step. 2. If the stopping criterion is met, go to step (k), else, continue.

step. 3. Calculate OF_1, OF_2, OF_3 for each particle.

step. 4. Calculate the fitness using (26) for each particle.

step. 5. Calculate the V_p^{iter}

step. 6. Calculate the $X_{g,Best}^{iter}, X_{p,Best}^{iter}$

step. 7. Construct a new population by moving the particles.

step. 8. Return to step (2).

step. 9. End.

The flowchart of the first stage of the proposed method is depicted in Fig.3.

2) Stage II (Selecting 'the best' solution): The decision maker (planner) needs a tool to select the final solution among the existing solutions of Pareto front. In this paper a fuzzy satisfying method is used for this purpose. The concept of this method is as follows: for each solution in the Pareto optimal front, X_n, a membership function is defined as $\mu^{f_k(X_n)}$. This value, which varies between 0 to 1, shows the level of which X_n belongs to the set that minimizes the objective function f_k. A linear membership function is used here for all objective functions as follows:

$$\mu^{f_k(X)} = \begin{cases} 0 & f_k(X) > f_k^{max} \\ \frac{f_k^{max} - f_k(X)}{f_k^{max} - f_k^{min}} & f_k^{min} \leq f_k(X) \leq f_k^{max} \\ 1 & f_k(X) < f_k^{min} \end{cases} \tag{31}$$

A conservative decision maker tries to maximize minimum satisfaction among all objectives or minimize the maximum dissatisfaction [23]. The final solution can then be found as:

$$\max_{n=1}^{N_p}(\min_{k=1}^{N_o}(\mu^{f_k(X_n)})) \tag{32}$$

IV. SIMULATION RESULTS

The proposed methodology is applied to a realistic 201-node 10 kV distribution system which is shown in Fig.4. The technical data of this network can be found in [24]. Three DG technology options, namely, Micro Turbine (MT), Wind Turbine (WT), Gas Turbine (GT) are considered here. It is also assumed that all buses are candidate for DG investment and more than one DG can be installed in a specific bus. The stopping criterion is reaching to a predefined maximum number of iterations. The Rayleigh parameter of the wind speed in each wind farm has been

assumed to be $c = 8.78$ and the other characteristics of wind turbine are given in Table II. Using the technique described in [19], the PDF of wind speed is divided into twelve states as given in Table III. The forecasted values of demand and price level factors are given in Table IV [25]. The σ value of each demand level is assumed to be 1% of its forecasted value. Other simulation assumptions and characteristics of the DG units [26], [27] are presented in Table V and Table VI respectively. The presented solution algorithm was implemented in MATLAB. The number of demand levels, i.e. N_{dl} is assumed to be 24 and the duration of each demand level is 365hr. In (18) the values of w_{avg} and w_{sev} are assumed to be 0.8 and 0.2, respectivley. Solving the (6) gives $7 \times 7 \times 12 = 588$ states for each demand level. It is clear that solving the evaluation process for all of these states (for all demand levels) imposes a heavy computational burden. In order to resolve this problem, a scenario reduction technique proposed in [28] has been used to reduce the number of states (see Appendix-II for more details). The scenario set is reduced into 110 states (this is chosen based on trial and error) using the described technique. The formulated problem is solved using the proposed two-stage algorithm and 80 non-inferior solutions are found. The planner can choose the best solution based on the planning criteria, as further discussed next. The Pareto optimal front of the search space, found in the first stage, is depicted in Fig.5. The variation ranges of all objective functions are given in Table VII. In the second stage, the planner can choose the most preferred solution using the fuzzy satisfaction method introduced in section III-B. The final solution is solution #68. The various costs related to the selected solution are given in Table VIII. The investment plan of this solution is described in Table IX. It would be interesting to know how much accuracy is lost if the scenarios are reduced. The final solution (which was already found using the $N_s = 110$) is reevaluated using various values of N_s. The exact values of this solution is obtained if all scenarios are considered. This value is taken as a reference for comparing the results obtained by various number of reduced scenarios. The computation error due to scenario reduction is depicted versus the number of reduced scenarios, i.e N_s, Fig.6. This figure shows that if the number of scenarios is chosen to be greater than 93, then the accuracy is acceptable and the fluctuation is highly reduced and the error will be less than 0.005%.

In order to analyze the performance of the proposed binary PSO methodology, it is compared with an standard real coded PSO as described in [29]. For this purpose, the problem is solved just for two objectives namely, technical satisfaction and cost (this is done just because the

comparison is much more easier). The initial solution (random solution) and the Pareto optimal fronts found by each PSO method are depicted in Fig. 7. Both algorithms are run for 100 iterations. It is obvious that the Pareto front obtained by binary PSO dominates the solutions of the read coded algorithm. This means for each solution in Pareto front of real coded PSO there exists at least one solution in the BPSO front that gives lower cost and technical dissatisfaction simultaneously.

The model can be directly used in power market model in which the DNO is authorized for DG investment. However, in power market models where the DG investment is done by independent investors instead of DNO, It can be easily modified to be used in such regulatory frameworks. The decisions related to investment and operating of DG units are made by private entities. In this case, the values of $\xi_{i,t}^{dg}$ are determined by DG owners. The decision variables of DNO are γ_t^ℓ and ψ_t^{tr} (network investment options). The provided information would also be useful as a technical, economical and environmental signal for regulators. It can be used for regulating the incentives to encourage the private section to invest in what DG technology and where to be more beneficial.

V. CONCLUSION

This paper presents a dynamic multi-objective model for DG planning problem and a binary PSO based method to solve the formulated problem. The proposed two-step algorithm finds the non-dominated solutions by simultaneous minimization of technical dissatisfaction, costs and emissions in the first stage and uses a fuzzy satisfying method to select the best solution from the candidate set in the second stage. The new planning model is applied to a realistic distribution network and its flexibility is demonstrated through different case studies. The solution set provides the planner with an insight into the problem and enables him to choose the best solution according to planning preferences.

APPENDIX-I: PARETO OPTIMALITY CONCEPT

In general, in a multi-objective optimization problem, more than one objective function needs to be simultaneously optimized as follows:

$$min \quad F(X) = [f_1(X), ..., f_{N_o}(X)] \tag{33}$$

Subject to:

$$\{G(X) = \bar{0}, H(X) \leq \bar{0}\}$$

$$X = [x_1, \cdots, x_m]$$

Suppose X_1 and X_2 belong to the solution space. X_1 dominates X_2 if:

$$\forall k \in \{1...N_O\} \, f_k(X_1) \leq f_k(X_2) \tag{34}$$

$$\exists k' \in \{1...N_O\} \, f_{k'}(X_1) < f_{k'}(X_2)$$

Any solution which is not dominated by any other is called to belong to a Pareto front which is referred to as the first Pareto front or optimal front or non-dominated front.

APPENDIX-II: SCENARIO REDUCTION TECHNIQUE

The purpose of scenario reduction is selection of a set, i.e. Ω_S, with the cardinality of N_{Ω_S}, from the original set, i.e. Ω_J [30]. This procedure should be done in a way that makes a trade off between the loss of the information and decreasing the computational burden [31]. The scenario reduction technique used in this paper is described as the following steps [28]:

step. 1 Construct the matrix containing the distance between each pair of scenarios $c(w, \acute{w})$

step. 2 Select the fist scenario w_1 as follows:

$$w_1 = arg \left\{ \min_{w' \in \Omega_J} \sum_{w \in \Omega_J} \pi_w c(w, w') \right\} \tag{35}$$

$$\Omega_S = \{w_1\}, \Omega_J = \Omega_J - \Omega_S$$

step. 3 Select the next scenario to be added to Ω_S as follows:

$$w_n = \tag{36}$$

$$arg \left\{ \min_{w' \in \Omega_J} \sum_{w \in \Omega_J - \{w'\}} \pi_w \min_{w'' \in \Omega_S \cup \{w\}} c(w, w'') \right\}$$

$$\Omega_S = \Omega_S \cup \{w_n\}, \Omega_J = \Omega_J - \Omega_S$$

step. 4 If the number of selected set is sufficient then end and go to step 2 ; else continue.

step. 5 The probabilities of each non-selected scenario will be added to its closest scenario in the selected set.

step. 6 End.

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List of figure captions:

TABLE I

DG PLANNING METHODS

Reference	Single/Multi objective	Static/ Dynamic	Uncertainty handling	Network reinforcement	Method
El-Khattam et al. [32]	S	S	N	Y	Classic MINLP
Jabr et al. [33]	S	S	N	Y(not exact)	Ordinal optimization
El-Khattam et al. [34]	S	S	N	N	Classic MINLP
Wang et al. [35]	S	D	N	Y	Greedy heuristic
Kumar et al. [36]	S	S	N	N	Classic MINLP
Wong et al. [37]	S	D	N	Y	Classic MINLP
Zangeneh et al. [38]	M	S	N	N	Normal boundary intersection
Haghifam et al. [17]	M	S	Y	N	Heuristic NSGA-II
Atwa et al. [19]	S	S	Y	N	Classic MINLP
Khalesi et al. [39]	S	S	N	N	Dynamic programming
Atwa et al. [40]	S	S	Y	N	Classic MINLP
Harrison et al. [41]	M	S	N	Y(not exact)	ϵ-constrained technique
Soroudi et al. [18]	M	D	N	Y(exact)	Immune algorithm
proposed	M	D	Y	Y(exact)	Heuristic BPSO

TABLE II

THE TECHNICAL CHARACTERISTICS OF WIND TURBINES

v_{in}^{cut} (m/s)	v_{rated} (m/s)	v_{out}^{cut} (m/s)	$P_{i,r}^{w}$ (MW)
3	13	25	0.5

TABLE III

WIND TURBINE STATES

State	wp_s (%)	$Prob_s^w$
1	0	0.1105
2	5	0.0772
3	15	0.0895
4	25	0.0961
5	35	0.0973
6	45	0.0936
7	55	0.0863
8	65	0.0764
9	75	0.0652
10	85	0.0537
11	95	0.0428
12	100	0.1115

TABLE IV

THE FORECASTED VALUES OF DEMAND AND PRICE LEVEL FACTORS IN EACH DEMAND LEVEL

dl	D_{dl}	λ_{dl}
1	0.8363	0.9128
2	0.7883	0.6372
3	0.7522	0.4841
4	0.7352	0.4849
5	0.7278	0.4808
6	0.7324	0.4849
7	0.7899	0.6449
8	0.8741	0.9655
9	0.8804	0.9391
10	0.9184	0.9662
11	0.9586	0.9690
12	1.0000	0.9798
13	0.9972	0.9742
14	0.9880	0.9683
15	0.9464	0.9582
16	0.9496	0.9582
17	0.9687	0.9798
18	0.9807	0.9856
19	0.9676	0.9798
20	0.9367	0.9587
21	0.9587	0.9813
22	0.9803	1.0000
23	0.9045	0.9511
24	0.8364	0.9152

TABLE V

DATA USED IN THE STUDY

Parameter	Unit	Value
T	year	8
$\overline{S}_{safe,t=0}^{tr}$	MVA	32
$\overline{S}_{crit,t=0}^{tr}$	MVA	40
E_{grid}	$kgCO_2/MWh$	632
ρ	\$/MWh.	60
α	%	1
d	%	12
V_{safe}^{max}	Pu	1.05
V_{crit}^{max}	Pu	$(1+5\%) \cdot V_{safe}^{max}$
V_{safe}^{min}	Pu	0.95
V_{crit}^{min}	Pu	$(1-5\%) \cdot V_{safe}^{min}$
$\overline{I}_{s,t}^{\ell}$	A	$0.9 \times \overline{I}_{crit,t}^{\ell}$
N_p		80
Maximum iteration		1000

TABLE VI

CHARACTERISTICS OF THE DG UNITS

DG Technology	Size MVA	Emission $\frac{kgCO_2}{MWh}$	IC $\frac{k\$}{MVA}$	OC $\frac{\$}{MWh}$	$cos\varphi$
Micro Turbine	0.5	502	1485	75	0.9
Gas Turbine	1	365	1030	67	0.9
Wind turbine	1	0	1225	45	1

TABLE VII

VARIATION RANGE OF OBJECTIVE FUNCTION FOR ALL SOLUTION IN PARETO OPTIMAL FRONT

	OF_1	OF_2 (\$)	OF_3 (Ton CO_2)
f_k^{min}	0.0026	9.7208×10^7	4.323×10^5
f_k^{max}	0.0689	2.4936×10^9	6.6973×10^5

TABLE VIII

THE INVESTMENT PLAN OBTAINED FOR THE FINAL SOLUTION

DGtech	$\xi_{i,t}^{dg}$	Year	Bus
Micro Turbine	4	1	201
	1	1	39,114,26
	1	2	164,201
	1	3	39
Gas Turbine	1	1	152,102
	1	2	14,177
	1	3	177,102
	1	4	76
	1	5	201
	1	7	14
	1	8	201
Wind Turbine	1	1	26,39,64,89,114,139
	3	1	201

TABLE IX

INVESTMENT/OPERATING COST IN FINAL SOLUTION (M$)

Year	GC	IC	sub	feeder
1	9.624959	18.4675	0	0
2	9.670223	2.485	0	34.5
3	9.662216	1.7425	0	89.25
4	9.680735	0.5	0	0
5	9.705028	0.5	0	73.5
6	9.813719	0	0	25.2
7	9.835628	0.5	0	90
8	9.861077	0.5	0.2	78
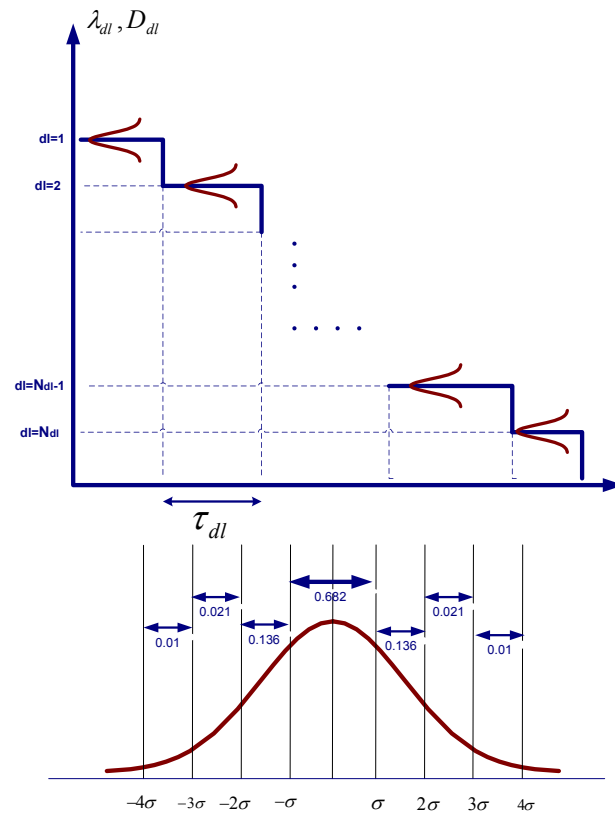
Fig. 1. Demand and price level factor uncertainty modeling
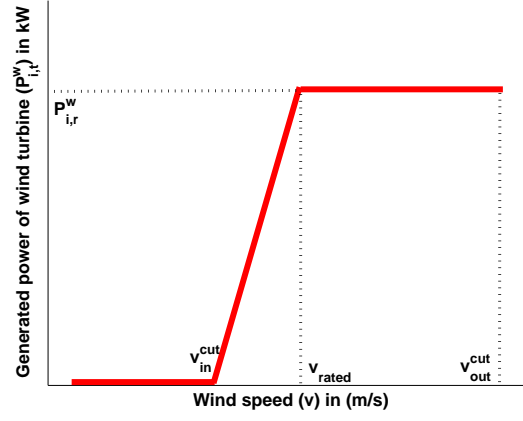
Fig. 2.   The idealized power curve of a wind turbine
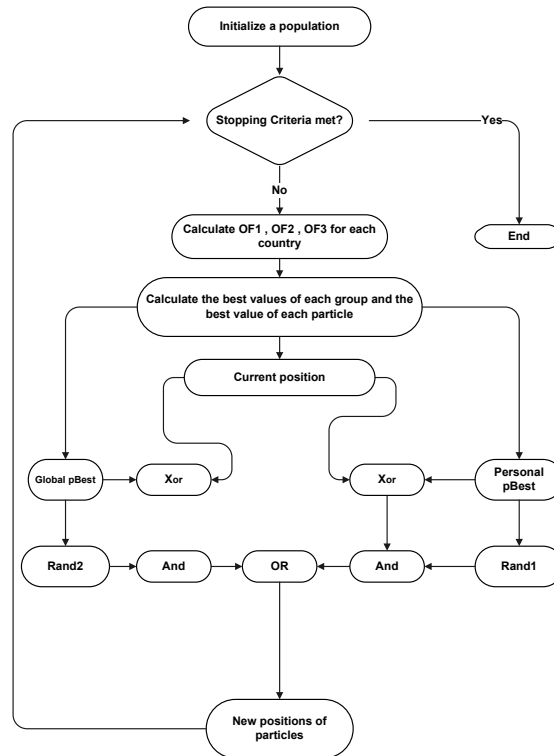

Fig. 3.   Flowchart of the first stage of the proposed method
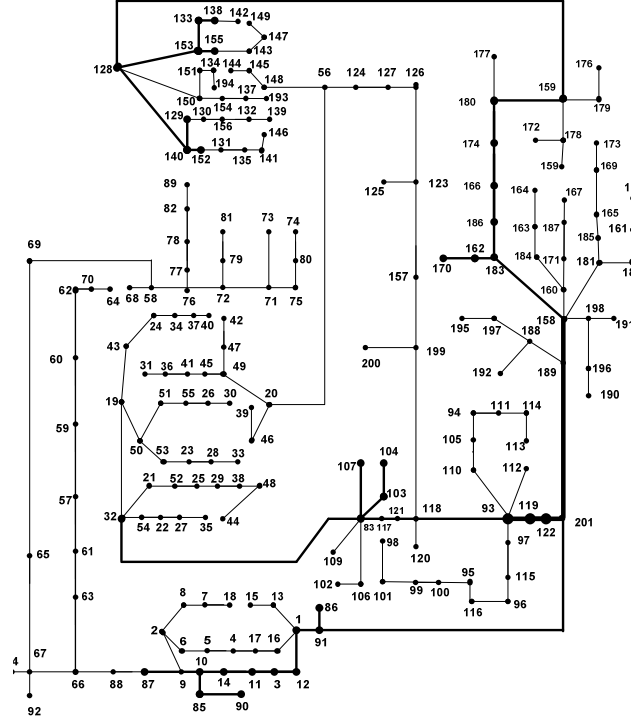
Fig. 4.   Single-Line Diagram of the real system under study
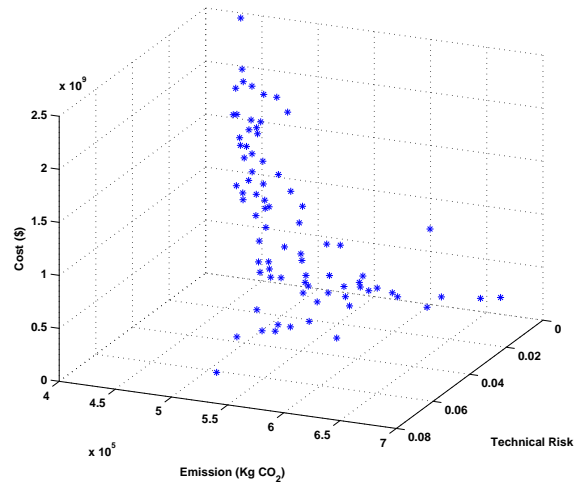

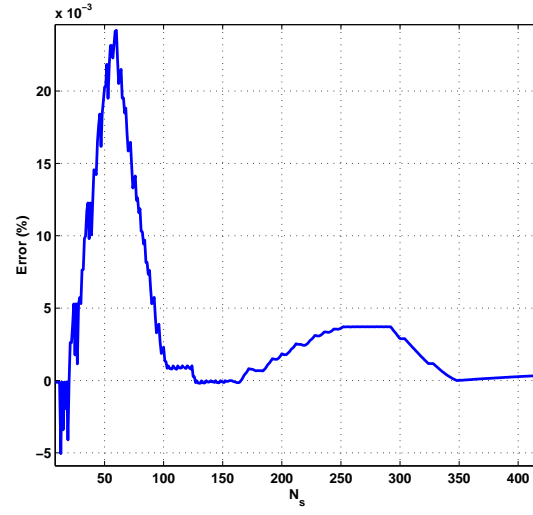Fig. 5.   Pareto optimal front found by the algorithm

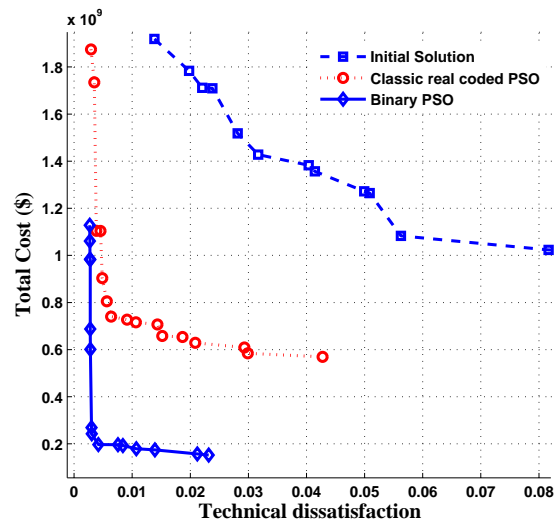Fig. 6. Sensitivity analysis of the computation accuracy versus the number of scenarios

Fig. 7. Comparison between Binary PSO and real coded classic PSO